

$$1 \quad a \quad \frac{2}{\cos x} = \frac{3}{\sin x}$$

$$\frac{\sin x}{\cos x} = \frac{3}{2}$$

$$\tan x = \frac{3}{2}$$

$$x = 56.3, 56.3 - 180$$

$$x = -123.7^\circ, 56.3^\circ$$

$$b \quad \cot^2 \theta - \cot \theta + 1 + \cot^2 \theta = 4$$

$$2 \cot^2 \theta - \cot \theta - 3 = 0$$

$$(2 \cot \theta - 3)(\cot \theta + 1) = 0$$

$$\cot \theta = -1 \quad \text{or} \quad \frac{3}{2}$$

$$\tan \theta = -1 \quad \text{or} \quad \frac{2}{3}$$

$$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4} \quad \text{or} \quad 0.5880, \pi + 0.5880$$

$$\theta = 0.59 \text{ (2dp)}, \frac{3\pi}{4}, 3.73 \text{ (2dp)}, \frac{7\pi}{4}$$

$$2 \quad 2 \sin \theta \cos 30 + 2 \cos \theta \sin 30$$

$$= \sin \theta \cos 30 - \cos \theta \sin 30$$

$$\sqrt{3} \sin \theta + \cos \theta = \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta$$

$$\frac{\sqrt{3}}{2} \sin \theta = -\frac{3}{2} \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = -\sqrt{3}$$

$$\tan \theta = -\sqrt{3}$$

$$\theta = 180 - 60, 360 - 60$$

$$\theta = 120^\circ, 300^\circ$$

$$3 \quad a \quad i \quad \operatorname{cosec} A = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$$

$$ii \quad \operatorname{cosec}^2 A = (2 + \sqrt{3})^2$$

$$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$$

$$\cot^2 A = \operatorname{cosec}^2 A - 1 = 6 + 4\sqrt{3}$$

$$b \quad 3(1 - 2 \sin^2 x) - 8 \sin x + 5 = 0$$

$$3 \sin^2 x + 4 \sin x - 4 = 0$$

$$(3 \sin x - 2)(\sin x + 2) = 0$$

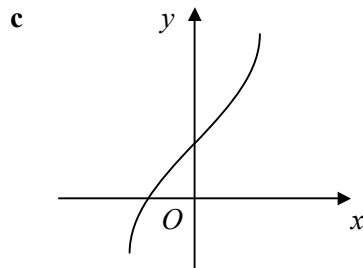
$$\sin x = \frac{2}{3} \quad \text{or} \quad -2 \text{ [no solutions]}$$

$$x = 41.8, 180 - 41.8$$

$$x = 41.8^\circ, 138.2^\circ$$

$$4 \quad a \quad = \frac{\pi}{2} + 2 \times \frac{\pi}{6} = \frac{5\pi}{6}$$

$$b \quad -\frac{\pi}{2} \leq f(x) \leq \frac{3\pi}{2}$$



$$d \quad \frac{\pi}{2} + 2 \arcsin x = 0$$

$$\arcsin x = -\frac{\pi}{4}$$

$$x = \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

5 a $2 \sin x - 3 \cos x$
 $= R \sin x \cos \alpha - R \cos x \sin \alpha$
 $\Rightarrow R \cos \alpha = 2, R \sin \alpha = 3$
 $\therefore R = \sqrt{4+9} = \sqrt{13} = 3.61$
 $\tan \alpha = \frac{3}{2}, \alpha = 0.983$
 $\therefore 2 \sin x - 3 \cos x = 3.61 \sin (x - 0.983)$

b min. value = -3.61 (3sf)
 when $x - 0.9828 = \frac{3\pi}{2}, x = 5.70$ (3sf)

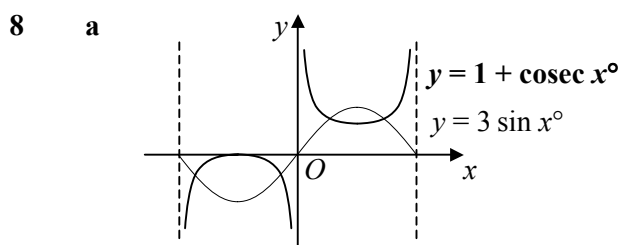
c $\sqrt{13} \sin (2x - 0.9828) + 1 = 0$
 $\sin (2x - 0.9828) = -\frac{1}{\sqrt{13}}$
 $2x - 0.983 = \pi + 0.2810, -0.2810$
 $= -0.2810, 3.4226$
 $2x = 0.7018, 4.4054$
 $x = 0.35, 2.20$

7 a LHS = $\frac{1}{\sin \theta} - \sin \theta$
 $= \frac{1 - \sin^2 \theta}{\sin \theta}$
 $= \frac{\cos^2 \theta}{\sin \theta}$
 $= \cos \theta \times \frac{\cos \theta}{\sin \theta}$
 $= \cos \theta \cot \theta$
 $= \text{RHS}$

b $\frac{2}{\cos x} + \frac{\sin x}{\cos x} = 2 \cos x$
 $2 + \sin x = 2 \cos^2 x$
 $2 + \sin x = 2(1 - \sin^2 x)$
 $2 \sin^2 x + \sin x = 0$
 $\sin x(2 \sin x + 1) = 0$
 $\sin x = -\frac{1}{2} \text{ or } 0$
 $x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \text{ or } 0, \pi, 2\pi$
 $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

6 a $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$
 let $A = B = \frac{x}{2}$
 $\cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$
 $\cos x \equiv \cos^2 \frac{x}{2} - (1 - \cos^2 \frac{x}{2})$
 $\cos x \equiv 2 \cos^2 \frac{x}{2} - 1$

b $\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{1 + (2 \cos^2 \frac{x}{2} - 1)} = 3 \cot \frac{x}{2}$
 $\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = 3 \cot \frac{x}{2}$
 $\tan \frac{x}{2} = \frac{3}{\tan \frac{x}{2}}$
 $\tan^2 \frac{x}{2} = 3$
 $\tan \frac{x}{2} = \pm \sqrt{3}$
 $\frac{x}{2} = 60 \text{ or } 180 - 60$
 $\frac{x}{2} = 60, 120$
 $x = 120^\circ, 240^\circ$



b $3 \sin x = 1 + \frac{1}{\sin x}$
 $3 \sin^2 x - \sin x - 1 = 0$
 $\sin x = \frac{1 \pm \sqrt{1+12}}{6} = \frac{1 \pm \sqrt{13}}{6}$
 $\sin x = -0.4343 \text{ or } 0.7676$
 $x = -25.7, 25.7 - 180 \text{ or } 50.1, 180 - 50.1$
 $x = -154.3, -25.7, 50.1, 129.9$

9 a LHS = $\sec x + \tan x - \tan x - \sin x \tan x$

$$= \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x}{\cos x}$$

$$= \cos x$$

$$= \text{RHS}$$

b $2(1 + \tan^2 2y) + \tan^2 2y = 3$

$$\tan^2 2y = \frac{1}{3}$$

$$\tan 2y = \pm \frac{1}{\sqrt{3}}$$

$$2y = \frac{\pi}{6}, \pi + \frac{\pi}{6} \text{ or } \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$y = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

10 a $4 \sin x - \cos x$

$$= R \sin x \cos \alpha - R \cos x \sin \alpha$$

$$\Rightarrow R \cos \alpha = 4, R \sin \alpha = 1$$

$$\therefore R = \sqrt{16+1} = \sqrt{17} = 4.12$$

$$\tan \alpha = \frac{1}{4}, \alpha = 14.0$$

$$\therefore 4 \sin x^\circ - \cos x^\circ = 4.12 \sin(x - 14.0)^\circ$$

b $\frac{2}{\sin x} - \frac{\cos x}{\sin x} + 4 = 0$

$$2 - \cos x + 4 \sin x = 0$$

$$\therefore 4 \sin x^\circ - \cos x^\circ + 2 = 0$$

c $\sqrt{17} \sin(x - 14.04) + 2 = 0$

$$\sin(x - 14.04) = -\frac{2}{\sqrt{17}}$$

$$x - 14.04 = 180 + 29.02, 360 - 29.02$$

$$= 209.02, 330.98$$

$$x = 223.1, 345.0 \text{ (1dp)}$$

11 a adding

$$\cos(A+B) + \cos(A-B) \equiv 2 \cos A \cos B$$

$$\text{let } P = A+B \text{ (1) and } Q = A-B \text{ (2)}$$

$$(1) + (2) \Rightarrow 2A = P+Q \Rightarrow A = \frac{P+Q}{2}$$

$$(1) - (2) \Rightarrow 2B = P-Q \Rightarrow B = \frac{P-Q}{2}$$

$$\therefore \cos P + \cos Q \equiv 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}$$

b $2 \cos \frac{x+3x}{2} \cos \frac{x-3x}{2} + \cos 2x = 0$

$$2 \cos 2x \cos(-x) + \cos 2x = 0$$

$$\cos 2x(2 \cos x + 1) = 0$$

$$\cos 2x = 0 \text{ or } \cos x = -\frac{1}{2}$$

$$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$$

$$\text{or } x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ or } x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}$$

12 a $3 \cos \theta + 4 \sin \theta$

$$= R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$$

$$\Rightarrow R \cos \alpha = 3, R \sin \alpha = 4$$

$$\therefore R = \sqrt{9+16} = 5$$

$$\tan \alpha = \frac{4}{3}, \alpha = 0.927 \text{ (3sf)}$$

$$\therefore 3 \cos \theta + 4 \sin \theta = 5 \cos(\theta - 0.927)$$

b i $-4 \leq f(\theta) \leq 6$

ii $1 - 5 \cos(2\theta - 0.9273) = 0$

$$\cos(2\theta - 0.9273) = \frac{1}{5}$$

$$2\theta - 0.9273 = 1.3694, 2\pi - 1.3694$$

$$= 1.3694, 4.9137$$

$$2\theta = 2.2967, 5.8410$$

$$\theta = 1.15, 2.92 \text{ (2dp)}$$

c $y = \frac{2}{5 \cos(x - 0.9273)}$

TP: $y = \frac{2}{5}$ when $x - 0.9273 = 0$

$$y = -\frac{2}{5} \text{ when } x - 0.9273 = \pi$$

$$\therefore (0.93, \frac{2}{5}) \text{ and } (4.07, -\frac{2}{5})$$