C3 RATIONAL EXPRESSIONS *Worksheet B*

1 Find the quotient and remainder obtained in dividing

a $(3x^3 - 10x^2 - 9x + 15)$ by $(x - 4)$

b $(2x^3 - 11x^2 - x + 3)$ by $(2x - 1)$

c $(4x^3 + 8x^2 + 7x + 32)$ by $(2x + 5)$

d $(1 - 22x^2 - 6x^3)$ by $(3x + 2)$

2 **a** Show that $(x + 2)$ is a factor of $(x^3 + 4x^2 + x - 6)$.

b Fully factorise $x^3 + 4x^2 + x - 6$.

c Simplify $\dfrac{x^3 + 4x^2 + x - 6}{x^2 - 9}$.

3 **a** Show that $(2x - 3)$ is a factor of $(2x^3 - 5x^2 + 13x - 15)$.

b Simplify $\dfrac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6}$.

4 **a** State a linear factor of $x^3 - 1$.

b Simplify $\dfrac{x^3 - 1}{x^2 + x - 2}$.

5 Find the integers A and B such that

$$\frac{2x + 5}{x + 3} \equiv A + \frac{B}{x + 3}.$$

6 Express each of the following in the form $A + \dfrac{B}{\mathrm{f}(x)}$, where $\mathrm{f}(x)$ is linear.

a $\dfrac{x + 2}{x + 1}$ **b** $\dfrac{x + 3}{x - 2}$ **c** $\dfrac{x}{1 - x}$ **d** $\dfrac{2x + 1}{x + 2}$ **e** $\dfrac{x - 1}{2x - 1}$ **f** $\dfrac{1 - 4x}{3 + 2x}$

7 Find the quotient and remainder obtained in dividing

a $(x^2 + 3x + 5)$ by $(x^2 + x + 2)$

b $(2x^2 + 3x - 8)$ by $(x^2 - x - 4)$

c $(x^2 + 7)$ by $(x^2 + 3x - 1)$

d $(3x^2 - x - 4)$ by $(x^2 + 2)$

e $(x^3 - 2x^2 - 5x + 8)$ by $(x^2 + x - 2)$

f $(2x^3 - 7x^2 + 1)$ by $(x^2 - 5x + 1)$

g $(3x^3 + 6x^2 - 2x + 5)$ by $(3x^2 + 4)$

h $(6x^3 - x^2 - 44x - 6)$ by $(2x^2 - 5x - 2)$

8 **a** Divide $(x^3 + 5x^2 + 7x - 13)$ by $(x^2 + 3x - 4)$.

b Hence show that

$$\frac{x^3 + 5x^2 + 7x - 13}{x^2 + 3x - 4} \equiv x + 2 + \frac{5}{x + 4}.$$

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$$\mathrm{f}(x) = \frac{x^3 - 2x^2 - 21x + 70}{x^2 + 2x - 15}, \quad x \neq 3.$$

a Express $\mathrm{f}(x)$ in the form $Ax + B + \dfrac{C}{\mathrm{g}(x)}$, where $\mathrm{g}(x)$ is linear.

b Hence, or otherwise, solve the equation $\mathrm{f}(x) = \dfrac{3x - 7}{x - 3}$.