C4 INTEGRATION *Answers - Worksheet I*

1
$$= \pi\int_{\frac{1}{2}}^{2} \left(\frac{2}{x}\right)^2 \, dx$$
$$= \pi\int_{\frac{1}{2}}^{2} 4x^{-2} \, dx$$
$$= \pi[-4x^{-1}]_{\frac{1}{2}}^{2}$$
$$= \pi[-2 - (-8)]$$
$$= 6\pi$$

2
$$= \pi\int_{0}^{2} (x^2 + 3)^2 \, dx$$
$$= \pi\int_{0}^{2} (x^4 + 6x^2 + 9) \, dx$$
$$= \pi[\tfrac{1}{5}x^5 + 2x^3 + 9x]_{0}^{2}$$
$$= \pi[(\tfrac{32}{5} + 16 + 18) - (0)]$$
$$= \tfrac{202}{5}\pi \approx 127$$

3 **a**
$$= \pi\int_{0}^{1} (2e^{\frac{x}{2}})^2 \, dx$$
$$= \pi\int_{0}^{1} 4e^{x} \, dx$$
$$= \pi[4e^{x}]_{0}^{1}$$
$$= \pi(4e - 4)$$
$$= 4\pi(e - 1)$$

b
$$= \pi\int_{-2}^{-1} \left(\frac{3}{x^2}\right)^2 \, dx$$
$$= \pi\int_{-2}^{-1} 9x^{-4} \, dx$$
$$= \pi[-3x^{-3}]_{-2}^{-1}$$
$$= \pi(3 - \tfrac{3}{8})$$
$$= \tfrac{21}{8}\pi$$

c
$$= \pi\int_{3}^{9} \left(1 + \frac{1}{x}\right)^2 \, dx$$
$$= \pi\int_{3}^{9} (1 + 2x^{-1} + x^{-2}) \, dx$$
$$= \pi[x + 2\ln|x| - x^{-1}]_{3}^{9}$$
$$= \pi[(9 + 2\ln 9 - \tfrac{1}{9}) - (3 + 2\ln 3 - \tfrac{1}{3})]$$
$$= \pi(6\tfrac{2}{9} + 2\ln 3)$$

d
$$= \pi\int_{1}^{2} \left(3x + \frac{1}{x}\right)^2 \, dx$$
$$= \pi\int_{1}^{2} (9x^2 + 6 + x^{-2}) \, dx$$
$$= \pi[3x^3 + 6x - x^{-1}]_{1}^{2}$$
$$= \pi[(24 + 12 - \tfrac{1}{2}) - (3 + 6 - 1)]$$
$$= \tfrac{55}{2}\pi$$

e
$$= \pi\int_{2}^{6} \left(\frac{1}{\sqrt{x+2}}\right)^2 \, dx$$
$$= \pi\int_{2}^{6} \frac{1}{x+2} \, dx$$
$$= \pi[\ln|x + 2|]_{2}^{6}$$
$$= \pi(\ln 8 - \ln 4)$$
$$= \pi\ln 2$$

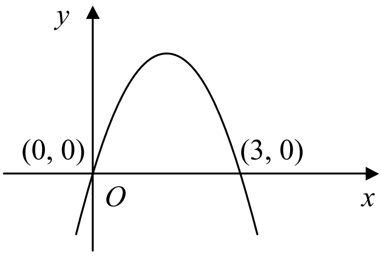
f
$$= \pi\int_{-1}^{1} (e^{1-x})^2 \, dx$$
$$= \pi\int_{-1}^{1} e^{2-2x} \, dx$$
$$= \pi[-\tfrac{1}{2}e^{2-2x}]_{-1}^{1}$$
$$= -\tfrac{1}{2}\pi(1 - e^4)$$
$$= \tfrac{1}{2}\pi(e^4 - 1)$$

4 **a**
$$= \int_{0}^{2} \frac{4}{x+2} \, dx$$
$$= [4\ln|x + 2|]_{0}^{2}$$
$$= 4(\ln 4 - \ln 2) = 4\ln 2$$

b
$$= \pi\int_{0}^{2} \left(\frac{4}{x+2}\right)^2 \, dx$$
$$= \pi\int_{0}^{2} 16(x + 2)^{-2} \, dx$$
$$= \pi[-16(x + 2)^{-1}]_{0}^{2}$$
$$= \pi[-4 - (-8)] = 4\pi$$

5
$$= \pi\int_{1}^{3} (2x^{\frac{1}{2}} + x^{-\frac{1}{2}})^2 \, dx$$
$$= \pi\int_{1}^{3} (4x + 4 + x^{-1}) \, dx$$
$$= \pi[2x^2 + 4x + \ln|x|]_{1}^{3}$$
$$= \pi[(18 + 12 + \ln 3) - (2 + 4 + 0)]$$
$$= \pi(24 + \ln 3)$$

6 **a**



b $= \pi\int_0^3 (3x - x^2)^2 \, dx$

$= \pi\int_0^3 (9x^2 - 6x^3 + x^4) \, dx$

$= \pi[3x^3 - \frac{3}{2}x^4 + \frac{1}{5}x^5]_0^3$

$= \pi[(81 - \frac{243}{2} + \frac{243}{5}) - (0)]$

$= \frac{81}{10}\pi$

7 **a** $y = 0 \;\therefore\; x = \frac{1}{3}$

$\therefore\; (\frac{1}{3}, 0)$

b $= \int_{\frac{1}{3}}^3 (3 - \frac{1}{x}) \, dx$

$= [3x - \ln|x|]_{\frac{1}{3}}^3$

$= (9 - \ln 3) - (1 - \ln \frac{1}{3})$

$= 8 - 2\ln 3$

c $= \pi\int_{\frac{1}{3}}^3 (3 - \frac{1}{x})^2 \, dx$

$= \pi\int_{\frac{1}{3}}^3 (9 - 6x^{-1} + x^{-2}) \, dx$

$= \pi[9x - 6\ln|x| - x^{-1}]_{\frac{1}{3}}^3$

$= \pi[(27 - 6\ln 3 - \frac{1}{3}) - (3 - 6\ln\frac{1}{3} - 3)]$

$= \pi(26\frac{2}{3} - 12\ln 3)$

8 **a** $x - \frac{1}{x} = 0$

$x^2 = 1$

$x = \pm 1 \;\therefore\; (-1, 0)$ and $(1, 0)$

b $= \int_1^3 (x - \frac{1}{x}) \, dx$

$= [\frac{1}{2}x^2 - \ln|x|]_1^3$

$= (\frac{9}{2} - \ln 3) - (\frac{1}{2} - 0)$

$= 4 - \ln 3$

c $= \pi\int_1^3 (x - \frac{1}{x})^2 \, dx$

$= \pi\int_1^3 (x^2 - 2 + x^{-2}) \, dx$

$= \pi[\frac{1}{3}x^3 - 2x - x^{-1}]_1^3$

$= \pi[(9 - 6 - \frac{1}{3}) - (\frac{1}{3} - 2 - 1)]$

$= \frac{16}{3}\pi$