

1 a $\frac{dx}{dt} = 2t, \frac{dy}{dt} = -2t^{-2}$
 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-2t^{-2}}{2t} = -\frac{1}{t^3}$
 b $t = 2 \therefore x = 4, y = 1$
 $\text{grad} = -\frac{1}{8}$
 $\therefore \text{grad of normal} = 8$
 $\therefore y - 1 = 8(x - 4)$
 $y = 8x - 31$

2 $x = 1 \therefore y = 4$
 $\frac{dy}{dx} = 4^x \ln 4$
 $\text{grad} = 4 \ln 4 = 4 \ln 2^2 = 8 \ln 2$
 $\therefore y - 4 = (8 \ln 2)(x - 1)$
 $y = 4 + 8(x - 1) \ln 2$

3 a $\frac{dx}{d\theta} = \sec \theta \tan \theta, \frac{dy}{d\theta} = -2 \sin 2\theta$
 $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{-2 \sin 2\theta}{\sec \theta \tan \theta}$
 $= -4 \sin \theta \cos \theta \times \cos \theta \times \frac{\cos \theta}{\sin \theta}$
 $= -4 \cos^3 \theta$
 b $\theta = \frac{\pi}{6} \therefore x = \frac{2}{\sqrt{3}}, y = \frac{1}{2}$
 $\text{grad} = -4 \times \left(\frac{\sqrt{3}}{2}\right)^3 = -\frac{3}{2}\sqrt{3}$
 $\therefore y - \frac{1}{2} = -\frac{3}{2}\sqrt{3} \left(x - \frac{2}{\sqrt{3}}\right)$
 $2y - 1 = -3\sqrt{3}x + 6$
 $3\sqrt{3}x + 2y = 7 \quad [k = 7]$

4 a $4x + 6 \times y + 6x \times \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$
 $4x + 6y = \frac{dy}{dx} (2y - 6x)$
 $\frac{dy}{dx} = \frac{2x + 3y}{y - 3x}$
 at P , $\text{grad} = 1$
 $\therefore \text{grad of normal} = -1$
 $\therefore y + 5 = -(x - 2)$
 $x + y + 3 = 0$

b sub. $y = -x - 3$ into eqn of curve
 $2x^2 + 6x(-x - 3) - (-x - 3)^2 + 77 = 0$
 $5x^2 + 24x - 68 = 0$
 $(5x + 34)(x - 2) = 0$
 $x = 2 \text{ (at } P) \text{ or } -\frac{34}{5} \therefore x = -6\frac{4}{5}$

5 a $y = 0 \Rightarrow \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$
 $\therefore \left(\frac{\pi}{2} - 1, 0\right), \left(\frac{3\pi}{2} + 1, 0\right)$
 b $\frac{dx}{d\theta} = 1 - \cos \theta, \frac{dy}{d\theta} = -\sin \theta$
 $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{-\sin \theta}{1 - \cos \theta}$
 $= -\frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{1 - (1 - 2 \sin^2 \frac{\theta}{2})}$
 $= -\frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = -\cot \frac{\theta}{2}$
 c $-\cot \frac{\theta}{2} = 0$
 $\frac{\theta}{2} = \frac{\pi}{2}, \theta = \pi$
 $\therefore (\pi, -1)$

6 a $\sin \theta = \frac{1}{2}$
 $\therefore \theta = \frac{\pi}{6}$
 b $\theta = \frac{\pi}{6} \therefore y = \frac{4}{3}$
 $\frac{dx}{d\theta} = \cos \theta, \frac{dy}{d\theta} = 2 \sec \theta \times \sec \theta \tan \theta,$
 $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{2 \sec^2 \theta \tan \theta}{\cos \theta} = \frac{2 \sin \theta}{\cos^4 \theta}$
 $\therefore \text{grad} = \frac{16}{9}$
 $\therefore y - \frac{4}{3} = \frac{16}{9} \left(x - \frac{1}{2}\right)$
 $9y - 12 = 16x - 8$
 $16x - 9y + 4 = 0$
 c $y = \sec^2 \theta = \frac{1}{\cos^2 \theta} = \frac{1}{1 - \sin^2 \theta}$
 $\therefore y = \frac{1}{1 - x^2}$

$$7 \quad \mathbf{a} \quad 2 \cos x - 2 \frac{dy}{dx} \sec^2 2y = 0$$

$$2 \cos x = \frac{2}{\cos^2 2y} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \cos x \cos^2 2y$$

$$\mathbf{b} \quad \text{grad} = \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

$$\therefore y - \frac{\pi}{6} = \frac{1}{8} \left(x - \frac{\pi}{3}\right)$$

$$24y - 4\pi = 3x - \pi$$

$$3x - 24y + 3\pi = 0$$

$$x - 8y + \pi = 0$$

$$8 \quad \mathbf{a} \quad \frac{dx}{dt} = -4 \sin t, \quad \frac{dy}{dt} = 3 \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{3 \cos t}{-4 \sin t} = -\frac{3}{4} \cot t$$

$$\mathbf{b} \quad y - 3 \sin t = -\frac{3 \cos t}{4 \sin t} (x - 4 \cos t)$$

$$4y \sin t - 12 \sin^2 t = -3x \cos t + 12 \cos^2 t$$

$$3x \cos t + 4y \sin t = 12(\sin^2 t + \cos^2 t)$$

$$3x \cos t + 4y \sin t = 12$$

$$\mathbf{c} \quad \cos t = \frac{x}{4}, \quad \sin t = \frac{y}{3}$$

$$\cos^2 t + \sin^2 t = 1$$

$$\therefore \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$9x^2 + 16y^2 = 144$$

$$9 \quad \mathbf{a} \quad \frac{dx}{dt} = \frac{1 \times (t+1) - t \times 1}{(t+1)^2} = \frac{1}{(t+1)^2}$$

$$\frac{dy}{dt} = \frac{2 \times (t-1) - 2t \times 1}{(t-1)^2} = \frac{-2}{(t-1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-2}{(t-1)^2} \div \frac{1}{(t+1)^2}$$

$$= \frac{-2(t+1)^2}{(t-1)^2} = -2 \left(\frac{t+1}{t-1}\right)^2$$

$$\mathbf{b} \quad \text{at } O, t = 0 \therefore \text{grad} = -2$$

$$\therefore \text{grad of normal} = \frac{1}{2}$$

$$\therefore y = \frac{1}{2}x$$

$$\mathbf{c} \quad \frac{2t}{t-1} = \frac{1}{2} \times \frac{t}{t+1}$$

$$4t(t+1) = t(t-1)$$

$$3t^2 + 5t = 0$$

$$t(3t+5) = 0$$

$$t = 0 \text{ (at } O) \text{ or } -\frac{5}{3}$$

$$\therefore \left(\frac{5}{2}, \frac{5}{4}\right)$$

$$\mathbf{d} \quad x = \frac{t}{t+1} \Rightarrow xt + x = t$$

$$x = t(1-x)$$

$$t = \frac{x}{1-x}$$

$$\therefore y = \frac{\frac{2x}{1-x}}{\frac{x}{1-x} - 1}$$

$$y = \frac{2x}{x - (1-x)}$$

$$y = \frac{2x}{2x-1}$$