

- 1** **a** $= 1 + \left(\frac{1}{2}\right)(-x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}(-x)^3 + \dots$
 $= 1 - \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \dots$
- b** when $x = 0.01$, $(1 - x)^{\frac{1}{2}} \approx 1 - \frac{1}{2}(0.01) - \frac{1}{8}(0.01)^2 - \frac{1}{16}(0.01)^3$
 $= 1 - 0.005 - 0.000\,012\,5 - 0.000\,000\,062\,5 = 0.994\,987\,437\,5$
 $(1 - 0.01)^{\frac{1}{2}} = \sqrt{0.99} = \sqrt{\frac{9 \times 11}{100}} = \frac{3}{10} \sqrt{11}$
 $\therefore \sqrt{11} = \frac{10}{3} \times 0.994\,987\,437\,5 = 3.316\,624\,79 \text{ (9sf)}$
- 2** **a** $a = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(8)^2 = -8$, $b = \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}(8)^3 = 32$
- b** when $x = 0.01$, $(1 + 8x)^{\frac{1}{2}} \approx 1 + 4(0.01) - 8(0.01)^2 + 32(0.01)^3$
 $= 1 + 0.04 - 0.000\,8 + 0.000\,032 = 1.039\,232$
 $(1 + 0.08)^{\frac{1}{2}} = \sqrt{1.08} = \sqrt{\frac{36 \times 3}{100}} = \frac{3}{5} \sqrt{3}$
 $\therefore \sqrt{3} = \frac{5}{3} \times 1.039\,232 = 1.732\,05 \text{ (5dp)}$
- 3** **a** $= 9^{\frac{1}{2}}\left(1 - \frac{2}{3}x\right)^{\frac{1}{2}} = 3\left(1 - \frac{2}{3}x\right)^{\frac{1}{2}}$
 $= 3\left[1 + \left(\frac{1}{2}\right)\left(-\frac{2}{3}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(-\frac{2}{3}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3 \times 2}\left(-\frac{2}{3}x\right)^3 + \dots\right]$
 $= 3 - x - \frac{1}{6}x^2 - \frac{1}{18}x^3 + \dots$
- b** let $x = 0.05$
 $\sqrt{8.7} \approx 3 - 0.05 - \frac{1}{6}(0.05)^2 - \frac{1}{18}(0.05)^3$
 $= 2.949\,576 \text{ (7sf)}$
- 4** **a** $= 1 + \left(\frac{1}{3}\right)(6x) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}(6x)^2 + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3 \times 2}(6x)^3 + \dots$
 $= 1 + 2x - 4x^2 + \frac{40}{3}x^3 + \dots$
- b** when $x = 0.004$, $(1 + 6x)^{\frac{1}{3}} \approx 1 + 2(0.004) - 4(0.004)^2 + \frac{40}{3}(0.004)^3$
 $= 1.007\,936\,853$
 $(1 + 0.024)^{\frac{1}{3}} = \sqrt[3]{1.024} = \sqrt[3]{\frac{512 \times 2}{1000}} = \frac{4}{5} \sqrt[3]{2}$
 $\therefore \sqrt[3]{2} = \frac{5}{4} \times 1.007\,936\,853 = 1.259\,921 \text{ (7sf)}$
- 5** **a** $= 1 + (-3)(2x) + \frac{(-3)(-4)}{2}(2x)^2 + \frac{(-3)(-4)(-5)}{3 \times 2}(2x)^3 + \dots$
 $= 1 - 6x + 24x^2 - 80x^3 + \dots$, $|2x| < 1 \therefore \text{valid for } |x| < \frac{1}{2}$
- b** $= (1 + 3x)(1 + 2x)^{-3} = (1 + 3x)(1 - 6x + 24x^2 - 80x^3 + \dots)$
 $= 1 - 6x + 24x^2 - 80x^3 + 3x - 18x^2 + 72x^3 + \dots$
 $= 1 - 3x + 6x^2 - 8x^3 + \dots$
- 6** $\frac{2+x}{\sqrt{4-2x}} = (2+x)(4-2x)^{-\frac{1}{2}} = (2+x) \times 4^{-\frac{1}{2}}\left(1 - \frac{1}{2}x\right)^{-\frac{1}{2}}$
 $= (2+x) \times \frac{1}{2} \left[1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{1}{2}x\right)^2 + \dots\right]$
 $= (2+x)\left(\frac{1}{2} + \frac{1}{8}x + \frac{3}{64}x^2 + \dots\right)$
 $\therefore \text{coeff of } x^2 = \left(2 \times \frac{3}{64}\right) + \left(1 \times \frac{1}{8}\right) = \frac{7}{32}$

7 a $\frac{2-11x}{1-5x+4x^2} \equiv \frac{A}{1-x} + \frac{B}{1-4x}$
 $2-11x \equiv A(1-4x) + B(1-x)$
 $x=1 \Rightarrow -9 = -3A \Rightarrow A=3$
 $x=\frac{1}{4} \Rightarrow -\frac{3}{4} = \frac{3}{4}B \Rightarrow B=-1$

b $\frac{2-11x}{1-5x+4x^2} \equiv \frac{3}{1-x} - \frac{1}{1-4x}$
 $\frac{3}{1-x} = 3(1-x)^{-1} = 3[1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-x)^3 + \dots]$
 $= 3 + 3x + 3x^2 + 3x^3 + \dots, | -x | < 1 \therefore |x| < 1$
 $\frac{1}{1-4x} = (1-4x)^{-1} = 1 + (-1)(-4x) + \frac{(-1)(-2)}{2}(-4x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-4x)^3 + \dots$
 $= 1 + 4x + 16x^2 + 64x^3 + \dots, | -4x | < 1 \therefore |x| < \frac{1}{4}$
 $\therefore \frac{2-11x}{1-5x+4x^2} = (3 + 3x + 3x^2 + 3x^3 + \dots) - (1 + 4x + 16x^2 + 64x^3 + \dots)$
 $= 2 - x - 13x^2 - 61x^3 + \dots, \text{ valid for } |x| < \frac{1}{4}$

8 a $\frac{4-17x}{(1+2x)(1-3x)^2} \equiv \frac{A}{1+2x} + \frac{B}{1-3x} + \frac{C}{(1-3x)^2}$
 $4-17x \equiv A(1-3x)^2 + B(1+2x)(1-3x) + C(1+2x)$
 $x = -\frac{1}{2} \Rightarrow \frac{25}{2} = \frac{25}{4}A \Rightarrow A=2$
 $x = \frac{1}{3} \Rightarrow -\frac{5}{3} = \frac{5}{3}C \Rightarrow C=-1$
coeffs of $x^2 \Rightarrow 0 = 9A - 6B \Rightarrow B=3$
 $\therefore f(x) \equiv \frac{2}{1+2x} + \frac{3}{1-3x} - \frac{1}{(1-3x)^2}$

b $\frac{2}{1+2x} = 2(1+2x)^{-1} = 2[1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(2x)^3 + \dots]$
 $= 2 - 4x + 8x^2 - 16x^3 + \dots$
 $\frac{3}{1-3x} = 3(1-3x)^{-1} = 3[1 + (-1)(-3x) + \frac{(-1)(-2)}{2}(-3x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-3x)^3 + \dots]$
 $= 3 + 9x + 27x^2 + 81x^3 + \dots$
 $\frac{1}{(1-3x)^2} = (1-3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(-3x)^3 + \dots$
 $= 1 + 6x + 27x^2 + 108x^3 + \dots$
 $f(x) = (2 - 4x + 8x^2 - 16x^3 + \dots) + (3 + 9x + 27x^2 + 81x^3 + \dots) - (1 + 6x + 27x^2 + 108x^3 + \dots)$
 $= 4 - x + 8x^2 - 43x^3 + \dots$

9 a $(1+ax)^b = 1 + b(ax) + \frac{b(b-1)}{2}(ax)^2 + \dots$
 $\therefore ab = -6 \quad (1)$
and $\frac{1}{2}a^2b(b-1) = 24 \quad (2)$
 $(1) \Rightarrow a = -\frac{6}{b}$
sub. (2) $\Rightarrow \frac{18}{b}(b-1) = 24$
 $18b - 18 = 24b$
 $b = -3$
 $a = 2$

b $= \frac{(-3)(-4)(-5)}{3 \times 2}(2)^3 = -80$