

$$1 \quad \begin{array}{llll} \mathbf{a} = 2 \sin x + c & \mathbf{b} = -\frac{1}{4} \cos 4x + c & \mathbf{c} = 2 \sin \frac{1}{2}x + c & \mathbf{d} = -\cos \left(x + \frac{\pi}{4}\right) + c \\ \mathbf{e} = \frac{1}{2} \sin (2x - 1) + c & \mathbf{f} = 3 \cos \left(\frac{\pi}{3} - x\right) + c & \mathbf{g} = \sec x + c & \mathbf{h} = -\cot x + c \\ \mathbf{i} = \frac{5}{2} \tan 2x + c & \mathbf{j} = -4 \operatorname{cosec} \frac{1}{4}x + c & \mathbf{k} = \int 4 \operatorname{cosec}^2 x \, dx & \mathbf{l} = \int \sec^2 (4x + 1) \, dx \\ & & = -4 \cot x + c & = \frac{1}{4} \tan (4x + 1) + c \end{array}$$

$$2 \quad \begin{array}{lll} \mathbf{a} = [\sin x]_0^{\frac{\pi}{2}} & \mathbf{b} = \left[-\frac{1}{2} \cos 2x\right]_0^{\frac{\pi}{6}} & \mathbf{c} = [4 \sec \frac{1}{2}x]_0^{\frac{\pi}{2}} \\ = 1 - 0 = 1 & = -\frac{1}{4} - \left(-\frac{1}{2}\right) = \frac{1}{4} & = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1) \\ \mathbf{d} = \left[\frac{1}{2} \sin \left(2x - \frac{\pi}{3}\right)\right]_0^{\frac{\pi}{3}} & \mathbf{e} = \left[\frac{1}{3} \tan 3x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} & \mathbf{f} = [-\operatorname{cosec} x]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \\ = \frac{\sqrt{3}}{4} - \left(-\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2} & = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3} & = -\frac{2}{\sqrt{3}} - (-1) = 1 - \frac{2}{3}\sqrt{3} \end{array}$$

$$3 \quad \begin{array}{l} \mathbf{a} \quad \tan^2 \theta = \sec^2 \theta - 1 \\ \mathbf{b} \quad \int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + c \end{array}$$

$$4 \quad \begin{array}{l} \mathbf{a} \quad \cos(A+B) \equiv \cos A \cos B - \sin A \sin B \\ \text{let } B = A \quad \Rightarrow \quad \cos 2A \equiv \cos^2 A - \sin^2 A \\ \cos 2A \equiv \cos^2 A - (1 - \cos^2 A) \\ \cos 2A \equiv 2 \cos^2 A - 1 \\ \cos^2 A \equiv \frac{1}{2}(1 + \cos 2A) \end{array}$$

$$\mathbf{b} \quad \int \cos^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) \, dx = \frac{1}{2}x + \frac{1}{4} \sin 2x + c$$

$$5 \quad \begin{array}{ll} \mathbf{a} = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x\right) \, dx & \mathbf{b} = \int (\operatorname{cosec}^2 2x - 1) \, dx \\ = \frac{1}{2}x - \frac{1}{4} \sin 2x + c & = -\frac{1}{2} \cot 2x - x + c \end{array}$$

$$\begin{array}{ll} \mathbf{c} = \int \frac{1}{2} \sin 2x \, dx & \mathbf{d} = \int \frac{1}{\cos x} \times \frac{\sin x}{\cos x} \, dx \\ = -\frac{1}{4} \cos 2x + c & = \int \sec x \tan x \, dx \\ & = \sec x + c \end{array}$$

$$\begin{array}{ll} \mathbf{e} = \int (2 + 2 \cos 6x) \, dx & \mathbf{f} = \int (1 + 2 \sin x + \sin^2 x) \, dx \\ = 2x + \frac{1}{3} \sin 6x + c & = \int \left(1 + 2 \sin x + \frac{1}{2} - \frac{1}{2} \cos 2x\right) \, dx \\ & = \int \left(\frac{3}{2} + 2 \sin x - \frac{1}{2} \cos 2x\right) \, dx \\ & = \frac{3}{2}x - 2 \cos x - \frac{1}{4} \sin 2x + c \end{array}$$

$$\begin{array}{ll} \mathbf{g} = \int (\sec^2 x - 2 \sec x \tan x + \tan^2 x) \, dx & \mathbf{h} = \int \frac{1}{\sin 2x} \times \frac{\cos x}{\sin x} \, dx \\ = \int (\sec^2 x - 2 \sec x \tan x + \sec^2 x - 1) \, dx & = \int \frac{1}{2 \sin x \cos x} \times \frac{\cos x}{\sin x} \, dx \\ = \int (2 \sec^2 x - 2 \sec x \tan x - 1) \, dx & = \int \frac{1}{2} \operatorname{cosec}^2 x \, dx \\ = 2 \tan x - 2 \sec x - x + c & = -\frac{1}{2} \cot x + c \end{array}$$

$$\begin{aligned}
 \text{i} &= \int (\cos^2 x)^2 dx \\
 &= \int \left[\frac{1}{2}(1 + \cos 2x)\right]^2 dx \\
 &= \frac{1}{4} \int (1 + 2 \cos 2x + \cos^2 2x) dx \\
 &= \frac{1}{4} \int \left[1 + 2 \cos 2x + \frac{1}{2}(1 + \cos 4x)\right] dx \\
 &= \frac{1}{8} \int (3 + 4 \cos 2x + \cos 4x) dx \\
 &= \frac{1}{8} (3x + 2 \sin 2x + \frac{1}{4} \sin 4x) + c \\
 &= \frac{3}{8}x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{6 a} &= \int_0^{\frac{\pi}{2}} (1 + \cos 2x) dx \\
 &= \left[x + \frac{1}{2} \sin 2x\right]_0^{\frac{\pi}{2}} \\
 &= \left(\frac{\pi}{2} + 0\right) - (0 + 0) = \frac{\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c} &= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sec^2 \frac{1}{2}x - 1) dx \\
 &= \left[2 \tan \frac{1}{2}x - x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \\
 &= \left(2 - \frac{\pi}{2}\right) - \left(\frac{2}{\sqrt{3}} - \frac{\pi}{3}\right) \\
 &= 2 - \frac{2}{3}\sqrt{3} - \frac{\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{e} &= \int_0^{\frac{\pi}{4}} (1 - 4 \sin x + 4 \sin^2 x) dx \\
 &= \int_0^{\frac{\pi}{4}} [1 - 4 \sin x + 2(1 - \cos 2x)] dx \\
 &= \int_0^{\frac{\pi}{4}} (3 - 4 \sin x - 2 \cos 2x) dx \\
 &= \left[3x + 4 \cos x - \sin 2x\right]_0^{\frac{\pi}{4}} \\
 &= \left(\frac{3\pi}{4} + 2\sqrt{2} - 1\right) - (0 + 4 - 0) \\
 &= \frac{3\pi}{4} + 2\sqrt{2} - 5
 \end{aligned}$$

$$\begin{aligned}
 \text{7 a} \quad \sin(A + B) &\equiv \sin A \cos B + \cos A \sin B \quad (1) \\
 \sin(A - B) &\equiv \sin A \cos B - \cos A \sin B \quad (2) \\
 (1) + (2) &\Rightarrow \sin(A + B) + \sin(A - B) \equiv 2 \sin A \cos B \\
 \sin A \cos B &\equiv \frac{1}{2} [\sin(A + B) + \sin(A - B)]
 \end{aligned}$$

$$\text{b} = \int \left(\frac{1}{2} \sin 4x + \frac{1}{2} \sin 2x\right) dx = -\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x + c$$

$$\begin{aligned}
 \text{8 a} &= \int (\cos 4x - \cos 6x) dx \\
 &= \frac{1}{4} \sin 4x - \frac{1}{6} \sin 6x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{c} &= \int [2 \sin 5x + 2 \sin(-3x)] dx \\
 &= \int (2 \sin 5x - 2 \sin 3x) dx \\
 &= -\frac{2}{5} \cos 5x + \frac{2}{3} \cos 3x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{b} &= \int_0^{\frac{\pi}{4}} \frac{1}{2} \sin 4x dx \\
 &= \left[-\frac{1}{8} \cos 4x\right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{8} - \left(-\frac{1}{8}\right) = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{d} &= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x} \times \frac{\cos 2x}{\sin 2x} dx \\
 &= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} 2x \cot 2x dx \\
 &= \left[-\frac{1}{2} \operatorname{cosec} 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} \\
 &= -\frac{1}{2} - \left(-\frac{1}{\sqrt{3}}\right) = \frac{1}{3}\sqrt{3} - \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{f} &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x} dx \\
 &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(\frac{1}{2} \sin 2x)^2} dx \\
 &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 4 \operatorname{cosec}^2 2x dx \\
 &= \left[-2 \cot 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\
 &= \frac{2}{\sqrt{3}} - \left(-\frac{2}{\sqrt{3}}\right) \\
 &= \frac{4}{3}\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} &= \int \left(\frac{1}{2} \cos 3x + \frac{1}{2} \cos x\right) dx \\
 &= \frac{1}{6} \sin 3x + \frac{1}{2} \sin x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{d} &= \int \left[\frac{1}{2} \sin\left(2x + \frac{\pi}{6}\right) - \frac{1}{2} \sin \frac{\pi}{6}\right] dx \\
 &= \int \left[\frac{1}{2} \sin\left(2x + \frac{\pi}{6}\right) - \frac{1}{4}\right] dx \\
 &= -\frac{1}{4} \cos\left(2x + \frac{\pi}{6}\right) - \frac{1}{4}x + c
 \end{aligned}$$