C3 TRIGONOMETRY Worksheet D

1 **a** Write down the identities for $\sin (A + B)$ and $\cos (A + B)$.

b Use these identities to obtain similar identities for $\sin (A - B)$ and $\cos (A - B)$.

c Use the above identities to obtain similar identities for $\tan (A + B)$ and $\tan (A - B)$.

2 Express each of the following in the form $\sin \alpha$, where α is acute.

a $\sin 10° \cos 30° + \cos 10° \sin 30°$ **b** $\sin 67° \cos 18° - \cos 67° \sin 18°$

c $\sin 62° \cos 74° + \cos 62° \sin 74°$ **d** $\cos 14° \cos 39° - \sin 14° \sin 39°$

3 Express as a single trigonometric ratio

a $\cos A \cos 2A - \sin A \sin 2A$ **b** $\sin 4A \cos B - \cos 4A \sin B$

c $\dfrac{\tan 2A + \tan 5A}{1 - \tan 2A \tan 5A}$ **d** $\cos A \cos 3A + \sin A \sin 3A$

4 Find in exact form, with a rational denominator, the value of

a $\sin 15°$ **b** $\sin 165°$ **c** $\operatorname{cosec} 15°$ **d** $\cos 75°$

e $\cos 15°$ **f** $\sec 195°$ **g** $\tan 75°$ **h** $\operatorname{cosec} 105°$

5 Find the maximum value that each expression can take and the smallest positive value of x, in degrees, for which this maximum occurs.

a $\cos x \cos 30° + \sin x \sin 30°$ **b** $3 \sin x \cos 45° + 3 \cos x \sin 45°$

c $\sin x \cos 67° - \cos x \sin 67°$ **d** $4 \sin x \sin 108° - 4 \cos x \cos 108°$

6 Find the minimum value that each expression can take and the smallest positive value of x, in radians in terms of π, for which this minimum occurs.

a $\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3}$ **b** $2 \cos x \cos \frac{\pi}{6} - 2 \sin x \sin \frac{\pi}{6}$

c $\cos 4x \cos x + \sin 4x \sin x$ **d** $6 \sin 2x \cos 3x - 6 \sin 3x \cos 2x$

7 Given that $\sin A = \frac{4}{5}$, $0 < A < 90°$ and that $\cos B = \frac{2}{3}$, $0 < B < 90°$, find without using a calculator the value of

a $\tan A$ **b** $\sin B$ **c** $\cos (A + B)$ **d** $\sin (A + B)$

8 Given that $\operatorname{cosec} C = \frac{5}{3}$, $0 < C < 90°$ and that $\sin D = \frac{5}{13}$, $90° < D < 180°$, find without using a calculator the value of

a $\cos C$ **b** $\cos D$ **c** $\sin (C - D)$ **d** $\sec (C - D)$

9 Solve each equation for θ in the interval $0 \le \theta \le 360$.
Give your answers to 1 decimal place where appropriate.

a $\sin \theta° \cos 15° + \cos \theta° \sin 15° = 0.4$ **b** $\dfrac{\tan 2\theta° - \tan 60°}{1 + \tan 2\theta° \tan 60°} = 1$

c $\cos (\theta - 60)° = \sin \theta°$ **d** $2 \sin \theta° + \sin (\theta + 45)° = 0$

e $\sin (\theta + 30)° = \cos (\theta - 45)°$ **f** $3 \cos (2\theta + 60)° - \sin (2\theta - 30)° = 0$

10 Find the value of k such that for all real values of x

$$\cos\left(x + \tfrac{\pi}{3}\right) - \cos\left(x - \tfrac{\pi}{3}\right) \equiv k \sin x.$$

11 Prove each identity.

a $\cos x - \cos\left(x - \tfrac{\pi}{3}\right) \equiv \cos\left(x + \tfrac{\pi}{3}\right)$

b $\sin\left(x - \tfrac{\pi}{6}\right) + \cos x \equiv \sin\left(x + \tfrac{\pi}{6}\right)$

12 **a** Use the identity for $\sin(A + B)$ to express $\sin 2A$ in terms of $\sin A$ and $\cos A$.

b Use the identity for $\cos(A + B)$ to express $\cos 2A$ in terms of $\sin A$ and $\cos A$.

c Hence, express $\cos 2A$ in terms of

i $\cos A$ **ii** $\sin A$

d Use the identity for $\tan(A + B)$ to express $\tan 2A$ in terms of $\tan A$.

13 Solve each equation for x in the interval $0 \le x \le 360°$.
Give your answers to 1 decimal place where appropriate.

a $\cos 2x + \cos x = 0$

b $\sin 2x + \cos x = 0$

c $2 \cos 2x = 7 \sin x$

d $11 \cos x = 4 + 3 \cos 2x$

e $\tan 2x - \tan x = 0$

f $\sec x - 4 \sin x = 0$

g $5 \sin 4x = 2 \sin 2x$

h $2 \sin^2 x - \cos 2x - \cos x = 0$

14 Prove each identity.

a $(\cos x + \sin x)^2 \equiv 1 + \sin 2x$

b $\tan x\,(1 + \cos 2x) \equiv \sin 2x$

c $\dfrac{2\sin x}{2\cos x - \sec x} \equiv \tan 2x$

d $\tan x + \cot x \equiv 2 \operatorname{cosec} 2x$

e $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$

f $(\cos x + \sin x)(\operatorname{cosec} x - \sec x) \equiv 2 \cot 2x$

g $\dfrac{1 - \sin 2x}{\operatorname{cosec} x - 2\cos x} \equiv \sin x$

h $\cos 3x \equiv 4 \cos^3 x - 3 \cos x$

15 Use the double angle identities to prove that

a $\cos x \equiv 2 \cos^2 \dfrac{x}{2} - 1$

b $\sin^2 \dfrac{x}{2} \equiv \tfrac{1}{2}(1 - \cos x)$

16 **a** Given that $\cos A = \tfrac{7}{9}$, $0 < A < 90°$, find the exact value of $\sin \dfrac{A}{2}$ without using a calculator.

b Given that $\cos B = -\tfrac{3}{8}$, $90° < B < 180°$, find the value of $\cos \dfrac{B}{2}$, giving your answer in the form $k\sqrt{5}$.

17 Prove each identity.

a $\dfrac{2}{1 + \cos x} \equiv \sec^2 \dfrac{x}{2}$

b $\dfrac{1 + \cos x}{1 - \cos x} \equiv \cot^2 \dfrac{x}{2}$