C4 DIFFERENTIATION — *Answers - Worksheet A*

1 **a** $(5, 2)$

b $\frac{4}{t} = -8 \ \therefore\ t = -\frac{1}{2}$

2 **a** $(2, 0)$

b $1 + \sin t = \frac{3}{2},\ \sin t = \frac{1}{2},\ t = \frac{\pi}{6}, \frac{5\pi}{6}$

$2\cos t = -\sqrt{3},\ \cos t = -\frac{\sqrt{3}}{2},\ t = \frac{5\pi}{6}, \frac{7\pi}{6}$

$\therefore\ t = \frac{5\pi}{6}$

3 **a** $t = \frac{x}{3} \ \therefore\ y = (\frac{x}{3})^2$

$y = \frac{1}{9}x^2$

b $t = \frac{x}{2} \ \therefore\ y = \frac{1}{(\frac{x}{2})}$

$y = \frac{2}{x}$

c $x^2 = t^6,\ y^3 = 8t^6$

$\therefore\ y^3 = 8x^2$

d $t = 4 - y$

$\therefore\ x = 1 - (4 - y)^2$

e $t = \frac{1}{2}(x + 1)$

$\therefore\ y = \frac{2}{\frac{1}{4}(x+1)^2}$

$y = \frac{8}{(x+1)^2}$

f $t = \frac{1}{x} + 1$

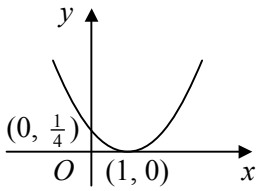
$\therefore\ y = \frac{1}{2 - (\frac{1}{x} + 1)} = \frac{1}{1 - \frac{1}{x}}$

$y = \frac{x}{x-1}$

4 **a** $t = \frac{1}{2}(x - 1)$

$\therefore\ y = \frac{1}{4}(x - 1)^2$

b



5 **a** $\cos^2\theta + \sin^2\theta = 1$

$\therefore\ x^2 + y^2 = 1$

b $\cos 2\theta = 1 - 2\sin^2\theta$

$\therefore\ y = 1 - 2x^2$

c $\cos\theta = \frac{x-3}{2},\ \sin\theta = \frac{y-1}{2}$

$\cos^2\theta + \sin^2\theta = 1$

$\therefore\ (\frac{x-3}{2})^2 + (\frac{y-1}{2})^2 = 1$

$(x - 3)^2 + (y - 1)^2 = 4$

d $\sec\theta = \frac{x}{2},\ \tan\theta = \frac{y}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + (\frac{y}{4})^2 = (\frac{x}{2})^2$

$16 + y^2 = 4x^2$

$y^2 = 4x^2 - 16$

e $\sin 2\theta = 2\sin\theta\cos\theta$

$\therefore\ y = 4\sin^2\theta\cos^2\theta$

$y = 4\sin^2\theta(1 - \sin^2\theta)$

$y = 4x^2(1 - x^2)$

f $\sec\theta = \frac{1}{x}$

$1 + \tan^2\theta = \sec^2\theta$

$\therefore\ 1 + y = (\frac{1}{x})^2$

$y = \frac{1}{x^2} - 1$

6 **a** $\cos\theta = \frac{x-1}{3}$, $\sin\theta = \frac{y-4}{3}$

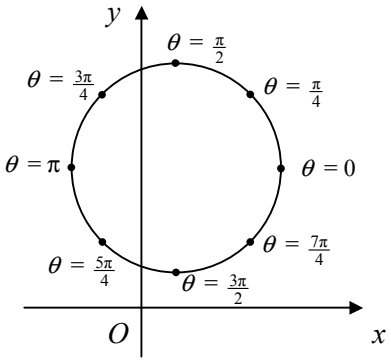
$\cos^2\theta + \sin^2\theta = 1$

$(\frac{x-1}{3})^2 + (\frac{y-4}{3})^2 = 1$

$(x-1)^2 + (y-4)^2 = 9$

b centre (1, 4) radius 3

c



7 **a** $x = 5\cos\theta$, $y = 5\sin\theta$, $0 \le \theta < 2\pi$

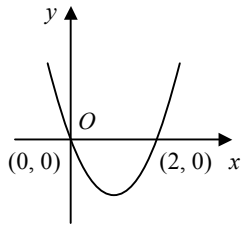
b $x = 6 + 2\cos\theta$, $y = -1 + 2\sin\theta$, $0 \le \theta < 2\pi$

c $x = a + r\cos\theta$, $y = b + r\sin\theta$, $0 \le \theta < 2\pi$

8 **a** $t = \frac{x}{2}$

$\therefore\ y = 4(\frac{x}{2})(\frac{x}{2} - 1)$

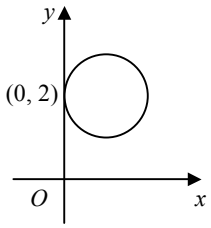
$y = x(x-2)$



b $\sin\theta = 1 - x$, $\cos\theta = 2 - y$

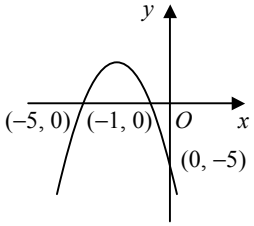
$\cos^2\theta + \sin^2\theta = 1$ $\therefore (2-y)^2 + (1-x)^2 = 1$

or $(x-1)^2 + (y-2)^2 = 1$



c $t = x + 3$

$\therefore\ y = 4 - (x+3)^2$



d $t = x - 1$

$\therefore\ y = \frac{2}{x-1}$