C4 DIFFERENTIATION *Worksheet E*

1 A curve has the equation

$$3x^2 + xy - y^2 + 9 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(5)**

2 A curve has parametric equations

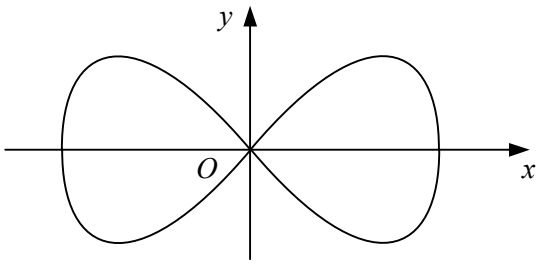
$$x = a\cos\theta, \quad y = a(\sin\theta - \theta), \quad 0 \leq \theta < \pi,$$

where a is a positive constant.

a Show that $\frac{dy}{dx} = \tan\frac{\theta}{2}$. **(5)**

b Find, in terms of a, an equation for the tangent to the curve at the point where it crosses the y-axis. **(3)**

3



The diagram shows the curve with parametric equations

$$x = \cos\theta, \quad y = \tfrac{1}{2}\sin 2\theta, \quad 0 \leq \theta < 2\pi.$$

a Find $\frac{dy}{dx}$ in terms of θ. **(3)**

b Find the two values of θ for which the curve passes through the origin. **(2)**

c Show that the two tangents to the curve at the origin are perpendicular to each other. **(2)**

d Find a cartesian equation for the curve. **(4)**

4 A curve has the equation

$$x^2 - 4xy + y^2 = 24.$$

a Show that $\frac{dy}{dx} = \frac{x - 2y}{2x - y}$. **(4)**

b Find an equation for the tangent to the curve at the point P (2, 10). **(3)**

The tangent to the curve at Q is parallel to the tangent at P.

c Find the coordinates of Q. **(4)**

5 A curve is given by the parametric equations

$$x = t^2 + 2, \quad y = t(t - 1).$$

a Find the coordinates of any points on the curve where the tangent to the curve is parallel to the x-axis. **(5)**

b Show that the tangent to the curve at the point (3, 2) has the equation

$$3x - 2y = 5.$$ **(5)**

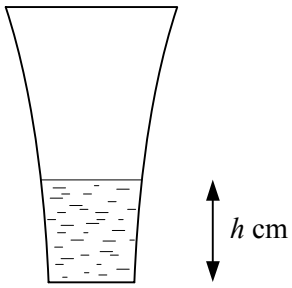
6 Find an equation for the normal to the curve with equation

$$x^3 - 3x + xy - 2y^2 + 3 = 0$$

at the point (1, 1).

Give your answer in the form $y = mx + c$. **(7)**

7



The diagram shows the cross-section of a vase. The volume of water in the vase, V cm^3, when the depth of water in the vase is h cm is given by

$$V = 40\pi(e^{0.1h} - 1).$$

The vase is initially empty and water is poured into it at a constant rate of 80 cm^3 s^{-1}.

Find the rate at which the depth of water in the vase is increasing

a when $h = 4$, **(5)**

b after 5 seconds of pouring water in. **(4)**

8 A curve is given by the parametric equations

$$x = \frac{t}{1+t}, \quad y = \frac{t}{1-t}, \quad t \neq \pm 1.$$

a Show that $\frac{dy}{dx} = \left(\frac{1+t}{1-t}\right)^2$. **(4)**

b Show that the normal to the curve at the point P, where $t = \frac{1}{2}$, has the equation

$$3x + 27y = 28.$$ **(4)**

The normal to the curve at P meets the curve again at the point Q.

c Find the exact value of the parameter t at Q. **(4)**

9 A curve has the equation

$$2x + x^2y - y^2 = 0.$$

Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. **(8)**

10 A curve has parametric equations

$$x = a \sec\theta, \quad y = 2a \tan\theta, \quad -\tfrac{\pi}{2} \leq \theta < \tfrac{\pi}{2},$$

where a is a positive constant.

a Find $\frac{dy}{dx}$ in terms of θ. **(3)**

b Show that the normal to the curve at the point where $\theta = \frac{\pi}{4}$ has the equation

$$x + 2\sqrt{2}\,y = 5\sqrt{2}\,a.$$ **(4)**

c Find a cartesian equation for the curve in the form $y^2 = f(x)$. **(3)**