SERIES

C4

- 1 Find the binomial expansion of each of the following in ascending powers of x up to and including the term in x^3 , for |x| < 1.
 - a $(1+x)^{-1}$ b $(1+x)^{\frac{1}{2}}$ c $2(1+x)^{-3}$ d $(1+x)^{\frac{2}{3}}$ e $\sqrt[3]{1-x}$ f $\frac{1}{(1+x)^2}$ g $\frac{1}{4(1-x)^4}$ h $\frac{3}{\sqrt{1-x}}$
- 2 Expand each of the following in ascending powers of x up to and including the term in x^3 and state the set of values of x for which each expansion is valid.
 - **a** $(1+2x)^{\frac{1}{2}}$ **b** $(1-3x)^{-1}$ **c** $(1-4x)^{-\frac{1}{2}}$ **d** $(1+\frac{1}{2}x)^{-3}$ **e** $(1-6x)^{\frac{1}{3}}$ **f** $(1+\frac{1}{4}x)^{-4}$ **g** $(1+2x)^{\frac{3}{2}}$ **h** $(1-3x)^{-\frac{4}{3}}$
- **3** a Expand $(1 2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3 .
 - **b** By substituting a suitable value of x in your expansion, find an estimate for $\sqrt{0.98}$
 - **c** Show that $\sqrt{0.98} = \frac{7}{10}\sqrt{2}$ and hence find the value of $\sqrt{2}$ correct to 8 significant figures.
- 4 Expand each of the following in ascending powers of x up to and including the term in x^3 and state the set of values of x for which each expansion is valid.
 - **a** $(2+x)^{-1}$ **b** $(4+x)^{\frac{1}{2}}$ **c** $(3-x)^{-3}$ **d** $(9+3x)^{\frac{1}{2}}$ **e** $(8-24x)^{\frac{1}{3}}$ **f** $(4-3x)^{-1}$ **g** $(4+6x)^{-\frac{1}{2}}$ **h** $(3+2x)^{-2}$
- 5 a Expand $(1 + 2x)^{-1}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3 .
 - **b** Hence find the series expansion of $\frac{1-x}{1+2x}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3 .
- 6 Find the first four terms in the series expansion in ascending powers of x of each of the following and state the set of values of x for which each expansion is valid.
 - **a** $\frac{1+3x}{1-x}$ **b** $\frac{2x-1}{(1+4x)^2}$ **c** $\frac{3+x}{2-x}$ **d** $\frac{1-x}{\sqrt{1+2x}}$
- 7 **a** Express $\frac{x-2}{(1-x)(1-2x)}$ in partial fractions.
 - **b** Hence find the series expansion of $\frac{x-2}{(1-x)(1-2x)}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid.
- 8 By first expressing f(x) in partial fractions, find the series expansion of f(x) in ascending powers of x up to and including the term in x^3 and state the set of values of x for which it is valid.

a
$$f(x) \equiv \frac{4}{(1+x)(1-3x)}$$

b $f(x) \equiv \frac{1-6x}{1+3x-4x^2}$
c $f(x) \equiv \frac{5}{2-3x-2x^2}$
d $f(x) \equiv \frac{7x-3}{x^2-4x+3}$
e $f(x) \equiv \frac{3+5x}{(1+3x)(1+x)^2}$
f $f(x) \equiv \frac{2x^2+4}{2x^2+x-1}$

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