

1 Using an appropriate method, integrate with respect to x

a $(2x - 3)^4$

b $\operatorname{cosec}^2 \frac{1}{2}x$

c $2e^{4x-1}$

d $\frac{2(x-1)}{x(x+1)}$

e $\frac{3}{\cos^2 2x}$

f $x(x^2 + 3)^3$

g $\sec^4 x \tan x$

h $\sqrt{7+2x}$

i xe^{3x}

j $\frac{x+2}{x^2-2x-3}$

k $\frac{1}{4(x+1)^3}$

l $\tan^2 3x$

m $4\cos^2(2x+1)$

n $\frac{3x}{1-x^2}$

o $x \sin 2x$

p $\frac{x+4}{x+2}$

2 Evaluate

a $\int_1^2 6e^{2x-3} dx$

b $\int_0^{\frac{\pi}{3}} \tan x dx$

c $\int_{-2}^2 \frac{2}{x-3} dx$

d $\int_2^3 \frac{6+x}{4+3x-x^2} dx$

e $\int_1^2 (1-2x)^3 dx$

f $\int_0^{\frac{\pi}{3}} \sin^2 x \sin 2x dx$

3 Using the given substitution, evaluate

a $\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}} dx$

$x = 3 \sin u$

b $\int_0^1 x(1-3x)^3 dx$

$u = 1 - 3x$

c $\int_2^{2\sqrt{3}} \frac{1}{4+x^2} dx$

$x = 2 \tan u$

d $\int_{-1}^0 x^2 \sqrt{x+1} dx$

$u^2 = x + 1$

4 Integrate with respect to x

a $\frac{2}{5-3x}$

b $(x+1)e^{x^2+2x}$

c $\frac{1-x}{2x+1}$

d $\sin 3x \cos 2x$

e $3x(x-1)^4$

f $\frac{3x^2+6x+2}{x^2+3x+2}$

g $\frac{5}{\sqrt[3]{2x-1}}$

h $\frac{\cos x}{2+3\sin x}$

i $\frac{x^2}{\sqrt{x^3-1}}$

j $(2 - \cot x)^2$

k $\frac{6x-5}{(x-1)(2x-1)^2}$

l $x^2 e^{-x}$

5 Evaluate

a $\int_2^4 \frac{1}{3x-4} dx$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x \cot^2 x dx$

c $\int_0^1 \frac{7-x^2}{(2-x)^2(3-x)} dx$

d $\int_0^{\frac{\pi}{2}} x \cos \frac{1}{2}x dx$

e $\int_1^5 \frac{1}{\sqrt{4x+5}} dx$

f $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 2 \cos x \cos 3x dx$

g $\int_0^2 x\sqrt{2x^2+1} dx$

h $\int_0^1 \frac{x^2+1}{x-2} dx$

i $\int_0^1 (x-2)(x+1)^3 dx$

6 Find the exact area of the region enclosed by the given curve, the x -axis and the given ordinates.

a $y = \frac{x}{(x^2+2)^3}, \quad x=1, \quad x=2$

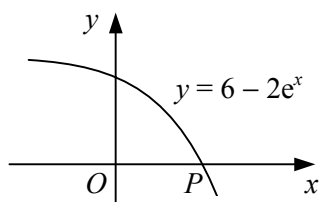
b $y = \ln x, \quad x=2, \quad x=4$

7 Given that

$$\int_3^6 \frac{ax^2+b}{x} dx = 18 + 5 \ln 2,$$

find the values of the rational constants a and b .

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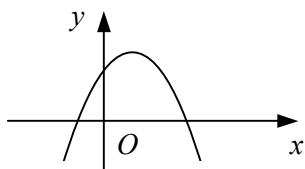
The diagram shows the curve with equation $y = 6 - 2e^x$.

- a Find the coordinates of the point P where the curve crosses the x -axis.
- b Show that the area of the region enclosed by the curve and the coordinate axes is $6 \ln 3 - 4$.

9 Using the substitution $u = \cot x$, show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot^2 x \operatorname{cosec}^4 x \, dx = \frac{2}{15} (21\sqrt{3} - 4).$$

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The diagram shows the curve with parametric equations

$$x = t + 1, \quad y = 4 - t^2.$$

- a Show that the area of the region bounded by the curve and the x -axis is given by

$$\int_{-2}^2 (4 - t^2) \, dt.$$

- b Hence, find the area of this region.

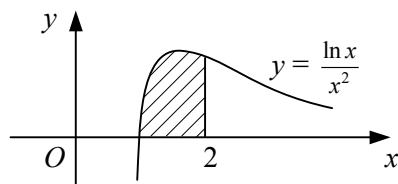
11 a Given that k is a constant, show that

$$\frac{d}{dx} (x^2 \sin 2x + 2kx \cos 2x - k \sin 2x) = 2x^2 \cos 2x + (2 - 4k)x \sin 2x.$$

- b Using your answer to part a with a suitable value of k , or otherwise, find

$$\int x^2 \cos 2x \, dx.$$

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The shaded region in the diagram is bounded by the curve with equation $y = \frac{\ln x}{x^2}$, the x -axis and the line $x = 2$. Use integration by parts to show that the area of the shaded region is $\frac{1}{2}(1 - \ln 2)$.

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$$f(x) \equiv \frac{x+16}{3x^3+11x^2+8x-4}$$

- a Factorise completely $3x^3 + 11x^2 + 8x - 4$.
- b Express $f(x)$ in partial fractions.
- c Show that $\int_{-1}^0 f(x) \, dx = -(1 + 3 \ln 2)$.