

$$\begin{aligned}
 1 \quad &= \frac{6}{(x+3)(x-3)} - \frac{7}{(2x+1)(x-3)} \\
 &= \frac{6(2x+1) - 7(x+3)}{(x+3)(x-3)(2x+1)} = \frac{5x-15}{(x+3)(x-3)(2x+1)} \\
 &= \frac{5(x-3)}{(x+3)(x-3)(2x+1)} = \frac{5}{(x+3)(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad f(x) &= \frac{3}{2x+3} - \frac{x+9}{(2x+3)(x+4)} \\
 &= \frac{3(x+4) - (x+9)}{(2x+3)(x+4)} \\
 &= \frac{2x+3}{(2x+3)(x+4)} = \frac{1}{x+4}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a \quad &= \frac{1}{x-6} - \frac{2}{(x+6)(x-6)} = \frac{(x+6)-2}{(x-6)(x+6)} \\
 &= \frac{x+4}{(x-6)(x+6)}
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{x+4}{(x-6)(x+6)} = \frac{1}{2} \\
 2(x+4) &= x^2 - 36 \\
 x^2 - 2x - 44 &= 0 \\
 x &= \frac{2 \pm \sqrt{4+176}}{2} = \frac{2 \pm 6\sqrt{5}}{2} = 1 \pm 3\sqrt{5}
 \end{aligned}$$

$$4 \quad a \quad f(5) = 250 - 125 - 115 - 10 = 0$$

$\therefore (x-5)$ is a factor of $f(x)$

$$\begin{array}{r}
 b \quad \begin{array}{r} 2x^2 + 5x + 2 \\ x-5 \overline{) 2x^3 - 5x^2 - 23x - 10} \\ \underline{2x^3 - 10x^2} \\ 5x^2 - 23x \\ \underline{5x^2 - 25x} \\ 2x - 10 \\ \underline{2x - 10} \end{array}
 \end{array}$$

$$\begin{aligned}
 \therefore f(x) &= (x-5)(2x^2 + 5x + 2) \\
 &= (x-5)(2x+1)(x+2) \\
 \therefore \frac{f(x)}{2x^2 - 9x - 5} &= \frac{(x-5)(2x+1)(x+2)}{(2x+1)(x-5)} \\
 &= x+2
 \end{aligned}$$

$$5 \quad \frac{x+6}{(x+3)(x+6)} + \frac{x-p}{x+7} = 0$$

$$(x+7) + (x-p)(x+3) = 0$$

$$x^2 + (4-p)x + 7 - 3p = 0$$

real, equal roots $\therefore b^2 - 4ac = 0$

$$(4-p)^2 - 4(7-3p) = 0$$

$$p^2 + 4p - 12 = 0$$

$$(p+6)(p-2) = 0$$

$$p = -6, 2$$

$$\begin{aligned}
 6 \quad &= \frac{1}{3x-1} - \frac{3x}{(3x-1)^2} - \frac{1}{x(3x-1)} \\
 &= \frac{x(3x-1) - 3x^2 - (3x-1)}{x(3x-1)^2} \\
 &= \frac{1-4x}{x(3x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad a \quad i \quad &= \frac{7(x+2)}{(2+x)(2-x)} = \frac{7}{2-x} \\
 ii \quad &= \frac{(2x-7)(x+4)}{3x(x+4)} = \frac{2x-7}{3x}
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{7}{2-x} = \frac{2x-7}{3x} \\
 21x &= (2-x)(2x-7) \\
 x^2 + 5x + 7 &= 0 \\
 b^2 - 4ac &= 25 - 28 = -3 \\
 b^2 - 4ac < 0 \quad \therefore &\text{no real roots}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad a \quad &\frac{1}{2} - \frac{1}{t-2} = \frac{4}{t^2-2t} - \frac{1}{2} \\
 \therefore \frac{4}{t^2-2t} + \frac{1}{t-2} &= 1
 \end{aligned}$$

$$\begin{aligned}
 b \quad &\frac{4}{t(t-2)} + \frac{1}{t-2} = 1 \\
 4+t &= t^2 - 2t \\
 t^2 - 3t - 4 &= 0 \\
 (t+1)(t-4) &= 0 \\
 t &= -1, 4 \\
 \text{but if } t &= 4, \text{ common difference} = 0 \\
 \therefore t &= -1, \text{ first term} = -\frac{1}{3}
 \end{aligned}$$