C4 VECTORS Worksheet D

1 Calculate

a $(\mathbf{i} + 2\mathbf{j}).(3\mathbf{i} + \mathbf{j})$ **b** $(4\mathbf{i} - \mathbf{j}).(3\mathbf{i} + 5\mathbf{j})$ **c** $(\mathbf{i} - 2\mathbf{j}).(-5\mathbf{i} - 2\mathbf{j})$

2 Show that the vectors $(\mathbf{i} + 4\mathbf{j})$ and $(8\mathbf{i} - 2\mathbf{j})$ are perpendicular.

3 Find in each case the value of the constant c for which the vectors **u** and **v** are perpendicular.

a $\mathbf{u} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \ \mathbf{v} = \begin{pmatrix} c \\ 3 \end{pmatrix}$ **b** $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \ \mathbf{v} = \begin{pmatrix} 3 \\ c \end{pmatrix}$ **c** $\mathbf{u} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}, \ \mathbf{v} = \begin{pmatrix} c \\ -4 \end{pmatrix}$

4 Find, in degrees to 1 decimal place, the angle between the vectors

a $(4\mathbf{i} - 3\mathbf{j})$ and $(8\mathbf{i} + 6\mathbf{j})$ **b** $(7\mathbf{i} + \mathbf{j})$ and $(2\mathbf{i} + 6\mathbf{j})$ **c** $(4\mathbf{i} + 2\mathbf{j})$ and $(-5\mathbf{i} + 2\mathbf{j})$

5 Relative to a fixed origin O, the points A, B and C have position vectors $(9\mathbf{i} + \mathbf{j})$, $(3\mathbf{i} - \mathbf{j})$ and $(5\mathbf{i} - 2\mathbf{j})$ respectively. Show that $\angle ABC = 45°$.

6 Calculate

a $(\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}).(3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ **b** $(6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$

c $(-5\mathbf{i} + 2\mathbf{k}).(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$ **d** $(3\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}).(-\mathbf{i} + 11\mathbf{j} - 4\mathbf{k})$

e $(3\mathbf{i} - 7\mathbf{j} + \mathbf{k}).(9\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ **f** $(7\mathbf{i} - 3\mathbf{j}).(-3\mathbf{j} + 6\mathbf{k})$

7 Given that $\mathbf{p} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{q} = \mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $\mathbf{r} = 6\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$,

a find the value of $\mathbf{p}.\mathbf{q}$,

b find the value of $\mathbf{p}.\mathbf{r}$,

c verify that $\mathbf{p}.(\mathbf{q} + \mathbf{r}) = \mathbf{p}.\mathbf{q} + \mathbf{p}.\mathbf{r}$

8 Simplify

a $\mathbf{p}.(\mathbf{q} + \mathbf{r}) + \mathbf{p}.(\mathbf{q} - \mathbf{r})$ **b** $\mathbf{p}.(\mathbf{q} + \mathbf{r}) + \mathbf{q}.(\mathbf{r} - \mathbf{p})$

9 Show that the vectors $(5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j} - 6\mathbf{k})$ are perpendicular.

10 Relative to a fixed origin O, the points A, B and C have position vectors $(3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$, $(\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$ and $(8\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ respectively. Show that $\angle ABC = 90°$.

11 Find in each case the value or values of the constant c for which the vectors **u** and **v** are perpendicular.

a $\mathbf{u} = (2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$, $\mathbf{v} = (c\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ **b** $\mathbf{u} = (-5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$, $\mathbf{v} = (c\mathbf{i} - \mathbf{j} + 3c\mathbf{k})$

c $\mathbf{u} = (c\mathbf{i} - 2\mathbf{j} + 8\mathbf{k})$, $\mathbf{v} = (c\mathbf{i} + c\mathbf{j} - 3\mathbf{k})$ **d** $\mathbf{u} = (3c\mathbf{i} + 2\mathbf{j} + c\mathbf{k})$, $\mathbf{v} = (5\mathbf{i} - 4\mathbf{j} + 2c\mathbf{k})$

12 Find the exact value of the cosine of the angle between the vectors

a $\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 1 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix}$ **c** $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix}$ **d** $\begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$

13 Find, in degrees to 1 decimal place, the angle between the vectors

a $(3\mathbf{i} - 4\mathbf{k})$ and $(7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k})$ **b** $(2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k})$ and $(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$

c $(6\mathbf{i} - 2\mathbf{j} - 9\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ **d** $(\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ and $(-3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$

14 The points A (7, 2, –2), B (–1, 6, –3) and C (–3, 1, 2) are the vertices of a triangle.

a Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of **i**, **j** and **k**.

b Show that $\angle ABC = 82.2°$ to 1 decimal place.

c Find the area of triangle ABC to 3 significant figures.

15 Relative to a fixed origin, the points A, B and C have position vectors $(3\mathbf{i} - 2\mathbf{j} - \mathbf{k})$, $(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ and $(2\mathbf{i} - \mathbf{j})$ respectively.

a Find the exact value of the cosine of angle BAC.

b Hence show that the area of triangle ABC is $3\sqrt{2}$.

16 Find, in degrees to 1 decimal place, the acute angle between each pair of lines.

a $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 8 \\ 0 \\ -6 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 0 \\ -3 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ -1 \\ -18 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ 6 \\ -3 \end{pmatrix} + \mu\begin{pmatrix} 4 \\ -12 \\ 3 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

d $\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ -9 \end{pmatrix} + \lambda\begin{pmatrix} -4 \\ -6 \\ 7 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 11 \\ 1 \\ -2 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ -1 \\ -8 \end{pmatrix}$

17 Relative to a fixed origin, the points A and B have position vectors $(5\mathbf{i} + 8\mathbf{j} - \mathbf{k})$ and $(6\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ respectively.

a Find a vector equation of the straight line l_1 which passes through A and B.

The line l_2 has the equation $\mathbf{r} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \mu(-5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

b Show that lines l_1 and l_2 intersect and find the position vector of their point of intersection.

c Find, in degrees, the acute angle between lines l_1 and l_2.

18 Find, in degrees to 1 decimal place, the acute angle between the lines with cartesian equations

$$\frac{x-2}{3} = \frac{y}{2} = \frac{z+5}{-6} \quad \text{and} \quad \frac{x-4}{-4} = \frac{y+1}{7} = \frac{z-3}{-4}.$$

19 The line l has the equation $\mathbf{r} = 7\mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and the line m has the equation $\mathbf{r} = -4\mathbf{i} + 7\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$.

a Find the coordinates of the point A where lines l and m intersect.

b Find, in degrees, the acute angle between lines l and m.

The point B has coordinates (5, 1, –4).

c Show that B lies on the line l.

d Find the distance of B from m.

20 Relative to a fixed origin O, the points A and B have position vectors $(9\mathbf{i} + 6\mathbf{j})$ and $(11\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ respectively.

a Show that for all values of λ, the point C with position vector $(9 + 2\lambda)\mathbf{i} + (6 - \lambda)\mathbf{j} + \lambda\mathbf{k}$ lies on the straight line l which passes through A and B.

b Find the value of λ for which OC is perpendicular to l.

c Hence, find the position vector of the foot of the perpendicular from O to l.

21 Find the coordinates of the point on each line which is closest to the origin.

a $\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$

b $\mathbf{r} = 7\mathbf{i} + 11\mathbf{j} - 9\mathbf{k} + \lambda(6\mathbf{i} - 9\mathbf{j} + 3\mathbf{k})$