# C4 DIFFERENTIATION *Worksheet F*

**1** A curve has parametric equations

$$x = t^2, \quad y = \frac{2}{t}.$$

**a** Find $\frac{dy}{dx}$ in terms of $t$. **(3)**

**b** Find an equation for the normal to the curve at the point where $t = 2$, giving your answer in the form $y = mx + c$. **(3)**

**2** A curve has the equation $y = 4^x$.

Show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 4 + 8(x - 1) \ln 2.$$ **(4)**

**3** A curve has parametric equations

$$x = \sec \theta, \quad y = \cos 2\theta, \quad 0 \leq \theta < \tfrac{\pi}{2}.$$

**a** Show that $\frac{dy}{dx} = -4 \cos^3 \theta$. **(4)**

**b** Show that the tangent to the curve at the point where $\theta = \frac{\pi}{6}$ has the equation

$$3\sqrt{3}\, x + 2y = k,$$

where $k$ is an integer to be found. **(4)**

**4** A curve has the equation

$$2x^2 + 6xy - y^2 + 77 = 0$$

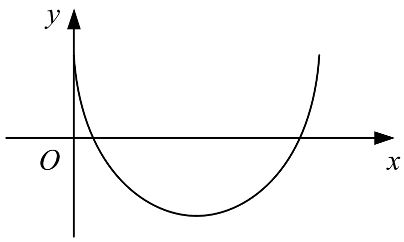
and passes through the point $P\,(2, -5)$.

**a** Show that the normal to the curve at $P$ has the equation

$$x + y + 3 = 0.$$ **(6)**

**b** Find the $x$-coordinate of the point where the normal to the curve at $P$ intersects the curve again. **(3)**

**5**




The diagram shows the curve with parametric equations

$$x = \theta - \sin \theta, \quad y = \cos \theta, \quad 0 \leq \theta \leq 2\pi.$$

**a** Find the exact coordinates of the points where the curve crosses the $x$-axis. **(3)**

**b** Show that $\frac{dy}{dx} = -\cot \frac{\theta}{2}$. **(5)**

**c** Find the exact coordinates of the point on the curve where the tangent to the curve is parallel to the $x$-axis. **(2)**

**6** A curve has parametric equations

$$x = \sin\theta, \quad y = \sec^2\theta, \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

The point $P$ on the curve has $x$-coordinate $\tfrac{1}{2}$.

**a** Write down the value of the parameter $\theta$ at $P$. **(1)**

**b** Show that the tangent to the curve at $P$ has the equation

$$16x - 9y + 4 = 0.$$ **(6)**

**c** Find a cartesian equation for the curve. **(2)**

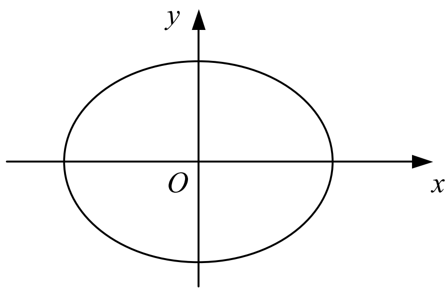
**7** A curve has the equation

$$2\sin x - \tan 2y = 0.$$

**a** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x \cos^2 2y$. **(4)**

**b** Find an equation for the tangent to the curve at the point $(\tfrac{\pi}{3}, \tfrac{\pi}{6})$, giving your answer in the form $ax + by + c = 0$. **(3)**

**8**




A particle moves on the ellipse shown in the diagram such that at time $t$ its coordinates are given by

$$x = 4\cos t, \quad y = 3\sin t, \quad t \geq 0.$$

**a** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$. **(3)**

**b** Show that at time $t$, the tangent to the path of the particle has the equation

$$3x\cos t + 4y\sin t = 12.$$ **(3)**

**c** Find a cartesian equation for the path of the particle. **(3)**

**9** The curve with parametric equations

$$x = \frac{t}{t+1}, \quad y = \frac{2t}{t-1},$$

passes through the origin, $O$.

**a** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\left(\frac{t+1}{t-1}\right)^2$. **(4)**

**b** Find an equation for the normal to the curve at $O$. **(2)**

**c** Find the coordinates of the point where the normal to the curve at $O$ meets the curve again. **(4)**

**d** Show that the cartesian equation of the curve can be written in the form

$$y = \frac{2x}{2x-1}.$$ **(4)**