**1** Given that $y = x^2 + 3x + 5$,

and that $x = (t - 4)^3$,

a find expressions for

  i $\dfrac{dy}{dx}$ in terms of x, **ii** $\dfrac{dx}{dt}$ in terms of t,

b find the value of $\dfrac{dy}{dt}$ when

  i $t = 5$, **ii** $x = 8$.

2 The variables x and y are related by the equation $y = x\sqrt{2x - 3}$.

Given that x is increasing at the rate of 0.3 units per second when $x = 6$, find the rate at which y is increasing at this instant.

3 The radius of a circle is increasing at a constant rate of 0.2 cm s^{-1}.

a Show that the perimeter of the circle is increasing at the rate of $0.4\pi \text{ cm s}^{-1}$.

b Find the rate at which the area of the circle is increasing when the radius is 10 cm.

c Find the radius of the circle when its area is increasing at the rate of $20 \text{ cm}^2 \text{ s}^{-1}$.

4 The area of a circle is decreasing at a constant rate of $0.5 \text{ cm}^2 \text{ s}^{-1}$.

a Find the rate at which the radius of the circle is decreasing when the radius is 8 cm.

b Find the rate at which the perimeter of the circle is decreasing when the radius is 8 cm.

5 The volume of a cube is increasing at a constant rate of $3.5 \text{ cm}^3 \text{ s}^{-1}$. Find

a the rate at which the length of one side of the cube is increasing when the volume is 200 cm^3,

b the volume of the cube when the length of one side is increasing at the rate of 2 mm s^{-1}.

6
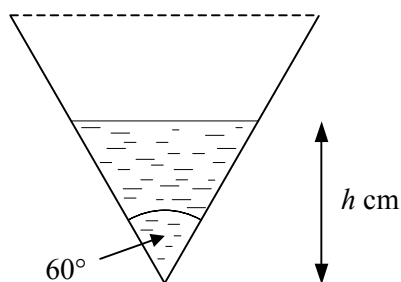
The diagram shows the cross-section of a right-circular paper cone being used as a filter funnel. The volume of liquid in the funnel is $V \text{ cm}^3$ when the depth of the liquid is h cm.

Given that the angle between the sides of the funnel in the cross-section is 60° as shown,

a show that $V = \frac{1}{9}\pi h^3$.

Given also that at time t seconds after liquid is put in the funnel

$$V = 600e^{-0.0005t},$$

b show that after two minutes, the depth of liquid in the funnel is approximately 11.7 cm,

c find the rate at which the depth of liquid is decreasing after two minutes.