# C4 INTEGRATION Worksheet L

**1** **a** Express $\dfrac{x+4}{(1+x)(2-x)}$ in partial fractions.

**b** Given that $y = 2$ when $x = 3$, solve the differential equation

$$\frac{dy}{dx} = \frac{y(x+4)}{(1+x)(2-x)}.$$

**2** Given that $y = 0$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^{x+y}\cos x.$$

**3** Given that $\dfrac{dy}{dx}$ is inversely proportional to $x$ and that $y = 4$ and $\dfrac{dy}{dx} = \frac{5}{3}$ when $x = 3$, find an expression for $y$ in terms of $x$.

**4** A quantity has the value $N$ at time $t$ hours and is increasing at a rate proportional to $N$.

**a** Write down a differential equation relating $N$ and $t$.

**b** By solving your differential equation, show that

$$N = Ae^{kt},$$

where $A$ and $k$ are constants and $k$ is positive.

Given that when $t = 0$, $N = 40$ and that when $t = 5$, $N = 60$,

**c** find the values of $A$ and $k$,

**d** find the value of $N$ when $t = 12$.

**5** A cube is increasing in size and has volume $V$ cm$^3$ and surface area $A$ cm$^2$ at time $t$ seconds.

**a** Show that

$$\frac{dV}{dA} = k\sqrt{A},$$

where $k$ is a positive constant.

Given that the rate at which the volume of the cube is increasing is proportional to its surface area and that when $t = 10$, $A = 100$ and $\dfrac{dA}{dt} = 5$,

**b** show that

$$A = \tfrac{1}{16}(t + 30)^2.$$

**6** At time $t = 0$, a piece of radioactive material has mass 24 g. Its mass after $t$ days is $m$ grams and is decreasing at a rate proportional to $m$.

**a** By forming and solving a suitable differential equation, show that

$$m = 24e^{-kt},$$

where $k$ is a positive constant.

After 20 days, the mass of the material is found to be 22.6 g.

**b** Find the value of $k$.

**c** Find the rate at which the mass is decreasing after 20 days.

**d** Find how long it takes for the mass of the material to be halved.

**7** A quantity has the value $P$ at time $t$ seconds and is decreasing at a rate proportional to $\sqrt{P}$ .

**a** By forming and solving a suitable differential equation, show that

$$P = (a - bt)^2,$$

where $a$ and $b$ are constants.

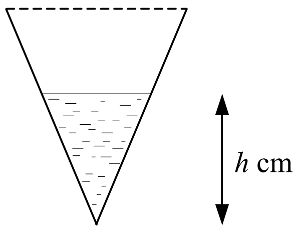
Given that when $t = 0$, $P = 400$,

**b** find the value of $a$.

Given also that when $t = 30$, $P = 100$,

**c** find the value of $P$ when $t = 50$.

**8**




The diagram shows a container in the shape of a right-circular cone. A quantity of water is poured into the container but this then leaks out from a small hole at its vertex.

In a model of the situation it is assumed that the rate at which the volume of water in the container, $V$ cm$^3$, decreases is proportional to $V$. Given that the depth of the water is $h$ cm at time $t$ minutes,

**a** show that

$$\frac{dh}{dt} = -kh,$$

where $k$ is a positive constant.

Given also that $h = 12$ when $t = 0$ and that $h = 10$ when $t = 20$,

**b** show that

$$h = 12e^{-kt},$$

and find the value of $k$,

**c** find the value of $t$ when $h = 6$.

**9** **a** Express $\frac{1}{(1+x)(1-x)}$ in partial fractions.

In an industrial process, the mass of a chemical, $m$ kg, produced after $t$ hours is modelled by the differential equation

$$\frac{dm}{dt} = ke^{-t}(1 + m)(1 - m),$$

where $k$ is a positive constant.

Given that when $t = 0$, $m = 0$ and that the initial rate at which the chemical is produced is 0.5 kg per hour,

**b** find the value of $k$,

**c** show that, for $0 \le m < 1$, $\ln\left(\frac{1+m}{1-m}\right) = 1 - e^{-t}$.

**d** find the time taken to produce 0.1 kg of the chemical,

**e** show that however long the process is allowed to run, the maximum amount of the chemical that will be produced is about 462 g.