C4 INTEGRATION *Answers - Worksheet A*

1 **a** $e^x + c$ **b** $4e^x + c$ **c** $\ln|x| + c$ **d** $6\ln|x| + c$

2 **a** $= 2t + 3e^t + c$ **b** $= \frac{1}{2}t^2 + \ln|t| + c$ **c** $= \frac{1}{3}t^3 - e^t + c$ **d** $= 9t - 2\ln|t| + c$

e $= \int (\frac{7}{t} + t^{\frac{1}{2}})\,dt$
$= 7\ln|t| + \frac{2}{3}t^{\frac{3}{2}} + c$

f $= \frac{1}{4}e^t - \ln|t| + c$

g $= \int (\frac{1}{3t} + t^{-2})\,dt$
$= \frac{1}{3}\ln|t| - t^{-1} + c$

h $= \frac{2}{5}\ln|t| - \frac{3}{7}e^t + c$

3 **a** $= 5x - 3\ln|x| + c$ **b** $= \ln|u| - u^{-1} + c$

c $= \int (\frac{2}{5}e^t + \frac{1}{5})\,dt$
$= \frac{2}{5}e^t + \frac{1}{5}t + c$

d $= \int (3 + \frac{1}{y})\,dy$
$= 3y + \ln|y| + c$

e $= \int (\frac{3}{4}e^t + 3t^{\frac{1}{2}})\,dt$
$= \frac{3}{4}e^t + 2t^{\frac{3}{2}} + c$

f $= \int (x^2 - 2 + x^{-2})\,dx$
$= \frac{1}{3}x^3 - 2x - x^{-1} + c$

4 $f'(x) = \frac{4x^2 - 4x + 1}{x} = 4x - 4 + \frac{1}{x}$
$f(x) = \int (4x - 4 + \frac{1}{x})\,dx = 2x^2 - 4x + \ln|x| + c$
$(1, -3) \Rightarrow -3 = 2 - 4 + 0 + c$
$\therefore\ c = -1$
$f(x) = 2x^2 - 4x + \ln|x| - 1$

5 **a** $= [e^x + 10x]_0^1$
$= (e + 10) - (1 + 0)$
$= e + 9$

b $= [\frac{1}{2}t^2 + \ln|t|]_2^5$
$= (\frac{25}{2} + \ln 5) - (2 + \ln 2)$
$= \frac{21}{2} + \ln\frac{5}{2}$

c $= \int_1^4 (\frac{5}{x} - x)\,dx$
$= [5\ln|x| - \frac{1}{2}x^2]_1^4$
$= (5\ln 4 - 8) - (0 - \frac{1}{2})$
$= 10\ln 2 - \frac{15}{2}$

d $= \int_{-2}^{-1} (2 + \frac{1}{3y})\,dy$
$= [2y + \frac{1}{3}\ln|y|]_{-2}^{-1}$
$= (-2 + 0) - (-4 + \frac{1}{3}\ln 2)$
$= 2 - \frac{1}{3}\ln 2$

e $= [e^x - \frac{1}{3}x^3]_{-3}^3$
$= (e^3 - 9) - (e^{-3} + 9)$
$= e^3 - e^{-3} - 18$

f $= \int_2^3 (4 - 3r^{-1} + 6r^{-2})\,dr$
$= [4r - 3\ln|r| - 6r^{-1}]_2^3$
$= (12 - 3\ln 3 - 2) - (8 - 3\ln 2 - 3)$
$= 5 - 3\ln\frac{3}{2}$

g $= [7u - e^u]_{\ln 2}^{\ln 4}$
$= (7\ln 4 - 4) - (7\ln 2 - 2)$
$= 7\ln 2 - 2$

h $= \int_6^{10} (2 + 9r^{-1})\,dr$
$= [2r + 9\ln|r|]_6^{10}$
$= (20 + 9\ln 10) - (12 + 9\ln 6)$
$= 8 + 9\ln\frac{5}{3}$

i $= \int_4^9 (x^{-\frac{1}{2}} + 3e^x)\,dx$
$= [2x^{\frac{1}{2}} + 3e^x]_4^9$
$= (6 + 3e^9) - (4 + 3e^4)$
$= 3e^9 - 3e^4 + 2$

6 $= \int_0^2 (3 + e^x)\,dx$
$= [3x + e^x]_0^2$
$= (6 + e^2) - (0 + 1)$
$= e^2 + 5$

7 $= \int_1^4 (2x + \frac{1}{x})\,dx$
$= [x^2 + \ln|x|]_1^4$
$= (16 + \ln 4) - (1 + 0)$
$= 15 + 2\ln 2$

8 **a** $= \int_0^1 (4x + 2e^x)\ dx$

$= [2x^2 + 2e^x]_0^1$

$= (2 + 2e) - (0 + 2) = 2e$

b $= \int_2^4 (1 + \frac{3}{x})\ dx$

$= [x + 3 \ln|x|]_2^4$

$= (4 + 3 \ln 4) - (2 + 3 \ln 2) = 2 + 3 \ln 2$

c $= \int_{-3}^{-1} (4 - \frac{1}{x})\ dx$

$= [4x - \ln|x|]_{-3}^{-1}$

$= (-4 - 0) - (-12 - \ln 3) = 8 + \ln 3$

d $= \int_0^{\ln 2} (2 - \frac{1}{2}e^x)\ dx$

$= [2x - \frac{1}{2}e^x]_0^{\ln 2}$

$= (2 \ln 2 - 1) - (0 - \frac{1}{2}) = 2 \ln 2 - \frac{1}{2}$

e $= \int_{\frac{1}{2}}^{2} (e^x + \frac{5}{x})\ dx$

$= [e^x + 5 \ln|x|]_{\frac{1}{2}}^{2}$

$= (e^2 + 5 \ln 2) - (e^{\frac{1}{2}} + 5 \ln \frac{1}{2})$

$= e^2 - e^{\frac{1}{2}} + 10 \ln 2$

f $= \int_2^3 (x^2 - \frac{2}{x})\ dx$

$= [\frac{1}{3}x^3 - 2 \ln|x|]_2^3$

$= (9 - 2 \ln 3) - (\frac{8}{3} - 2 \ln 2)$

$= \frac{19}{3} - 2 \ln \frac{3}{2}$

9 **a** $9 - \frac{7}{x} - 2x = 0$

$2x^2 - 9x + 7 = 0$

$(2x - 7)(x - 1) = 0$

$x = 1, \frac{7}{2}$

$\therefore (1, 0)$ and $(\frac{7}{2}, 0)$

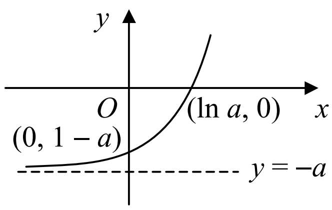
b $= \int_1^{\frac{7}{2}} (9 - \frac{7}{x} - 2x)\ dx$

$= [9x - 7 \ln|x| - x^2]_1^{\frac{7}{2}}$

$= (\frac{63}{2} - 7 \ln \frac{7}{2} - \frac{49}{4}) - (9 - 0 - 1)$

$= 11\frac{1}{4} - 7 \ln \frac{7}{2}$

10 **a**



b $= -\int_0^{\ln a} (e^x - a)\ dx = -[e^x - ax]_0^{\ln a}$

$= -[(a - a \ln a) - (1 - 0)] = 1 - a + a \ln a$

c $1 - a + a \ln a = 1 + a$

$a \ln a = 2a, \ \ln a = 2, \ a = e^2$

11 **a** $x = 3 \ \therefore y = e^3$

$\frac{dy}{dx} = e^x \ \therefore$ grad $= e^3$

$\therefore \ y - e^3 = e^3(x - 3) \quad [\ y = e^3(x - 2)\ ]$

b at Q, $y = 0 \ \therefore x = 2$

at R, $x = 0 \ \therefore y = -2e^3$

$\therefore \ Q\ (2, 0), \ R\ (0, -2e^3)$

c area under curve, $0 \leq x \leq 3$

$= \int_0^3 e^x\ dx = [e^x]_0^3 = e^3 - 1$

area of triangle under PQ

$= \frac{1}{2} \times 1 \times e^3 = \frac{1}{2}e^3$

area of triangle above QR

$= \frac{1}{2} \times 2 \times 2e^3 = 2e^3$

shaded area

$= (e^3 - 1) - \frac{1}{2}e^3 + 2e^3 = \frac{5}{2}e^3 - 1$

12 **a** $(\frac{3}{\sqrt{x}} - 4)^2 = 0$

$\sqrt{x} = \frac{3}{4}$

$x = \frac{9}{16} \ \therefore (\frac{9}{16}, 0)$

b $= \int_{\frac{9}{16}}^{1} (\frac{3}{\sqrt{x}} - 4)^2\ dx$

$= \int_{\frac{9}{16}}^{1} (9x^{-1} - 24x^{-\frac{1}{2}} + 16)\ dx$

$= [9 \ln|x| - 48x^{\frac{1}{2}} + 16x]_{\frac{9}{16}}^{1}$

$= (0 - 48 + 16) - (9 \ln \frac{9}{16} - 36 + 9)$

$= -5 - 9 \ln \frac{9}{16} \approx 0.178$