C4 INTEGRATION Worksheet M

1 Use the trapezium rule with n intervals of equal width to estimate the value of each integral.

a $\int_1^5 x \ln(x+1)\, dx$ $n = 2$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\, dx$ $n = 2$

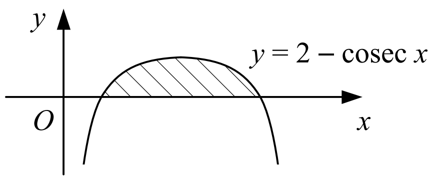
c $\int_{-2}^{2} e^{\frac{x^2}{10}}\, dx$ $n = 4$

d $\int_0^1 \arccos(x^2 - 1)\, dx$ $n = 4$

e $\int_0^{0.5} \sec^2(2x - 1)\, dx$ $n = 5$

f $\int_0^6 x^3 e^{-x}\, dx$ $n = 6$

2



The diagram shows the curve with equation $y = 2 - \text{cosec}\, x$, $0 < x < \pi$.

a Find the exact x-coordinates of the points where the curve crosses the x-axis.

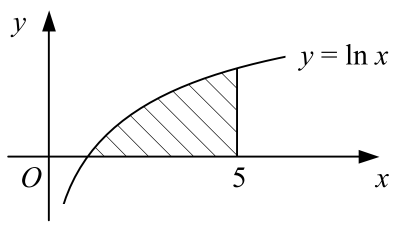
b Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region bounded by the curve and the x-axis.

3 $\text{f}(x) \equiv \frac{\pi}{6} + \arcsin(\frac{1}{2}x)$, $x \in \mathbb{R}$, $-2 \le x \le 2$.

a Use the trapezium rule with three strips to estimate the value of the integral $I = \int_{-1}^{2} \text{f}(x)\, dx$.

b Use the trapezium rule with six strips to find an improved estimate for I.

4



The shaded region in the diagram is bounded by the curve $y = \ln x$, the x-axis and the line $x = 5$.

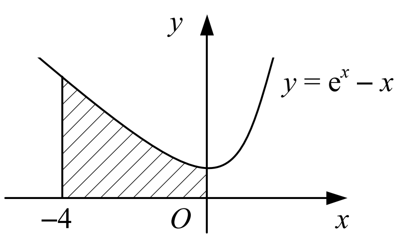
a Estimate the area of the shaded region to 3 decimal places using the trapezium rule with

i 2 strips **ii** 4 strips **iii** 8 strips

b By considering your answers to part **a**, suggest a more accurate value for the area of the shaded region correct to 3 decimal places.

c Use integration to find the true value of the area correct to 3 decimal places.

5



The shaded region in the diagram is bounded by the curve $y = e^x - x$, the coordinate axes and the line $x = -4$. Use the trapezium rule with five equally-spaced ordinates to estimate the volume of the solid formed when the shaded region is rotated completely about the x-axis.