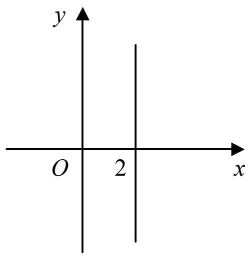
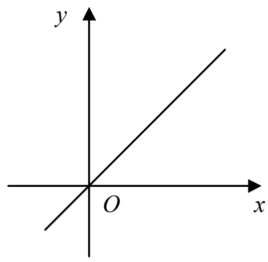
C4 VECTORS — Answers - Worksheet C

Note: For this worksheet especially, there may be alternative correct answers

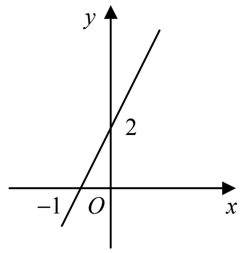
1 **a**



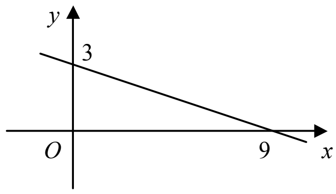
b



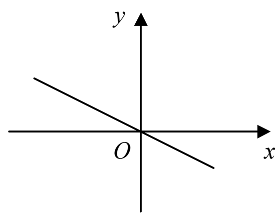
c



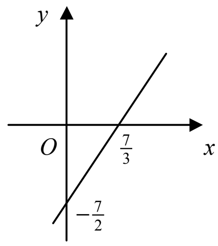
d



e



f $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + s(2\mathbf{i} + 3\mathbf{j})$



2 **a** $\mathbf{r} = -\mathbf{i} + \mathbf{j} + s(3\mathbf{i} - 2\mathbf{j})$

b $\mathbf{r} = 4\mathbf{j} + s\mathbf{i}$

c $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + s(\mathbf{i} + 5\mathbf{j})$

3 **a** $\text{dir}^n = \begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$\therefore\ \mathbf{r} = \begin{pmatrix}1\\0\end{pmatrix} + s\begin{pmatrix}2\\1\end{pmatrix}$

b $\text{dir}^n = \begin{pmatrix}-1\\1\end{pmatrix} - \begin{pmatrix}-3\\4\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}$

$\therefore\ \mathbf{r} = \begin{pmatrix}-3\\4\end{pmatrix} + s\begin{pmatrix}2\\-3\end{pmatrix}$

c $\text{dir}^n = \begin{pmatrix}-2\\3\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}-4\\5\end{pmatrix}$

$\therefore\ \mathbf{r} = \begin{pmatrix}2\\-2\end{pmatrix} + s\begin{pmatrix}-4\\5\end{pmatrix}$

4 **a** $-1 + 2\lambda = 5\ \therefore\ \lambda = 3$

$3 + c\lambda = 3 + 3c = 0\ \therefore\ c = -1$

b $c\mathbf{i} + 2\mathbf{j} = k(6\mathbf{i} + 3\mathbf{j})$

$\therefore\ k = \frac{2}{3}$

$\therefore\ c = 4$

5 **a** $\mathbf{r} = -\mathbf{i} + s\mathbf{j}$

b $\mathbf{r} = s(\mathbf{i} + 2\mathbf{j})$

c $\mathbf{r} = \mathbf{j} + s(\mathbf{i} + 3\mathbf{j})$

d $\mathbf{r} = -2\mathbf{j} + s(4\mathbf{i} + 3\mathbf{j})$

e $\mathbf{r} = 5\mathbf{j} + s(2\mathbf{i} - \mathbf{j})$

f $y = \frac{1}{4}x + 2$

$\therefore\ \mathbf{r} = 2\mathbf{j} + s(4\mathbf{i} + \mathbf{j})$

6 **a** $x = 2 + 3\lambda,\quad y = 1 + 2\lambda$

b $\lambda = \dfrac{x-2}{3} = \dfrac{y-1}{2}$

$2(x - 2) = 3(y - 1)$

$2x - 4 = 3y - 3$

$2x - 3y - 1 = 0$

7 **a** $x = 3 + \lambda,\quad y = 2\lambda$
$\lambda = x - 3 = \frac{y}{2}$
$2(x - 3) = y$
$2x - y - 6 = 0$

b $x = 1 + 3\lambda,\quad y = 4 + \lambda$
$\lambda = \frac{x-1}{3} = y - 4$
$x - 1 = 3(y - 4)$
$x - 3y + 11 = 0$

c $x = 4\lambda,\quad y = 2 - \lambda$
$\lambda = \frac{x}{4} = 2 - y$
$x = 4(2 - y)$
$x + 4y - 8 = 0$

d $x = -2 + 5\lambda,\quad y = 1 + 2\lambda$
$\lambda = \frac{x+2}{5} = \frac{y-1}{2}$
$2(x + 2) = 5(y - 1)$
$2x - 5y + 9 = 0$

e $x = 2 - 3\lambda,\quad y = -3 + 4\lambda$
$\lambda = \frac{x-2}{-3} = \frac{y+3}{4}$
$4(x - 2) = -3(y + 3)$
$4x + 3y + 1 = 0$

f $x = \lambda + 3,\quad y = -2\lambda - 1$
$\lambda = x - 3 = \frac{y+1}{-2}$
$-2(x - 3) = y + 1$
$2x + y - 5 = 0$

8 **a** $\begin{pmatrix} 3 \\ -1 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} -6 \\ 2 \end{pmatrix}$
$\therefore$ parallel
(1, 2) lies on first line
when $x = 1$ on second line
$-2 - 6t = 1 \Rightarrow t = -\frac{1}{2}$
$\Rightarrow y = 3 + 2(-\frac{1}{2}) = 2$
parallel and common point
$\therefore$ identical

b $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \neq k\begin{pmatrix} 4 \\ 1 \end{pmatrix}$
$\therefore$ not parallel

c $\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix}$
$\therefore$ parallel
(2, –5) lies on first line
when $x = 2$ on second line
$-1 + 3t = 2 \Rightarrow t = 1$
$\Rightarrow y = 1 + 6(1) = 7$
$\therefore$ (2, –5) not on second line
$\therefore$ parallel but not identical

9 **a** $1 + \lambda = 2 + 3\mu$ (1)
$2 = 1 + \mu$ (2)
(2) $\Rightarrow$ $\mu = 1$
$\therefore$ $5\mathbf{i} + 2\mathbf{j}$

b $4 - \lambda = 5 + 2\mu$ (1)
$1 + \lambda = -2 - 3\mu$ (2)
(1) + (2) $\Rightarrow$ $5 = 3 - \mu$
$\mu = -2$
$\therefore$ $\mathbf{i} + 4\mathbf{j}$

c $2\lambda = 2 - \mu$ (1)
$1 - \lambda = 10 + 3\mu$ (2)
(1) + 2×(2) $\Rightarrow$ $2 = 22 + 5\mu$
$\mu = -4$
$\therefore$ $6\mathbf{i} - 2\mathbf{j}$

d $-1 - 4\lambda = 2 - \mu$ (1)
$5 + 6\lambda = -2 + 2\mu$ (2)
2×(1) + (2) $\Rightarrow$ $3 - 2\lambda = 2$
$\lambda = \frac{1}{2}$
$\therefore$ $-3\mathbf{i} + 8\mathbf{j}$

e $-2 - 3\lambda = -3 + 5\mu$ (1)
$11 + 4\lambda = -7 + 3\mu$ (2)
4×(1) + 3×(2)
$\Rightarrow$ $25 = -33 + 29\mu$
$\mu = 2$
$\therefore$ $7\mathbf{i} - \mathbf{j}$

f $1 + 3\lambda = 3 + \mu$ (1)
$2 + 2\lambda = 5 + 4\mu$ (2)
2×(1) – 3×(2)
$\Rightarrow$ $-4 = -9 - 10\mu$
$\mu = -\frac{1}{2}$
$\therefore$ $\frac{5}{2}\mathbf{i} + 3\mathbf{j}$

10 **a** $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$

b $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + s\mathbf{k}$

c $\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$

11 **a** $\overrightarrow{AB} = (6\mathbf{i} - 3\mathbf{j} + \mathbf{k}) - (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = \mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$

b $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + s(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$

c $5 + s = 3 \Rightarrow s = -2$
when $s = -2$, $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) - 2(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} + 9\mathbf{j} - 8\mathbf{k}$
$\therefore$ l passes through (3, 9, –8)

12 **a** direction = $(5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) - (\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$
$= 4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
$\therefore\ \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + s(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

b direction = $(\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) - (3\mathbf{i} - 2\mathbf{k})$
$= -2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$
$\therefore\ \mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + s(-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$

c $\mathbf{r} = s(6\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

d direction = $(4\mathbf{i} - 7\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$
$= 5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$
$\therefore\ \mathbf{r} = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + s(5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$

13 **a** $3 + 2\lambda = 9\ \therefore\ \lambda = 3$
$-5 + a\lambda = -5 + 3a = -2\ \therefore\ a = 1$
$1 + b\lambda = 1 + 3b = -8\ \therefore\ b = -3$

b $2\mathbf{i} + a\mathbf{j} + b\mathbf{k} = k(8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$
$\therefore\ k = \frac{1}{4}$
$\therefore\ a = -1,\ b = \frac{1}{2}$

14 **a** $x = 2 + 3\lambda$,
$y = 3 + 5\lambda$,
$z = 2\lambda$,
$(\lambda =)\ \frac{x-2}{3} = \frac{y-3}{5} = \frac{z}{2}$

b $x = 4 + \lambda$,
$y = -1 + 6\lambda$,
$z = 3 + 3\lambda$,
$(\lambda =)\ x - 4 = \frac{y+1}{6} = \frac{z-3}{3}$

c $x = -1 + 4\lambda$,
$y = 5 - 2\lambda$,
$z = -2 - \lambda$,
$(\lambda =)\ \frac{x+1}{4} = \frac{y-5}{-2} = \frac{z+2}{-1}$

15 **a** $s = \frac{x-1}{3} = \frac{y+4}{2} = z - 5$
$x = 1 + 3s$,
$y = -4 + 2s$,
$z = 5 + s$,
$\mathbf{r} = \mathbf{i} - 4\mathbf{j} + 5\mathbf{k} + s(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

b $s = \frac{x}{4} = \frac{y-1}{-2} = \frac{z+7}{3}$
$x = 4s$,
$y = 1 - 2s$,
$z = -7 + 3s$,
$\mathbf{r} = \mathbf{j} - 7\mathbf{k} + s(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$

c $s = \frac{x+5}{-4} = y + 3 = z$
$x = -5 - 4s$,
$y = -3 + s$,
$z = s$,
$\mathbf{r} = -5\mathbf{i} - 3\mathbf{j} + s(-4\mathbf{i} + \mathbf{j} + \mathbf{k})$

16 $4 + s = 7 - 3t$ (1)
$-2s = 2 + 2t$ (2)
$3 + 2s = -5 + t$ (3)
(2) + (3) $\Rightarrow$ $3 = -3 + 3t$
$t = 2,\ s = -3$
check (1) $4 + (-3) = 7 - 3(2)$
true $\therefore$ intersect
point of intersection: $(1, 6, -3)$

17 $2 + \lambda = 1 + \mu$ (1)
$-1 + \lambda = 4 - 2\mu$ (2)
$4 + 3\lambda = 3 + \mu$ (3)
(1) − (2) $\Rightarrow$ $3 = -3 + 3\mu$
$\mu = 2,\ \lambda = 1$
check (3) $4 + 3(1) = 3 + (2)$
false $\therefore$ do not intersect
$(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \neq k(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\therefore$ not parallel
$\therefore$ skew

18 **a** $3 + 4\lambda = 3 + \mu$ (1)

$1 + \lambda = 2$ (2)

$5 - \lambda = -4 + 2\mu$ (3)

(2) $\Rightarrow$ $\lambda = 1$

sub. (1) $\mu = 4$

check (3) $5 - (1) = -4 + 2(4)$

true $\therefore$ intersect

position vector of intersection: $\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$

b $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$

$\therefore$ parallel

c $8 + \lambda = -2 + 4\mu$ (1)

$2 + 3\lambda = 2 - 3\mu$ (2)

$-4 - 2\lambda = 8 - 4\mu$ (3)

(1) + (3) $\Rightarrow$ $4 - \lambda = 6$

$\lambda = -2,\ \mu = 2$

check (2) $2 + 3(-2) = 2 - 3(2)$

true $\therefore$ intersect

position vector of intersection: $\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$

d $1 + \lambda = 7 + 2\mu$ (1)

$5 + 4\lambda = -6 + \mu$ (2)

$2 - 2\lambda = -5 - 3\mu$ (3)

2×(1) + (3) $\Rightarrow$ $4 = 9 + \mu$

$\mu = -5,\ \lambda = -4$

check (2) $5 + 4(-4) = -6 + (-5)$

true $\therefore$ intersect

position vector of intersection: $\begin{pmatrix} -3 \\ -11 \\ 10 \end{pmatrix}$

e $4 + 2\lambda = 3 + 5\mu$ (1)

$-1 + 5\lambda = -2 - 3\mu$ (2)

$3 - 3\lambda = 1 - 4\mu$ (3)

3×(1) + 2×(3) $\Rightarrow$ $18 = 11 + 7\mu$

$\mu = 1,\ \lambda = 2$

check (2) $-1 + 5(2) = -2 - 3(1)$

false $\therefore$ do not intersect

$\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \neq k\begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$

$\therefore$ skew

f $6\lambda = -12 + 5\mu$ (1)

$7 - 4\lambda = -1 + 2\mu$ (2)

$-2 + 8\lambda = 11 - 3\mu$ (3)

2×(2) + (3) $\Rightarrow$ $12 = 9 + \mu$

$\mu = 3,\ \lambda = \frac{1}{2}$

check (1) $6(\frac{1}{2}) = -12 + 5(3)$

true $\therefore$ intersect

position vector of intersection: $\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$