

$$\begin{aligned} 1 \quad \mathbf{a} &= \frac{1}{2} \times \frac{1}{5} (2x-3)^5 + c \\ &= \frac{1}{10} (2x-3)^5 + c \end{aligned}$$

$$\mathbf{c} = \frac{1}{2} e^{4x-1} + c$$

$$\begin{aligned} \mathbf{e} &= \int 3 \sec^2 2x \, dx \\ &= \frac{3}{2} \tan 2x + c \end{aligned}$$

$$\begin{aligned} \mathbf{g} &= \int (\sec x \tan x) \sec^3 x \, dx \\ &= \frac{1}{4} \sec^4 x + c \end{aligned}$$

$$\begin{aligned} \mathbf{i} \quad u &= x, \frac{du}{dx} = 1; \frac{dv}{dx} = e^{3x}, v = \frac{1}{3} e^{3x} \\ \int x e^{3x} \, dx &= \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} \, dx \\ &= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + c \\ &= \frac{1}{9} e^{3x} (3x-1) + c \end{aligned}$$

$$\begin{aligned} \mathbf{k} &= \frac{1}{4} \times \left(-\frac{1}{2}\right) (x+1)^{-2} + c \\ &= -\frac{1}{8(x+1)^2} + c \end{aligned}$$

$$\begin{aligned} \mathbf{m} &= \int [2 + 2 \cos (4x+2)] \, dx \\ &= 2x + \frac{1}{2} \sin (4x+2) + c \end{aligned}$$

$$\begin{aligned} \mathbf{o} \quad u &= x, \frac{du}{dx} = 1; \frac{dv}{dx} = \sin 2x, v = -\frac{1}{2} \cos 2x \\ \int x \sin 2x \, dx &= -\frac{1}{2} x \cos 2x + \int \frac{1}{2} \cos 2x \, dx \\ &= -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + c \end{aligned}$$

$$\mathbf{b} = -2 \cot \frac{1}{2}x + c$$

$$\begin{aligned} \mathbf{d} \quad \frac{2(x-1)}{x(x+1)} &\equiv \frac{A}{x} + \frac{B}{x+1}, 2(x-1) \equiv A(x+1) + Bx \\ x=0 &\Rightarrow A=-2, x=-1 \Rightarrow B=4 \\ \int \frac{2(x-1)}{x(x+1)} \, dx &= \int \left(\frac{4}{x+1} - \frac{2}{x} \right) \, dx \\ &= 4 \ln |x+1| - 2 \ln |x| + c \end{aligned}$$

$$\begin{aligned} \mathbf{f} &= \frac{1}{2} \int 2x(x^2+3)^3 \, dx \\ &= \frac{1}{2} \times \frac{1}{4} (x^2+3)^4 + c \\ &= \frac{1}{8} (x^2+3)^4 + c \end{aligned}$$

$$\begin{aligned} \mathbf{h} &= \frac{1}{2} \times \frac{2}{3} (7+2x)^{\frac{3}{2}} + c \\ &= \frac{1}{3} (7+2x)^{\frac{3}{2}} + c \end{aligned}$$

$$\begin{aligned} \mathbf{j} \quad \frac{x+2}{(x-3)(x+1)} &\equiv \frac{A}{x-3} + \frac{B}{x+1}, x+2 \equiv A(x+1) + B(x-3) \\ x=3 &\Rightarrow A=\frac{5}{4}, x=-1 \Rightarrow B=-\frac{1}{4} \\ \int \frac{x+2}{x^2-2x-3} \, dx &= \int \left(\frac{\frac{5}{4}}{x-3} - \frac{\frac{1}{4}}{x+1} \right) \, dx \\ &= \frac{5}{4} \ln |x-3| - \frac{1}{4} \ln |x+1| + c \end{aligned}$$

$$\begin{aligned} \mathbf{l} &= \int (\sec^2 3x - 1) \, dx \\ &= \frac{1}{3} \tan 3x - x + c \end{aligned}$$

$$\begin{aligned} \mathbf{n} &= -\frac{3}{2} \int \frac{-2x}{1-x^2} \, dx \\ &= -\frac{3}{2} \ln |1-x^2| + c \end{aligned}$$

$$\begin{aligned} \mathbf{p} &= \int \frac{(x+2)+2}{x+2} \, dx \\ &= \int \left(1 + \frac{2}{x+2} \right) \, dx \\ &= x + 2 \ln |x+2| + c \end{aligned}$$

$$\begin{aligned}
 2 \quad a \quad \int_1^2 6e^{2x-3} dx &= [3e^{2x-3}]_1^2 \\
 &= 3(e - e^{-1})
 \end{aligned}$$

$$\begin{aligned}
 c \quad \int_{-2}^2 \frac{2}{x-3} dx &= [2 \ln |x-3|]_{-2}^2 \\
 &= 0 - 2 \ln 5 \\
 &= -2 \ln 5
 \end{aligned}$$

$$\begin{aligned}
 e \quad \int_1^2 (1-2x)^3 dx &= [-\frac{1}{2} \times \frac{1}{4} (1-2x)^4]_1^2 \\
 &= -\frac{1}{8} (81 - 1) \\
 &= -10
 \end{aligned}$$

$$\begin{aligned}
 3 \quad a \quad x = 3 \sin u \quad \therefore \frac{dx}{du} &= 3 \cos u \\
 x = 0 &\Rightarrow u = 0 \\
 x = \frac{3}{2} &\Rightarrow u = \frac{\pi}{6} \\
 \int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}} dx &= \int_0^{\frac{\pi}{6}} \frac{1}{3 \cos u} \times 3 \cos u du \\
 &= \int_0^{\frac{\pi}{6}} du \\
 &= [u]_0^{\frac{\pi}{6}} \\
 &= \frac{\pi}{6} - 0 \\
 &= \frac{\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 c \quad x = 2 \tan u \quad \therefore \frac{dx}{du} &= 2 \sec^2 u \\
 x = 2 &\Rightarrow u = \frac{\pi}{4} \\
 x = 2\sqrt{3} &\Rightarrow u = \frac{\pi}{3} \\
 \int_2^{2\sqrt{3}} \frac{1}{4+x^2} dx &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{4 \sec^2 u} \times 2 \sec^2 u du \\
 &= \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} du \\
 &= \frac{1}{2} [u]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\
 &= \frac{1}{2} \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\
 &= \frac{1}{24} \pi
 \end{aligned}$$

$$\begin{aligned}
 b \quad \int_0^{\frac{\pi}{3}} \tan x dx &= -\int_0^{\frac{\pi}{3}} \frac{-\sin x}{\cos x} dx \\
 &= -[\ln |\cos x|]_0^{\frac{\pi}{3}} \\
 &= -(\ln \frac{1}{2} - 0) \\
 &= \ln 2
 \end{aligned}$$

$$\begin{aligned}
 d \quad \frac{6+x}{(4-x)(1+x)} &\equiv \frac{A}{4-x} + \frac{B}{1+x}, \quad 6+x \equiv A(1+x) + B(4-x) \\
 x = 4 &\Rightarrow A = 2, \quad x = -1 \Rightarrow B = 1 \\
 \int_2^3 \frac{6+x}{4+3x-x^2} dx &= \int_2^3 \left(\frac{2}{4-x} + \frac{1}{1+x} \right) dx \\
 &= [-2 \ln |4-x| + \ln |1+x|]_2^3 \\
 &= (0 + \ln 4) - (-2 \ln 2 + \ln 3) \\
 &= 4 \ln 2 - \ln 3
 \end{aligned}$$

$$\begin{aligned}
 f \quad \int_0^{\frac{\pi}{3}} \sin^2 x \sin 2x dx &= \int_0^{\frac{\pi}{3}} 2 \sin^3 x \cos x dx \\
 &= [\frac{1}{2} \sin^4 x]_0^{\frac{\pi}{3}} \\
 &= \frac{1}{2} \left(\frac{\sqrt{3}}{2} \right)^4 - 0 \\
 &= \frac{9}{32}
 \end{aligned}$$

$$\begin{aligned}
 b \quad u = 1 - 3x \quad \therefore x &= \frac{1}{3}(1-u), \quad \frac{du}{dx} = -3 \\
 x = 0 &\Rightarrow u = 1 \\
 x = 1 &\Rightarrow u = -2 \\
 \int_0^1 x(1-3x)^3 dx &= \int_1^{-2} \frac{1}{3}(1-u)u^3 \times \left(-\frac{1}{3}\right) du \\
 &= \frac{1}{9} \int_{-2}^1 (u^3 - u^4) du \\
 &= \frac{1}{9} \left[\frac{1}{4} u^4 - \frac{1}{5} u^5 \right]_{-2}^1 \\
 &= \frac{1}{9} \left[\left(\frac{1}{4} - \frac{1}{5} \right) - \left(4 + \frac{32}{5} \right) \right] \\
 &= -\frac{23}{20}
 \end{aligned}$$

$$\begin{aligned}
 d \quad u^2 = x + 1 \quad \therefore x &= u^2 - 1, \quad \frac{dx}{du} = 2u \\
 x = -1 &\Rightarrow u = 0 \\
 x = 0 &\Rightarrow u = 1 \\
 \int_{-1}^0 x^2 \sqrt{x+1} dx &= \int_0^1 (u^2 - 1)^2 u \times 2u du \\
 &= \int_0^1 2u^2(u^4 - 2u^2 + 1) du \\
 &= \int_0^1 (2u^6 - 4u^4 + 2u^2) du \\
 &= \left[\frac{2}{7} u^7 - \frac{4}{5} u^5 + \frac{2}{3} u^3 \right]_0^1 \\
 &= \left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3} \right) - (0) \\
 &= \frac{16}{105}
 \end{aligned}$$

$$4 \quad \mathbf{a} = -\frac{2}{3} \ln |5 - 3x| + c$$

$$\begin{aligned} \mathbf{b} &= \frac{1}{2} \int (2x + 2) e^{x^2 + 2x} dx \\ &= \frac{1}{2} e^{x^2 + 2x} + c \end{aligned}$$

$$\begin{aligned} \mathbf{c} &= \int \frac{-\frac{1}{2}(2x+1) + \frac{3}{2}}{2x+1} dx \\ &= \int \left(\frac{\frac{3}{2}}{2x+1} - \frac{1}{2} \right) dx \\ &= \frac{3}{4} \ln |2x+1| - \frac{1}{2}x + c \end{aligned}$$

$$\begin{aligned} \mathbf{d} &= \frac{1}{2} \int (\sin 5x + \sin x) dx \\ &= \frac{1}{2} \left(-\frac{1}{5} \cos 5x - \cos x \right) + c \\ &= -\frac{1}{10} (\cos 5x + 5 \cos x) + c \end{aligned}$$

$$\begin{aligned} \mathbf{e} \quad u = 3x, \frac{du}{dx} = 3; \frac{dv}{dx} = (x-1)^4, v = \frac{1}{5}(x-1)^5 \quad \int 3x(x-1)^4 dx \\ = \frac{3}{5}x(x-1)^5 - \int \frac{3}{5}(x-1)^5 dx \\ = \frac{3}{5}x(x-1)^5 - \frac{1}{10}(x-1)^6 + c \\ = \frac{1}{10}(x-1)^5[6x - (x-1)] + c \\ = \frac{1}{10}(5x+1)(x-1)^5 + c \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad \frac{3x^2+6x+2}{(x+1)(x+2)} &\equiv 3 + \frac{A}{x+1} + \frac{B}{x+2} \\ 3x^2+6x+2 &\equiv 3(x+1)(x+2) + A(x+2) + B(x+1) \\ x = -1 &\Rightarrow A = -1, \quad x = -2 \Rightarrow B = -2 \end{aligned}$$

$$\begin{aligned} \int \frac{3x^2+6x+2}{x^2+3x+2} dx &= \int \left(3 - \frac{1}{x+1} - \frac{2}{x+2} \right) dx \\ &= 3x - \ln |x+1| - 2 \ln |x+2| + c \end{aligned}$$

$$\begin{aligned} \mathbf{g} &= \int 5(2x-1)^{-\frac{1}{3}} dx \\ &= \frac{1}{2} \times \frac{15}{2} (2x-1)^{\frac{2}{3}} + c \\ &= \frac{15}{4} (2x-1)^{\frac{2}{3}} + c \end{aligned}$$

$$\begin{aligned} \mathbf{h} &= \frac{1}{3} \int \frac{3 \cos x}{2 + 3 \sin x} dx \\ &= \frac{1}{3} \ln |2 + 3 \sin x| + c \end{aligned}$$

$$\begin{aligned} \mathbf{i} &= \frac{1}{3} \int 3x^2(x^3-1)^{-\frac{1}{2}} dx \\ &= \frac{1}{3} \times 2(x^3-1)^{\frac{1}{2}} + c \\ &= \frac{2}{3} \sqrt{x^3-1} + c \end{aligned}$$

$$\begin{aligned} \mathbf{j} &= \int (4 - 4 \cot x + \cot^2 x) dx \\ &= \int \left(4 - 4 \frac{\cos x}{\sin x} + \operatorname{cosec}^2 x - 1 \right) dx \\ &= 3x - 4 \ln |\sin x| - \cot x + c \end{aligned}$$

$$\begin{aligned} \mathbf{k} \quad \frac{6x-5}{(x-1)(2x-1)^2} &\equiv \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2} \\ 6x-5 &\equiv A(2x-1)^2 + B(x-1)(2x-1) + C(x-1) \\ x = 1 &\Rightarrow A = 1, \quad x = \frac{1}{2} \Rightarrow C = 4 \\ \text{coeffs of } x^2 &\Rightarrow B = -2 \\ \int \frac{6x-5}{(x-1)(2x-1)^2} dx \\ &= \int \left(\frac{1}{x-1} - \frac{2}{2x-1} + \frac{4}{(2x-1)^2} \right) dx \\ &= \ln |x-1| - \ln |2x-1| - 2(2x-1)^{-1} + c \\ &= \ln \left| \frac{x-1}{2x-1} \right| - \frac{2}{2x-1} + c \end{aligned}$$

$$\begin{aligned} \mathbf{l} \quad u = x^2, \frac{du}{dx} = 2x; \frac{dv}{dx} = e^{-x}, v = -e^{-x} \\ \int x^2 e^{-x} dx &= -x^2 e^{-x} + \int 2x e^{-x} dx \\ u = 2x, \frac{du}{dx} = 2; \frac{dv}{dx} = e^{-x}, v = -e^{-x} \\ \int x^2 e^{-x} dx &= -x^2 e^{-x} - 2x e^{-x} + \int 2e^{-x} dx \\ &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c \\ &= -e^{-x}(x^2 + 2x + 2) + c \end{aligned}$$

$$5 \quad \mathbf{a} \quad \int_2^4 \frac{1}{3x-4} \, dx$$

$$= \left[\frac{1}{3} \ln |3x-4| \right]_2^4$$

$$= \frac{1}{3} (\ln 8 - \ln 2)$$

$$= \frac{2}{3} \ln 2$$

$$\mathbf{c} \quad \frac{7-x^2}{(2-x)^2(3-x)} \equiv \frac{A}{2-x} + \frac{B}{(2-x)^2} + \frac{C}{3-x}$$

$$7-x^2 \equiv A(2-x)(3-x) + B(3-x) + C(2-x)^2$$

$$x=2 \Rightarrow B=3, \quad x=3 \Rightarrow C=-2$$

$$\text{coeffs of } x^2 \Rightarrow A=1$$

$$\int_0^1 \frac{7-x^2}{(2-x)^2(3-x)} \, dx$$

$$= \int_0^1 \left(\frac{1}{2-x} + \frac{3}{(2-x)^2} - \frac{2}{3-x} \right) \, dx$$

$$= [-\ln |2-x| + 3(2-x)^{-1} + 2 \ln |3-x|]_0^1$$

$$= (0+3+2 \ln 2) - (-\ln 2 + \frac{3}{2} + 2 \ln 3)$$

$$= \frac{3}{2} + 3 \ln 2 - 2 \ln 3$$

$$\mathbf{e} \quad \int_1^5 \frac{1}{\sqrt{4x+5}} \, dx$$

$$= \left[\frac{1}{4} \times 2(4x+5)^{\frac{1}{2}} \right]_1^5$$

$$= \frac{1}{2} (5-3)$$

$$= 1$$

$$\mathbf{g} \quad \int_0^2 x\sqrt{2x^2+1} \, dx = \frac{1}{4} \int_0^2 4x\sqrt{2x^2+1} \, dx \quad \mathbf{h}$$

$$= \frac{1}{4} \left[\frac{2}{3} (2x^2+1)^{\frac{3}{2}} \right]_0^2$$

$$= \frac{1}{6} (27-1)$$

$$= 4\frac{1}{3}$$

$$\mathbf{b} \quad \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x \cot^2 x \, dx = - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (-\operatorname{cosec}^2 x) \cot^2 x \, dx$$

$$= - \left[\frac{1}{3} \cot^3 x \right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= -\frac{1}{3} [1 - (\sqrt{3})^3]$$

$$= \sqrt{3} - \frac{1}{3}$$

$$\mathbf{d} \quad u=x, \quad \frac{du}{dx}=1; \quad \frac{dv}{dx}=\cos \frac{1}{2}x, \quad v=2 \sin \frac{1}{2}x$$

$$\int_0^{\frac{\pi}{2}} x \cos \frac{1}{2}x \, dx$$

$$= [2x \sin \frac{1}{2}x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2 \sin \frac{1}{2}x \, dx$$

$$= [2x \sin \frac{1}{2}x + 4 \cos \frac{1}{2}x]_0^{\frac{\pi}{2}}$$

$$= [\pi(\frac{1}{\sqrt{2}}) - 4(\frac{1}{\sqrt{2}})] - [0+4]$$

$$= \frac{1}{2}\sqrt{2}(\pi-4) - 4$$

$$\mathbf{f} \quad \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 2 \cos x \cos 3x \, dx$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} [\cos 4x + \cos (-2x)] \, dx$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\cos 4x + \cos 2x) \, dx$$

$$= \left[\frac{1}{4} \sin 4x + \frac{1}{2} \sin 2x \right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

$$= \left[\frac{1}{4} \left(\frac{\sqrt{3}}{2} \right) + \frac{1}{2} \left(\frac{\sqrt{3}}{2} \right) \right] - \left[\frac{1}{4} \left(-\frac{\sqrt{3}}{2} \right) + \frac{1}{2} \left(-\frac{\sqrt{3}}{2} \right) \right]$$

$$= \frac{3}{4}\sqrt{3}$$

$$\begin{array}{r} x+2 \\ x-2 \overline{) x^2+0x+1} \\ \underline{x^2-2x} \\ 2x+1 \\ \underline{2x-4} \\ 5 \end{array}$$

$$\int_0^1 \frac{x^2+1}{x-2} \, dx = \int_0^1 \left(x+2 + \frac{5}{x-2} \right) \, dx$$

$$= \left[\frac{1}{2}x^2 + 2x + 5 \ln |x-2| \right]_0^1$$

$$= \left(\frac{1}{2} + 2 + 0 \right) - (0 + 0 + 5 \ln 2)$$

$$= \frac{5}{2} - 5 \ln 2$$

$$\mathbf{i} \quad u = x - 2, \frac{du}{dx} = 1; \quad \frac{dv}{dx} = (x + 1)^3, v = \frac{1}{4}(x + 1)^4$$

$$\begin{aligned} \int_0^1 (x - 2)(x + 1)^3 dx &= \left[\frac{1}{4}(x - 2)(x + 1)^4 \right]_0^1 - \int_0^1 \frac{1}{4}(x + 1)^4 dx \\ &= \left[\frac{1}{4}(x - 2)(x + 1)^4 - \frac{1}{20}(x + 1)^5 \right]_0^1 \\ &= \left(-4 - \frac{8}{5} \right) - \left(-\frac{1}{2} - \frac{1}{20} \right) \\ &= -5\frac{1}{20} \end{aligned}$$

$$\begin{aligned} \mathbf{6} \quad \mathbf{a} \quad &= \int_1^2 \frac{x}{(x^2 + 2)^3} dx \\ &= \frac{1}{2} \int_1^2 \frac{2x}{(x^2 + 2)^3} dx \\ &= \frac{1}{2} \left[-\frac{1}{2}(x^2 + 2)^{-2} \right]_1^2 \\ &= -\frac{1}{4} \left(\frac{1}{36} - \frac{1}{9} \right) \\ &= \frac{1}{48} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad &= \int_2^4 \ln x dx \\ u &= \ln x, \frac{du}{dx} = \frac{1}{x}; \quad \frac{dv}{dx} = 1, v = x \\ &= [x \ln x]_2^4 - \int_2^4 dx \\ &= [x \ln x - x]_2^4 \\ &= (4 \ln 4 - 4) - (2 \ln 2 - 2) \\ &= 6 \ln 2 - 2 \end{aligned}$$

$$\begin{aligned} \mathbf{7} \quad &\int_3^6 \frac{ax^2 + b}{x} dx = \int_3^6 \left(ax + \frac{b}{x} \right) dx \\ &= \left[\frac{1}{2}ax^2 + b \ln |x| \right]_3^6 \\ &= (18a + b \ln 6) - \left(\frac{9}{2}a + b \ln 3 \right) \\ &\therefore \frac{27}{2}a + b \ln 2 = 18 + 5 \ln 2 \\ &a, b \text{ rational} \\ &\therefore b = 5, \frac{27}{2}a = 18 \\ &a = \frac{4}{3}, b = 5 \end{aligned}$$

$$\begin{aligned} \mathbf{8} \quad \mathbf{a} \quad &6 - 2e^x = 0 \\ x &= \ln 3 \quad \therefore (\ln 3, 0) \\ \mathbf{b} \quad &= \int_0^{\ln 3} (6 - 2e^x) dx \\ &= [6x - 2e^x]_0^{\ln 3} \\ &= (6 \ln 3 - 6) - (0 - 2) \\ &= 6 \ln 3 - 4 \end{aligned}$$

$$\begin{aligned} \mathbf{9} \quad u &= \cot x \quad \therefore \frac{du}{dx} = -\operatorname{cosec}^2 x \\ x = \frac{\pi}{6} &\Rightarrow u = \sqrt{3} \\ x = \frac{\pi}{4} &\Rightarrow u = 1 \\ \operatorname{cosec}^2 x &= 1 + \cot^2 x = 1 + u^2 \\ \therefore \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot^2 x \operatorname{cosec}^4 x dx &= \int_{\sqrt{3}}^1 u^2(1 + u^2) \times (-1) du \\ &= \int_1^{\sqrt{3}} (u^2 + u^4) du \\ &= \left[\frac{1}{3}u^3 + \frac{1}{5}u^5 \right]_1^{\sqrt{3}} \\ &= \left(\sqrt{3} + \frac{9}{5}\sqrt{3} \right) - \left(\frac{1}{3} + \frac{1}{5} \right) \\ &= \frac{14}{5}\sqrt{3} - \frac{8}{15} \\ &= \frac{2}{15}(21\sqrt{3} - 4) \end{aligned}$$

$$\begin{aligned} \mathbf{10} \quad \mathbf{a} \quad y = 0 &\Rightarrow 4 - t^2 = 0 \\ t &= \pm 2 \\ x = t + 1 &\therefore \frac{dx}{dt} = 1 \\ \therefore \text{area} &= \int_{-2}^2 y \times 1 dt \\ &= \int_{-2}^2 (4 - t^2) dt \\ \mathbf{b} \quad &= \left[4t - \frac{1}{3}t^3 \right]_{-2}^2 \\ &= \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) \\ &= 10\frac{2}{3} \end{aligned}$$

$$11 \quad a \quad \frac{d}{dx} (x^2 \sin 2x + 2kx \cos 2x - k \sin 2x)$$

$$= 2x \sin 2x + 2x^2 \cos 2x + 2k \cos 2x$$

$$-4kx \sin 2x - 2k \sin 2x$$

$$= 2x^2 \cos 2x + (2 - 4k)x \sin 2x$$

$$b \quad \text{let } k = \frac{1}{2}$$

$$\frac{d}{dx} (x^2 \sin 2x + x \cos 2x - \frac{1}{2} \sin 2x)$$

$$= 2x^2 \cos 2x$$

$$\therefore \int x^2 \cos 2x \, dx$$

$$= \frac{1}{2} (x^2 \sin 2x + x \cos 2x - \frac{1}{2} \sin 2x) + c$$

$$= \frac{1}{4} (2x^2 \sin 2x + 2x \cos 2x - \sin 2x) + c$$

$$12 \quad \text{curve meets } x\text{-axis when } \frac{\ln x}{x^2} = 0 \quad \therefore x = 1$$

$$\text{area} = \int_1^2 \frac{\ln x}{x^2} \, dx$$

$$u = \ln x, \quad \frac{du}{dx} = \frac{1}{x}; \quad \frac{dv}{dx} = x^{-2}, \quad v = -x^{-1}$$

$$\text{area} = \left[-\frac{\ln x}{x} \right]_1^2 + \int_1^2 x^{-2} \, dx$$

$$= \left[-\frac{\ln x}{x} - x^{-1} \right]_1^2$$

$$= \left(-\frac{1}{2} \ln 2 - \frac{1}{2} \right) - (0 - 1)$$

$$= \frac{1}{2} - \frac{1}{2} \ln 2$$

$$= \frac{1}{2} (1 - \ln 2)$$

$$13 \quad a \quad f(1) = 18, f(2) = 80, \\ f(-1) = -4, f(-2) = 0 \\ \therefore (x+2) \text{ is a factor}$$

$$\begin{array}{r} 3x^2 + 5x - 4 \\ x+2 \overline{) 3x^3 + 11x^2 + 8x - 4} \\ \underline{3x^3 + 6x^2} \\ 5x^2 + 8x \\ \underline{5x^2 + 10x} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

$$\therefore 3x^3 + 11x^2 + 8x - 4 = (x+2)(3x^2 + 5x - 2) \\ = (3x-1)(x+2)^2$$

$$b \quad \frac{x+16}{3x^3+11x^2+8x-4} \equiv \frac{A}{3x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$x+16 \equiv A(x+2)^2 + B(3x-1)(x+2) + C(3x-1)$$

$$x = \frac{1}{3} \quad \Rightarrow \quad \frac{49}{3} = \frac{49}{9}A \quad \Rightarrow \quad A = 3$$

$$x = -2 \quad \Rightarrow \quad 14 = -7C \quad \Rightarrow \quad C = -2$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + 3B \quad \Rightarrow \quad B = -1$$

$$\therefore f(x) \equiv \frac{3}{3x-1} - \frac{1}{x+2} - \frac{2}{(x+2)^2}$$

$$c \quad = \int_{-1}^0 \left(\frac{3}{3x-1} - \frac{1}{x+2} - \frac{2}{(x+2)^2} \right) dx$$

$$= [\ln |3x-1| - \ln |x+2| + 2(x+2)^{-1}]_{-1}^0$$

$$= (0 - \ln 2 + 1) - (\ln 4 - 0 + 2)$$

$$= -1 - \ln 2 - \ln 2^2$$

$$= -1 - 3 \ln 2$$

$$= -(1 + 3 \ln 2)$$