C3 TRIGONOMETRY Worksheet F

1 Express each of the following in the form $R\cos(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.
Give the values of R and α correct to 1 decimal place where appropriate.

a $\cos x° + \sin x°$ **b** $3\cos x° + 4\sin x°$

c $2\sin x° + \cos x°$ **d** $\cos x° + \sqrt{3}\sin x°$

2 Express each of the following in the given form, where $R > 0$ and $0 < \alpha < 90$.
Give the exact value of R and the value of α correct to 1 decimal place.

a $5\cos x° - 12\sin x°$, $R\cos(x + \alpha)°$ **b** $4\sin x° + 2\cos x°$, $R\sin(x + \alpha)°$

c $\sin x° - 7\cos x°$, $R\sin(x - \alpha)°$ **d** $8\cos 2x° - 15\sin 2x°$, $R\cos(2x + \alpha)°$

3 Express each of the following in the given form, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give the exact value of R and the value of α correct to 2 decimal places where appropriate.

a $3\sin x - 2\cos x$, $R\sin(x - \alpha)$ **b** $3\cos x + \sqrt{3}\sin x$, $R\cos(x - \alpha)$

c $8\sin 3x + 6\cos 3x$, $R\sin(3x + \alpha)$ **d** $\cos x + \frac{1}{2}\sin x$, $R\cos(x - \alpha)$

4 Find the maximum value that each expression can take and the smallest positive value of x, in degrees, for which this occurs.

a $24\sin x - 7\cos x$ **b** $4\cos 2x + 4\sin 2x$

c $3\cos x - 5\sin x$ **d** $5\sin 3x + \cos 3x$

5 **a** Express $3\sin x° - 3\cos x°$ in the form $R\sin(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$.

b Hence, describe two transformations that would map the graph of $y = \sin x°$ onto the graph of $y = 3\sin x° - 3\cos x°$.

6 By first expressing each curve in an appropriate form, sketch each of the following for x in the interval $0 \le x \le 360°$, showing the coordinates of any turning points.

a $y = 12\cos x + 5\sin x$ **b** $y = \sin x - 2\cos x$

c $y = 2\sqrt{3}\cos x - 6\sin x$ **d** $y = 9\sin x + 4\cos x$

7 **a** Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

b Solve the equation $\sqrt{3}\cos x - \sin x = 1$ for x in the interval $0 \le x \le 2\pi$, giving your answers in terms of π.

8 Solve each equation for x in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places.

a $6\sin x + 8\cos x = 5$ **b** $2\cos x - 2\sin x = 1$

c $7\sin x - 24\cos x - 10 = 0$ **d** $3\cos x + \sin x + 1 = 0$

e $\cos 2x + 4\sin 2x = 3$ **f** $5\sin x - 8\cos x + 7 = 0$

9 Solve each equation for x in the interval $-180° \le x \le 180°$, giving your answers to 1 decimal place where appropriate.

a $\sin x + \cos x = 1$ **b** $4\cos x - \sin x + 2 = 0$

c $\cos\frac{x}{2} + 5\sin\frac{x}{2} - 4 = 0$ **d** $6\sin x = 5 - 3\cos x$

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