1   **a** Solve the equation

$$2 \sec x - 3 \operatorname{cosec} x = 0,$$

for x in the interval $-180° \le x \le 180°$.   **(4)**

b Find all values of θ in the interval $0 \le \theta \le 2\pi$ for which

$$\cot^2 \theta - \cot \theta + \operatorname{cosec}^2 \theta = 4.$$ **(6)**

2   For values of θ in the interval $0 \le \theta \le 360°$, solve the equation

$$2 \sin (\theta + 30°) = \sin (\theta - 30°).$$ **(6)**

3   **a** Given that $\sin A = 2 - \sqrt{3}$, find in the form $a + b\sqrt{3}$ the exact value of

i $\operatorname{cosec} A$,

ii $\cot^2 A$.   **(5)**

b Solve the equation

$$3 \cos 2x - 8 \sin x + 5 = 0,$$

for values of x in the interval $0 \le x \le 360°$, giving your answers to 1 decimal place.   **(5)**

4 $$f : x \to \frac{\pi}{2} + 2 \arcsin x, \ x \in \mathbb{R}, \ -1 \le x \le 1.$$

a Find the exact value of $f(\frac{1}{2})$.   **(2)**

b State the range of f.   **(2)**

c Sketch the curve $y = f(x)$.   **(2)**

d Solve the equation $f(x) = 0$.   **(3)**

5   **a** Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the values of R and α to 3 significant figures.   **(4)**

b State the minimum value of $2 \sin x - 3 \cos x$ and the smallest positive value of x for which this minimum occurs.   **(3)**

c Solve the equation

$$2 \sin 2x - 3 \cos 2x + 1 = 0,$$

for x in the interval $0 \le x \le \pi$, giving your answers to 2 decimal places.   **(5)**

6   **a** Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 2 \cos^2 \frac{x}{2} - 1.$$ **(3)**

b Solve the equation

$$\frac{\sin x}{1 + \cos x} = 3 \cot \frac{x}{2},$$

for values of x in the interval $0 \le x \le 360°$.   **(7)**

7 **a** Prove the identity

$$\text{cosec } \theta - \sin \theta \equiv \cos \theta \cot \theta, \quad \theta \neq n\pi, \ n \in \mathbb{Z} \ . \tag{3}$$

b Find the values of x in the interval $0 \le x \le 2\pi$ for which

$$2 \sec x + \tan x = 2 \cos x,$$

giving your answers in terms of π. **(6)**

8 **a** Sketch on the same diagram the curves $y = 3 \sin x°$ and $y = 1 + \text{cosec } x°$ for x in the interval $-180 \le x \le 180$. **(4)**

b Find the x-coordinate of each point where the curves intersect in this interval, giving your answers correct to 1 decimal place. **(6)**

9 **a** Prove the identity

$$(1 - \sin x)(\sec x + \tan x) \equiv \cos x, \quad x \neq \frac{(2n+1)\pi}{2}, \ n \in \mathbb{Z} \ . \tag{4}$$

b Find the values of y in the interval $0 \le y \le \pi$ for which

$$2 \sec^2 2y + \tan^2 2y = 3,$$

giving your answers in terms of π. **(6)**

10 **a** Express $4 \sin x° - \cos x°$ in the form $R \sin (x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$. Give the values of R and α to 3 significant figures. **(4)**

b Show that the equation

$$2 \text{ cosec } x° - \cot x° + 4 = 0 \qquad \text{(I)}$$

can be written in the form

$$4 \sin x° - \cos x° + 2 = 0. \tag{2}$$

c Using your answers to parts **a** and **b**, solve equation (I) for x in the interval $0 \le x \le 360$. **(4)**

11 **a** Use the identities

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

and $\qquad \cos (A - B) \equiv \cos A \cos B + \sin A \sin B$

to prove that

$$\cos P + \cos Q \equiv 2 \cos \frac{P+Q}{2} \cos \frac{P-Q}{2}. \tag{4}$$

b Find, in terms of π, the values of x in the interval $0 \le x \le 2\pi$ for which

$$\cos x + \cos 2x + \cos 3x = 0. \tag{7}$$

12 **a** Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

b Given that the function f is defined by

$$\text{f}(\theta) \equiv 1 - 3 \cos 2\theta - 4 \sin 2\theta, \ \theta \in \mathbb{R}, \ 0 \le \theta \le \pi,$$

i state the range of f,

ii solve the equation $\text{f}(\theta) = 0$. **(6)**

c Find the coordinates of the turning points of the curve with equation $y = \dfrac{2}{3 \cos x + 4 \sin x}$

in the interval $[0, 2\pi]$. **(3)**