C3 TRIGONOMETRY — Answers - Worksheet D

1 **a** $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$ (1)
$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$ (2)

b let $B = -B$ in (1) $\Rightarrow$ $\sin[A + (-B)] \equiv \sin A \cos(-B) + \cos A \sin(-B)$
$\sin(A - B) \equiv \sin A \cos B + \cos A(-\sin B)$
$\sin(A - B) \equiv \sin A \cos B - \cos A \sin B$

let $B = -B$ in (2) $\Rightarrow$ $\cos[A + (-B)] \equiv \cos A \cos(-B) - \sin A \sin(-B)$
$\cos(A - B) \equiv \cos A \cos B - \sin A(-\sin B)$
$\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$

c (1) ÷ (2) $\Rightarrow$ $\dfrac{\sin(A+B)}{\cos(A+B)} \equiv \dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$

$\tan(A + B) \equiv \dfrac{\dfrac{\sin A\cos B}{\cos A\cos B} + \dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B} - \dfrac{\sin A\sin B}{\cos A\cos B}}$

$\tan(A + B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

let $B = -B$ $\Rightarrow$ $\tan[A + (-B)] \equiv \dfrac{\tan A + \tan(-B)}{1 - \tan A\tan(-B)}$

$\tan(A - B) \equiv \dfrac{\tan A + (-\tan B)}{1 - \tan A(-\tan B)}$

$\tan(A - B) \equiv \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

2 **a** $= \sin(10 + 30)°$
$= \sin 40°$

b $= \sin(67 - 18)°$
$= \sin 49°$

c $= \sin(62 + 74)°$
$= \sin 136°$
$= \sin(180 - 136)°$
$= \sin 44°$

d $= \cos(14 + 39)°$
$= \cos 53°$
$= \sin(90 - 53)°$
$= \sin 37°$

3 **a** $= \cos(A + 2A)$
$= \cos 3A$

b $= \sin(4A - B)$

c $= \tan(2A + 5A)$
$= \tan 7A$

d $= \cos(A - 3A)$
$= \cos(-2A)$
$= \cos 2A$

4 **a** $= \sin(45 - 30)°$
$= \sin 45° \cos 30° - \cos 45° \sin 30°$
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

b $= \sin 15°$
$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

c $= \frac{1}{\sin 15°}$
$= \frac{2\sqrt{2}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
$= \frac{2\sqrt{2}(\sqrt{3}+1)}{3-1}$
$= \sqrt{6} + \sqrt{2}$

d $= \sin(90 - 75)°$
$= \sin 15°$
$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

e $= \cos(45 - 30)°$
$= \cos 45° \cos 30° + \sin 45° \sin 30°$
$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$
$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{1}{4}(\sqrt{6} + \sqrt{2})$

f $= \frac{1}{\cos 195°}$
$= \frac{1}{-\cos 15°}$
$= -\frac{2\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$
$= -\frac{2\sqrt{2}(\sqrt{3}-1)}{3-1}$
$= \sqrt{2} - \sqrt{6}$

g $= \tan(30 + 45)°$
$= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$
$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1}$
$= \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
$= \frac{1+2\sqrt{3}+3}{3-1} = 2 + \sqrt{3}$

h $= \frac{1}{\sin 105°}$
$= \frac{1}{\sin 75°}$
$= \frac{1}{\cos 15°}$
$= \sqrt{6} - \sqrt{2}$

5 **a** $= \cos(x - 30°)$
$\therefore$ max. = 1 when $x = 30°$

b $= 3\sin(x + 45°)$
$\therefore$ max. = 3 when $x = 45°$

c $= \sin(x - 67°)$
$\therefore$ max. = 1 when $x = 157°$

d $= -4(\cos x \cos 108° - \sin x \sin 108°)$
$= -4\cos(x + 108°)$
$\therefore$ max. = 4 when $x = 72°$

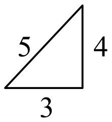
6 **a** $= \sin(x - \frac{\pi}{3})$
$\therefore$ min. = –1 when $x = \frac{11\pi}{6}$

b $= 2\cos(x + \frac{\pi}{6})$
$\therefore$ min. = –2 when $x = \frac{5\pi}{6}$

c $= \cos(4x - x)$
$= \cos 3x$
$\therefore$ min. = –1 when $x = \frac{\pi}{3}$

d $= 6\sin(2x - 3x)$
$= 6\sin(-x)$
$= -6\sin x$
$\therefore$ min. = –6 when $x = \frac{\pi}{2}$

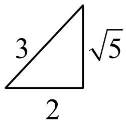
7 **a**



$\therefore\ \tan A = \pm\frac{4}{3}$

$0 < A < 90° \Rightarrow \tan A = \frac{4}{3}$

b



$\therefore\ \sin B = \pm\frac{\sqrt{5}}{3}$

$0 < B < 90° \Rightarrow \sin B = \frac{\sqrt{5}}{3}$

c $= \cos A \cos B - \sin A \sin B$

$= \frac{3}{5}\times\frac{2}{3} - \frac{4}{5}\times\frac{\sqrt{5}}{3}$

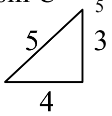
$= \frac{2}{15}(3 - 2\sqrt{5})$

d $= \sin A \cos B + \cos A \sin B$

$= \frac{4}{5}\times\frac{2}{3} + \frac{3}{5}\times\frac{\sqrt{5}}{3}$

$= \frac{1}{15}(8 + 3\sqrt{5})$

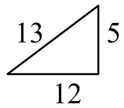
8 **a** $\sin C = \frac{3}{5}$



$\therefore\ \cos C = \pm\frac{4}{5}$

$0 < C < 90° \Rightarrow \cos C = \frac{4}{5}$

b



$\therefore\ \cos D = \pm\frac{12}{13}$

$90° < D < 180° \Rightarrow \cos D = -\frac{12}{13}$

c $= \sin C \cos D - \cos C \sin D$

$= \frac{3}{5}\times(-\frac{12}{13}) - \frac{4}{5}\times\frac{5}{13}$

$= -\frac{56}{65}$

d $\cos(C - D) = \cos C \cos D + \sin C \sin D$

$= \frac{4}{5}\times(-\frac{12}{13}) + \frac{3}{5}\times\frac{5}{13}$

$= -\frac{33}{65}$

$\therefore\ \sec(C - D) = -\frac{65}{33}$

9 **a** $\sin(\theta + 15) = 0.4$

$\theta + 15 = 23.6,\ 180 - 23.6$

$= 23.6,\ 156.4$

$\theta = 8.6,\ 141.4$

b $\tan(2\theta - 60) = 1$

$2\theta - 60 = 45,\ 180 + 45,\ 360 + 45,\ 540 + 45$

$= 45,\ 225,\ 405,\ 585$

$2\theta = 105,\ 285,\ 465,\ 645$

$\theta = 52.5,\ 142.5,\ 232.5,\ 322.5$

c $\cos\theta \cos 60 + \sin\theta \sin 60 = \sin\theta$

$\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = \sin\theta$

$(1 - \frac{\sqrt{3}}{2})\sin\theta = \frac{1}{2}\cos\theta$

$\tan\theta = \frac{1}{2} \div (1 - \frac{\sqrt{3}}{2}) = 3.7321$

$\theta = 75,\ 180 + 75$

$\theta = 75,\ 255$

d $2\sin\theta + \sin\theta\cos 45 + \cos\theta\sin 45 = 0$

$2\sin\theta + \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 0$

$(2 + \frac{1}{\sqrt{2}})\sin\theta = -\frac{1}{\sqrt{2}}\cos\theta$

$\tan\theta = -\frac{1}{\sqrt{2}} \div (2 + \frac{1}{\sqrt{2}}) = -0.2612$

$\theta = 180 - 14.6,\ 360 - 14.6$

$\theta = 165.4,\ 345.4$

e $\sin\theta\cos 30 + \cos\theta\sin 30$

$= \cos\theta\cos 45 + \sin\theta\sin 45$

$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta$

$(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}})\sin\theta = (\frac{1}{\sqrt{2}} - \frac{1}{2})\cos\theta$

$\tan\theta = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \div (\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) = 1.3032$

$\theta = 52.2,\ 180 + 52.5$

$\theta = 52.5,\ 232.5$

f $3(\cos 2\theta\cos 60 - \sin 2\theta\sin 60)$

$-(\sin 2\theta\cos 30 - \cos 2\theta\sin 30) = 0$

$\frac{3}{2}\cos 2\theta - \frac{3\sqrt{3}}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta = 0$

$2\sqrt{3}\sin 2\theta = 2\cos 2\theta$

$\tan 2\theta = \frac{1}{\sqrt{3}}$

$2\theta = 30,\ 180 + 30,\ 360 + 30,\ 540 + 30$

$= 30,\ 210,\ 390,\ 570$

$\theta = 15,\ 105,\ 195,\ 285$

10 LHS $= \cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3} - (\cos x\cos\frac{\pi}{3} + \sin x\sin\frac{\pi}{3})$

$= -2\sin x\sin\frac{\pi}{3}$

$= -\sqrt{3}\sin x \quad \therefore\ k = -\sqrt{3}$

11 **a** LHS = $\cos x - (\cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3})$

$= \cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$= \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

$= \cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}$

$= \cos(x + \frac{\pi}{3})$ = RHS

b LHS = $\sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} + \cos x$

$= \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x + \cos x$

$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$

$= \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}$

$= \sin(x + \frac{\pi}{6})$ = RHS

12 **a** $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$

let $B = A \Rightarrow \sin(A + A) \equiv \sin A \cos A + \cos A \sin A$

$\sin 2A \equiv 2 \sin A \cos A$

b $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

let $B = A \Rightarrow \cos(A + A) \equiv \cos A \cos A - \sin A \sin A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

c **i** $\cos 2A \equiv \cos^2 A - (1 - \cos^2 A)$

$\cos 2A \equiv 2\cos^2 A - 1$

ii $\cos 2A \equiv 1 - \sin^2 A - \sin^2 A$

$\cos 2A \equiv 1 - 2\sin^2 A$

d $\tan(A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$

let $B = A \Rightarrow \tan(A + A) \equiv \frac{\tan A + \tan A}{1 - \tan A \tan A}$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

13 **a** $2\cos^2 x - 1 + \cos x = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = -1$ or $\frac{1}{2}$

$x = 180$ or $60, 360 - 60$

$x = 60°, 180°, 300°$

b $2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$x = 90, 360 - 90$ or $180 + 30, 360 - 30$

$x = 90°, 210°, 270°, 330°$

c $2(1 - 2\sin^2 x) = 7\sin x$

$4\sin^2 x + 7\sin x - 2 = 0$

$(4\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{4}$ or -2 [no solutions]

$x = 14.5, 180 - 14.5$

$x = 14.5°, 165.5°$

d $11\cos x = 4 + 3(2\cos^2 x - 1)$

$6\cos^2 x - 11\cos x + 1 = 0$

$\cos x = \frac{11 \pm \sqrt{121 - 24}}{12} = \frac{11 \pm \sqrt{97}}{12}$

$\cos x = 0.09593$ or 1.7374 [no solutions]

$x = 84.5, 360 - 84.5$

$x = 84.5°, 275.5°$

e $\frac{2\tan x}{1-\tan^2 x} - \tan x = 0$

$2\tan x = \tan x(1 - \tan^2 x)$

$\tan^3 x + \tan x = 0$

$\tan x(\tan^2 x + 1) = 0$

$\tan x = 0$ or $\tan^2 x = -1$ [no solutions]

$x = 0, 180°, 360°$

f $\frac{1}{\cos x} = 4\sin x$

$1 = 4\sin x\cos x$

$1 = 2\sin 2x$

$\sin 2x = \frac{1}{2}$

$2x = 30, 180 - 30, 360 + 30, 540 - 30$

$= 30, 150, 390, 510$

$x = 15°, 75°, 195°, 255°$

g $10\sin 2x\cos 2x = 2\sin 2x$

$2\sin 2x(5\cos 2x - 1) = 0$

$\sin 2x = 0$ or $\cos 2x = \frac{1}{5}$

$2x = 0, 180, 360, 540, 720$

or $78.463, 360 - 78.463,$

$360 + 78.463, 720 - 78.463$

$= 0, 78.463, 180, 281.537, 360$

$438.463, 540, 641.537, 720$

$x = 0, 39.2°, 90°, 140.8°, 180°,$

$219.2°, 270°, 320.8°, 360°$

h $2(1 - \cos^2 x) - (2\cos^2 x - 1) - \cos x = 0$

$4\cos^2 x + \cos x - 3 = 0$

$(4\cos x - 3)(\cos x + 1) = 0$

$\cos x = -1$ or $\frac{3}{4}$

$x = 180$ or $41.4, 360 - 41.4$

$x = 41.4°, 180, 318.6°$

14 **a** LHS $= \cos^2 x + 2\sin x\cos x + \sin^2 x$

$= \cos^2 x + \sin^2 x + \sin 2x$

$= 1 + \sin 2x$

$=$ RHS

b LHS $= \tan x(1 + 2\cos^2 x - 1)$

$= \frac{\sin x}{\cos x} \times 2\cos^2 x$

$= 2\sin x\cos x$

$= \sin 2x$

$=$ RHS

c LHS $= \frac{2\sin x\cos x}{\cos x(2\cos x - \sec x)}$

$= \frac{2\sin x\cos x}{2\cos^2 x - 1}$

$= \frac{\sin 2x}{\cos 2x}$

$= \tan 2x$

$=$ RHS

d LHS $= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x\sin x}$

$= \frac{1}{\frac{1}{2}\sin 2x}$

$= 2\operatorname{cosec} 2x$

$=$ RHS

e LHS $= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$

$= \frac{1 - \cos 2x}{\sin 2x}$

$= \frac{1 - (1 - 2\sin^2 x)}{\sin 2x}$

$= \frac{2\sin^2 x}{2\sin x\cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

$=$ RHS

f LHS $= \cos x\operatorname{cosec} x - 1 + 1 - \sin x\sec x$

$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x - \sin^2 x}{\sin x\cos x}$

$= \frac{\cos 2x}{\frac{1}{2}\sin 2x}$

$= 2\cot 2x$

$=$ RHS

g LHS = $\dfrac{\sin x(1-\sin 2x)}{\sin x(\operatorname{cosec} x - 2\cos x)}$

$= \dfrac{\sin x(1-\sin 2x)}{1 - 2\sin x \cos x}$

$= \dfrac{\sin x(1-\sin 2x)}{1-\sin 2x}$

$= \sin x$

= RHS

h LHS = $\cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= \cos x(2\cos^2 x - 1) - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$

= RHS

15 **a** $\cos 2A \equiv 2\cos^2 A - 1$

let $A = \frac{x}{2}$

$\cos x \equiv 2\cos^2 \frac{x}{2} - 1$

b $\cos 2A \equiv 1 - 2\sin^2 A$

let $A = \frac{x}{2}$

$\cos x \equiv 1 - 2\sin^2 \frac{x}{2}$

$\sin^2 \frac{x}{2} \equiv \frac{1}{2}(1 - \cos x)$

16 **a** $\sin^2 \frac{A}{2} = \frac{1}{2}(1 - \frac{7}{9}) = \frac{1}{9}$

$\sin \frac{A}{2} = \pm\frac{1}{3}$

$0 < \frac{A}{2} < 45° \;\; \therefore \sin \frac{A}{2} = \frac{1}{3}$

b $-\frac{3}{8} = 2\cos^2 \frac{B}{2} - 1$

$\cos^2 \frac{B}{2} = \frac{1}{2}(-\frac{3}{8} + 1) = \frac{5}{16}$

$\cos \frac{B}{2} = \pm\frac{1}{4}\sqrt{5}$

$45° < \frac{B}{2} < 90° \;\; \therefore \cos \frac{B}{2} = \frac{1}{4}\sqrt{5}$

17 **a** LHS = $\dfrac{2}{1 + (2\cos^2 \frac{x}{2} - 1)}$

$= \dfrac{2}{2\cos^2 \frac{x}{2}}$

$= \sec^2 \frac{x}{2}$

= RHS

b LHS = $\dfrac{1 + (2\cos^2 \frac{x}{2} - 1)}{1 - (1 - 2\sin^2 \frac{x}{2})}$

$= \dfrac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}}$

$= \cot^2 \frac{x}{2}$

= RHS