C4 INTEGRATION Worksheet N

1 Show that

$$\int_2^7 \frac{8}{4x-3}\ \mathrm{d}x = \ln 25. \qquad \textbf{(4)}$$

2 Given that $y = \frac{\pi}{4}$ when $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x \sec y \operatorname{cosec}^3 y. \qquad \textbf{(7)}$$

3 **a** Use the trapezium rule with three intervals of equal width to find an approximate value for the integral

$$\int_0^{1.5} \mathrm{e}^{x^2-1}\ \mathrm{d}x. \qquad \textbf{(4)}$$

b Use the trapezium rule with six intervals of equal width to find an improved approximation for the above integral. **(2)**

4

$$\mathrm{f}(x) \equiv \frac{3(2-x)}{(1-2x)^2(1+x)}.$$

a Express $\mathrm{f}(x)$ in partial fractions. **(4)**

b Show that

$$\int_1^2 \mathrm{f}(x)\ \mathrm{d}x = 1 - \ln 2. \qquad \textbf{(6)}$$

5 The rate of growth in the number of yeast cells, N, present in a culture after t hours is proportional to N.

a By forming and solving a differential equation, show that

$$N = A\mathrm{e}^{kt},$$

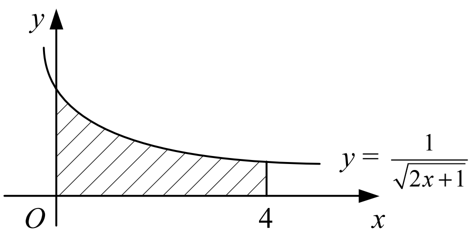
where A and k are positive constants. **(4)**

Initially there are 200 yeast cells in the culture and after 2 hours there are 3000 yeast cells in the culture. Find, to the nearest minute, after how long

b there are 10 000 yeast cells in the culture, **(5)**

c the number of yeast cells is increasing at the rate of 5 per second. **(4)**

6



The diagram shows part of the curve with equation $y = \frac{1}{\sqrt{2x+1}}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 4$.

a Find the area of the shaded region. **(4)**

The shaded region is rotated through four right angles about the x-axis.

b Find the volume of the solid formed, giving your answer in the form $\pi \ln k$. **(5)**

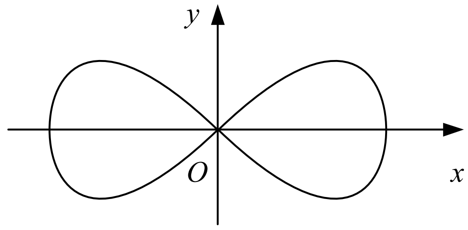
7 Using the substitution $u^2 = x + 3$, show that

$$\int_0^1 x\sqrt{x+3}\ \mathrm{d}x = k(3\sqrt{3} - 4),$$

where k is a rational number to be found. **(7)**

8 **a** Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$2\sin A\cos B \equiv \sin(A + B) + \sin(A - B).$$ **(2)**



The diagram shows the curve given by the parametric equations

$$x = 2\sin 2t, \quad y = \sin 4t, \quad 0 \le t < \pi.$$

b Show that the total area enclosed by the two loops of the curve is given by

$$\int_0^{\frac{\pi}{4}} 16\sin 4t\cos 2t\ \mathrm{d}t.$$ **(4)**

c Evaluate this integral. **(5)**

9 $$\mathrm{f}(x) \equiv \frac{x^2 - 22}{(x+2)(x-4)}.$$

a Find the values of the constants A, B and C such that

$$\mathrm{f}(x) \equiv A + \frac{B}{x+2} + \frac{C}{x-4}.$$ **(3)**

The finite region R is bounded by the curve $y = \mathrm{f}(x)$, the coordinate axes and the line $x = 2$.

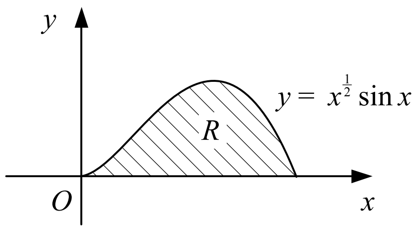
b Find the area of R, giving your answer in the form $p + \ln q$, where p and q are integers. **(5)**

10 **a** Find $\int \sin^2 x\ \mathrm{d}x$. **(4)**

b Use integration by parts to show that

$$\int x\sin^2 x\ \mathrm{d}x = \tfrac{1}{8}(2x^2 - 2x\sin 2x - \cos 2x) + c,$$

where c is an arbitrary constant. **(4)**



The diagram shows the curve with equation $y = x^{\frac{1}{2}}\sin x$, $0 \le x \le \pi$.

The finite region R, bounded by the curve and the x-axis, is rotated through 2π radians about the x-axis.

c Find the volume of the solid formed, giving your answer in terms of π. **(3)**