C4 SERIES Worksheet C

1 **a** Expand $(1 - 4x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid. **(4)**

b By substituting $x = 0.01$ in your expansion, find the value of $\sqrt{6}$ to 6 significant figures. **(3)**

2
$$\text{f}(x) \equiv \frac{4}{1+2x-3x^2}.$$

a Express f(x) in partial fractions. **(3)**

b Hence, or otherwise, find the series expansion of f(x) in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid. **(5)**

3 **a** Expand $(2 - x)^{-2}$, $|x| < 2$, in ascending powers of x up to and including the term in x^3. **(4)**

b Hence, find the coefficient of x^3 in the series expansion of $\dfrac{3-x}{(2-x)^2}$. **(2)**

4
$$\text{f}(x) \equiv \frac{4}{\sqrt{1+\frac{2}{3}x}}, \quad -\tfrac{3}{2} < x < \tfrac{3}{2}.$$

a Show that $\text{f}(\frac{1}{10}) = \sqrt{15}$. **(2)**

b Expand f(x) in ascending powers of x up to and including the term in x^2. **(3)**

c Use your expansion to obtain an approximation for $\sqrt{15}$, giving your answer as an exact, simplified fraction. **(2)**

d Show that $3\frac{55}{63}$ is a more accurate approximation for $\sqrt{15}$. **(2)**

5 **a** Expand $(1 - x)^{\frac{1}{3}}$, $|x| < 1$, in ascending powers of x up to and including the term in x^2. **(3)**

b By substituting $x = 10^{-3}$ in your expansion, find the cube root of 37 correct to 9 significant figures. **(3)**

6 The series expansion of $(1 + 5x)^{\frac{3}{5}}$, in ascending powers of x up to and including the term in x^3, is
$$1 + 3x + px^2 + qx^3, \quad |x| < \tfrac{1}{5}.$$

a Find the values of the constants p and q. **(4)**

b Use the expansion with a suitable value of x to find an approximate value for $(1.1)^{\frac{3}{5}}$. **(2)**

c Obtain the value of $(1.1)^{\frac{3}{5}}$ from your calculator and hence find the percentage error in your answer to part **b**. **(2)**

7 **a** Find the values of A, B and C such that
$$\frac{8-6x^2}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}.$$ **(4)**

b Hence find the series expansion of $\dfrac{8-6x^2}{(1+x)(2+x)^2}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(7)**

8 **a** Expand $(1 - 2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^2. **(3)**

b By substituting $x = 0.0008$ in your expansion, find the square root of 39 correct to 7 significant figures. **(4)**

9 **a** Find the series expansion of $(1 + 8x)^{\frac{1}{3}}$, $|x| < \frac{1}{8}$, in ascending powers of x up to and including the term in x^2, simplifying each term. **(3)**

b Find the exact fraction k such that

$$\sqrt[3]{5} = k\sqrt[3]{1.08}$$ **(2)**

c Hence, use your answer to part **a** together with a suitable value of x to obtain an estimate for $\sqrt[3]{5}$, giving your answer to 4 significant figures. **(3)**

10

$$\text{f}(x) \equiv \frac{6x}{x^2 - 4x + 3}, \quad |x| < 1.$$

a Express f(x) in partial fractions. **(3)**

b Show that for small values of x,

$$\text{f}(x) \approx 2x + \tfrac{8}{3}x^2 + \tfrac{26}{9}x^3.$$ **(5)**

11 **a** Find the binomial expansion of $(4 + x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 and state the set of values of x for which the expansion is valid. **(4)**

b By substituting $x = \frac{1}{20}$ in your expansion, find an estimate for $\sqrt{5}$, giving your answer to 9 significant figures. **(3)**

c Obtain the value of $\sqrt{5}$ from your calculator and hence comment on the accuracy of the estimate found in part **b**. **(2)**

12 **a** Expand $(1 + 2x)^{-\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3. **(4)**

b Hence, show that for small values of x,

$$\frac{2 - 5x}{\sqrt{1 + 2x}} \approx 2 - 7x + 8x^2 - \tfrac{25}{2}x^3.$$ **(3)**

c Solve the equation

$$\frac{2 - 5x}{\sqrt{1 + 2x}} = \sqrt{3}.$$ **(3)**

d Use your answers to parts **b** and **c** to find an approximate value for $\sqrt{3}$. **(2)**

13 **a** Expand $(1 + x)^{-1}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3. **(2)**

b Hence, write down the first four terms in the expansion in ascending powers of x of $(1 + bx)^{-1}$, where b is a constant, for $|bx| < 1$. **(1)**

Given that in the series expansion of

$$\frac{1 + ax}{1 + bx}, \quad |bx| < 1,$$

the coefficient of x is -4 and the coefficient of x^2 is 12,

c find the values of the constants a and b, **(5)**

d find the coefficient of x^3 in the expansion. **(2)**