

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions
Exercise A, Question 1

Question:

Express the following as a single fraction:

$$\frac{1}{3} + \frac{1}{4}$$

Solution:

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

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Partial fractions
Exercise A, Question 2

Question:

Express the following as a single fraction:

$$\frac{3}{4} - \frac{2}{5}$$

Solution:

$$\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$$

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Partial fractions Exercise A, Question 3

Question:

Express the following as a single fraction:

$$\frac{3}{x} - \frac{2}{x+1}$$

Solution:

$$\begin{aligned}\frac{3}{x} - \frac{2}{x+1} \\&= \frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)} \\&= \frac{3(x+1) - 2x}{x(x+1)} \\&= \frac{3x+3-2x}{x(x+1)} \\&= \frac{x+3}{x(x+1)}\end{aligned}$$

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Partial fractions Exercise A, Question 4

Question:

Express the following as a single fraction:

$$\frac{2}{(x-1)} + \frac{3}{(x+2)}$$

Solution:

$$\begin{aligned} & \frac{2}{(x-1)} + \frac{3}{(x+2)} \\ &= \frac{2(x+2)}{(x-1)(x+2)} + \frac{3(x-1)}{(x-1)(x+2)} \\ &= \frac{2(x+2) + 3(x-1)}{(x-1)(x+2)} \\ &= \frac{2x+4+3x-3}{(x-1)(x+2)} \\ &= \frac{5x+1}{(x-1)(x+2)} \end{aligned}$$

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Partial fractions
Exercise A, Question 5

Question:

Express the following as a single fraction:

$$\frac{4}{(2x+1)} + \frac{2}{(x-1)}$$

Solution:

$$\begin{aligned} & \frac{4}{(2x+1)} + \frac{2}{(x-1)} \\ &= \frac{4(x-1)}{(2x+1)(x-1)} + \frac{2(2x+1)}{(2x+1)(x-1)} \\ &= \frac{4(x-1) + 2(2x+1)}{(2x+1)(x-1)} \\ &= \frac{4x-4+4x+2}{(2x+1)(x-1)} \\ &= \frac{8x-2}{(2x+1)(x-1)} \end{aligned}$$

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Partial fractions Exercise A, Question 6

Question:

Express the following as a single fraction:

$$\frac{7}{(x-3)} - \frac{2}{(x+4)}$$

Solution:

$$\begin{aligned} & \frac{7}{(x-3)} - \frac{2}{(x+4)} \\ &= \frac{7(x+4)}{(x-3)(x+4)} - \frac{2(x-3)}{(x-3)(x+4)} \\ &= \frac{7(x+4) - 2(x-3)}{(x-3)(x+4)} \\ &= \frac{7x + 28 - 2x + 6}{(x-3)(x+4)} \\ &= \frac{5x + 34}{(x-3)(x+4)} \end{aligned}$$

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Partial fractions
Exercise A, Question 7

Question:

Express the following as a single fraction:

$$\frac{3}{2x} - \frac{6}{(x-1)}$$

Solution:

$$\begin{aligned} & \frac{3}{2x} - \frac{6}{(x-1)} \\ &= \frac{3(x-1)}{2x(x-1)} - \frac{6 \times 2x}{2x(x-1)} \\ &= \frac{3(x-1) - 12x}{2x(x-1)} \\ &= \frac{3x - 3 - 12x}{2x(x-1)} \\ &= \frac{-9x - 3}{2x(x-1)} \\ \text{or } & - \frac{9x + 3}{2x(x-1)} \\ \text{or } & - \frac{3(3x + 1)}{2x(x-1)} \end{aligned}$$

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Partial fractions Exercise A, Question 8

Question:

Express the following as a single fraction:

$$\frac{3}{x} + \frac{2}{(x+1)} + \frac{1}{(x+2)}$$

Solution:

$$\begin{aligned} & \frac{3}{x} + \frac{2}{(x+1)} + \frac{1}{(x+2)} \\ &= \frac{3(x+1)(x+2)}{x(x+1)(x+2)} + \frac{2x(x+2)}{x(x+1)(x+2)} + \frac{1x(x+1)}{x(x+1)(x+2)} \\ &= \frac{3(x+1)(x+2) + 2x(x+2) + 1x(x+1)}{x(x+1)(x+2)} && \text{Add numerators} \\ &= \frac{3(x^2 + 3x + 2) + 2x^2 + 4x + x^2 + x}{x(x+1)(x+2)} && \text{Expand brackets} \\ &= \frac{3x^2 + 9x + 6 + 2x^2 + 4x + x^2 + x}{x(x+1)(x+2)} && \text{Simplify terms} \\ &= \frac{6x^2 + 14x + 6}{x(x+1)(x+2)} && \text{Add like terms} \end{aligned}$$

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Partial fractions

Exercise A, Question 9

Question:

Express the following as a single fraction:

$$\frac{4}{3x} - \frac{2}{(x-2)} + \frac{1}{(2x+1)}$$

Solution:

$$\begin{aligned} & \frac{4}{3x} - \frac{2}{(x-2)} + \frac{1}{(2x+1)} \\ &= \frac{4(x-2)(2x+1)}{3x(x-2)(2x+1)} - \frac{2 \times 3x(2x+1)}{3x(x-2)(2x+1)} + \frac{3x(x-2)}{3x(x-2)(2x+1)} \\ &= \frac{4(x-2)(2x+1) - 2 \times 3x(2x+1) + 3x(x-2)}{3x(x-2)(2x+1)} \quad \text{Add numerators} \\ &= \frac{4(2x^2 - 3x - 2) - 6x(2x+1) + 3x^2 - 6x}{3x(x-2)(2x+1)} \quad \text{Expand brackets} \\ &= \frac{8x^2 - 12x - 8 - 12x^2 - 6x + 3x^2 - 6x}{3x(x-2)(2x+1)} \quad \text{Simplify terms} \\ &= \frac{-1x^2 - 24x - 8}{3x(x-2)(2x+1)} \quad \text{Add like terms} \\ \text{or } & \frac{x^2 + 24x + 8}{3x(x-2)(2x+1)} \end{aligned}$$

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Partial fractions Exercise A, Question 10

Question:

Express the following as a single fraction:

$$\frac{3}{(x-1)} + \frac{2}{(x+1)} + \frac{4}{(x-3)}$$

Solution:

$$\begin{aligned} & \frac{3}{(x-1)} + \frac{2}{(x+1)} + \frac{4}{(x-3)} \\ &= \frac{3(x+1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{2(x-1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{4(x-1)(x+1)}{(x-1)(x+1)(x-3)} \\ &= \frac{3(x+1)(x-3) + 2(x-1)(x-3) + 4(x-1)(x+1)}{(x-1)(x+1)(x-3)} && \text{Add numerators} \\ &= \frac{3(x^2 - 2x - 3) + 2(x^2 - 4x + 3) + 4(x^2 - 1)}{(x-1)(x+1)(x-3)} && \text{Expand brackets} \\ &= \frac{3x^2 - 6x - 9 + 2x^2 - 8x + 6 + 4x^2 - 4}{(x-1)(x+1)(x-3)} && \text{Simplify terms} \\ &= \frac{9x^2 - 14x - 7}{(x-1)(x+1)(x-3)} && \text{Add like terms} \end{aligned}$$

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Partial fractions Exercise B, Question 1

Question:

Express the following as partial fractions:

(a) $\frac{6x - 2}{(x - 2)(x + 3)}$

(b) $\frac{2x + 11}{(x + 1)(x + 4)}$

(c) $\frac{-7x - 12}{2x(x - 4)}$

(d) $\frac{2x - 13}{(2x + 1)(x - 3)}$

(e) $\frac{6x + 6}{x^2 - 9}$

(f) $\frac{7 - 3x}{x^2 - 3x - 4}$

(g) $\frac{8 - x}{x^2 + 4x}$

(h) $\frac{2x - 14}{x^2 + 2x - 15}$

Solution:

(a) Let $\frac{6x - 2}{(x - 2)(x + 3)} \equiv \frac{A}{(x - 2)} + \frac{B}{(x + 3)}$ Add the fractions

$$\Rightarrow \frac{6x - 2}{(x - 2)(x + 3)} \equiv \frac{A(x + 3) + B(x - 2)}{(x - 2)(x + 3)}$$

So $6x - 2 \equiv A(x + 3) + B(x - 2)$ Set numerators equal

Substitute $x = 2 \Rightarrow 6 \times 2 - 2 = A(2 + 3) + B(2 - 2)$

$$\Rightarrow 10 = 5A$$

$$\Rightarrow A = 2$$

$$\text{Substitute } x = -3 \Rightarrow 6 \times (-3) - 2 = A(-3 + 3) + B(-3 - 2)$$

$$\Rightarrow -20 = B \times -5$$

$$\Rightarrow B = 4$$

$$\text{Hence } \frac{6x - 2}{(x - 2)(x + 3)} \equiv \frac{2}{(x - 2)} + \frac{4}{(x + 3)}$$

$$(b) \text{ Let } \frac{2x + 11}{(x + 1)(x + 4)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 4)} \quad \text{Add the fractions}$$

$$\Rightarrow \frac{2x + 11}{(x + 1)(x + 4)} \equiv \frac{A(x + 4) + B(x + 1)}{(x + 1)(x + 4)}$$

$$\text{So } 2x + 11 \equiv A(x + 4) + B(x + 1) \quad \text{Set numerators equal}$$

$$\text{Substitute } x = -4 \Rightarrow 2 \times (-4) + 11 = A(-4 + 4) + B(-4 + 1)$$

$$\Rightarrow 3 = -3B$$

$$\Rightarrow B = -1$$

$$\text{Substitute } x = -1 \Rightarrow 2 \times -1 + 11 = A(-1 + 4) + B(-1 + 1)$$

$$\Rightarrow 9 = 3A$$

$$\Rightarrow A = 3$$

$$\text{Hence } \frac{2x + 11}{(x + 1)(x + 4)} \equiv \frac{3}{(x + 1)} + \frac{(-1)}{(x + 4)} \equiv \frac{3}{(x + 1)} - \frac{1}{(x + 4)}$$

$$(c) \text{ Let } \frac{-7x - 12}{2x(x - 4)} \equiv \frac{A}{2x} + \frac{B}{(x - 4)} \quad \text{Add the fractions}$$

$$\Rightarrow \frac{-7x - 12}{2x(x - 4)} \equiv \frac{A(x - 4) + B \times 2x}{2x(x - 4)}$$

$$\text{So } -7x - 12 \equiv A(x - 4) + 2Bx \quad \text{Set numerators equal}$$

$$\text{Substitute } x = 4 \Rightarrow -7 \times 4 - 12 = A(4 - 4) + 2B \times 4$$

$$\Rightarrow -40 = 8B$$

$$\Rightarrow B = -5$$

$$\text{Substitute } x = 0 \Rightarrow -7 \times 0 - 12 = A(0 - 4) + 2B \times 0$$

$$\Rightarrow -12 = -4A$$

$$\Rightarrow A = 3$$

$$\text{Hence } \frac{-7x - 12}{2x(x - 4)} \equiv \frac{3}{2x} + \frac{-5}{(x - 4)} \equiv \frac{3}{2x} - \frac{5}{(x - 4)}$$

(d) Let $\frac{2x-13}{(2x+1)(x-3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(x-3)}$ Add the fractions

$$\Rightarrow \frac{2x-13}{(2x+1)(x-3)} \equiv \frac{A(x-3) + B(2x+1)}{(2x+1)(x-3)}$$

So $2x-13 \equiv A(x-3) + B(2x+1)$ Set numerators equal

Substitute $x = 3 \Rightarrow 2 \times 3 - 13 = A(3-3) + B(2 \times 3 + 1)$

$$\Rightarrow -7 = B \times 7$$

$$\Rightarrow B = -1$$

Substitute $x = -\frac{1}{2} \Rightarrow 2 \times \left(-\frac{1}{2}\right) - 13 = A\left(-\frac{1}{2} - 3\right) + B$

$$\left(2 \times \left(-\frac{1}{2}\right) + 1\right)$$

$$\Rightarrow -14 = A \times -3\frac{1}{2}$$

$$\Rightarrow A = 4$$

Hence $\frac{2x-13}{(2x+1)(x-3)} \equiv \frac{4}{(2x+1)} + \frac{-1}{(x-3)} \equiv \frac{4}{(2x+1)} - \frac{1}{(x-3)}$

(e) $\frac{6x+6}{x^2+9} \equiv \frac{6x+6}{(x+3)(x-3)}$ Factorise denominator

Let $\frac{6x+6}{(x+3)(x-3)} \equiv \frac{A}{(x+3)} + \frac{B}{(x-3)}$ Add fractions

$$\Rightarrow \frac{6x+6}{(x+3)(x-3)} \equiv \frac{A(x-3) + B(x+3)}{(x+3)(x-3)}$$

So $6x+6 \equiv A(x-3) + B(x+3)$ Set numerators equal

Substitute $x = 3 \Rightarrow 6 \times 3 + 6 = A(3-3) + B(3+3)$

$$\Rightarrow 24 = B \times 6$$

$$\Rightarrow B = 4$$

Substitute $x = -3 \Rightarrow 6 \times (-3) + 6 = A(-3-3) + B(-3+3)$

$$\Rightarrow -12 = A \times -6$$

$$\Rightarrow A = 2$$

Hence $\frac{6x+6}{x^2-9} \equiv \frac{2}{(x+3)} + \frac{4}{(x-3)}$

$$(f) \frac{7-3x}{x^2-3x-4} \equiv \frac{7-3x}{(x-4)(x+1)} \quad \text{Factorise denominator}$$

$$\text{Let } \frac{7-3x}{(x-4)(x+1)} \equiv \frac{A}{(x-4)} + \frac{B}{(x+1)} \quad \text{Add fractions}$$

$$\Rightarrow \frac{7-3x}{(x-4)(x+1)} \equiv \frac{A(x+1) + B(x-4)}{(x-4)(x+1)}$$

$$\text{So } 7-3x \equiv A(x+1) + B(x-4) \quad \text{Set numerators equal}$$

$$\text{Substitute } x = -1 \Rightarrow 7-3 \times (-1) = A(-1+1) + B(-1-4)$$

$$\Rightarrow 10 = B \times -5$$

$$\Rightarrow B = -2$$

$$\text{Substitute } x = 4 \Rightarrow 7-3 \times 4 = A(4+1) + B(4-4)$$

$$\Rightarrow -5 = A \times 5$$

$$\Rightarrow A = -1$$

$$\text{Hence } \frac{7-3x}{x^2-3x-4} \equiv \frac{-1}{(x-4)} + \frac{-2}{(x+1)} \equiv -\frac{1}{(x-4)} - \frac{2}{(x+1)}$$

$$(g) \frac{8-x}{x^2+4x} \equiv \frac{8-x}{x(x+4)} \quad \text{Factorise denominator}$$

$$\text{Let } \frac{8-x}{x(x+4)} \equiv \frac{A}{x} + \frac{B}{(x+4)} \quad \text{Add fractions}$$

$$\Rightarrow \frac{8-x}{x(x+4)} \equiv \frac{A(x+4) + Bx}{x(x+4)}$$

$$\text{So } 8-x \equiv A(x+4) + Bx \quad \text{Set numerators equal}$$

$$\text{Substitute } x = 0 \Rightarrow 8-0 = A(0+4) + B \times 0$$

$$\Rightarrow 8 = 4A$$

$$\Rightarrow A = 2 \quad \text{Substitute } x = -4 \Rightarrow 8 - \left(-4 \right) = A \left(-4 + 4 \right)$$

$$+ B \times \left(-4 \right)$$

$$\Rightarrow 12 = -4B$$

$$\Rightarrow B = -3$$

$$\text{Hence } \frac{8-x}{x^2+4x} \equiv \frac{2}{x} + \frac{-3}{(x+4)} \equiv \frac{2}{x} - \frac{3}{(x+4)}$$

$$(h) \frac{2x-14}{x^2+2x-15} \equiv \frac{2x-14}{(x+5)(x-3)} \quad \text{Factorise denominator}$$

Let $\frac{2x - 14}{(x + 5)(x - 3)} \equiv \frac{A}{(x + 5)} + \frac{B}{(x - 3)}$ Add fractions

$$\Rightarrow \frac{2x - 14}{(x + 5)(x - 3)} \equiv \frac{A(x - 3) + B(x + 5)}{(x + 5)(x - 3)}$$

So $2x - 14 \equiv A(x - 3) + B(x + 5)$ Set numerators equal

Substitute $x = 3 \Rightarrow 2 \times 3 - 14 = A(3 - 3) + B(3 + 5)$

$$\Rightarrow -8 = B \times 8$$

$$\Rightarrow B = -1$$

Substitute $x = -5 \Rightarrow 2 \times (-5) - 14 = A(-5 - 3) + B(-5 + 5)$

$$\Rightarrow -24 = A \times (-8)$$

$$\Rightarrow A = 3$$

Hence $\frac{2x - 14}{x^2 + 2x - 15} \equiv \frac{3}{(x + 5)} + \frac{-1}{(x - 3)} \equiv \frac{3}{(x + 5)} - \frac{1}{(x - 3)}$

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Partial fractions Exercise B, Question 2

Question:

Show that $\frac{-2x-5}{(4+x)(2-x)}$ can be written in the form $\frac{A}{(4+x)} + \frac{B}{(2-x)}$ where A and B are constants to be found.

Solution:

$$\text{Let } \frac{-2x-5}{(4+x)(2-x)} \equiv \frac{A}{(4+x)} + \frac{B}{(2-x)} \equiv \frac{A(2-x) + B(4+x)}{(4+x)(2-x)}$$

$$\text{So } -2x-5 \equiv A(2-x) + B(4+x)$$

$$\text{Substitute } x = 2 \Rightarrow -2 \times 2 - 5 = A(2-2) + B(4+2)$$

$$\Rightarrow -9 = B \times 6$$

$$\Rightarrow B = \frac{-3}{2}$$

$$\text{Substitute } x = -4 \Rightarrow -2 \times (-4) - 5 = A \left(2 - (-4) \right) + B$$

$$\left(4 + (-4) \right)$$

$$\Rightarrow 3 = A \times 6$$

$$\Rightarrow \frac{1}{2} = A$$

$$\text{Hence } \frac{-2x-5}{(4+x)(2-x)} \equiv \frac{A}{(4+x)} + \frac{B}{(2-x)} \text{ when } A = \frac{1}{2} \text{ and } B = \frac{-3}{2}.$$

$$\text{or } \frac{-2x-5}{(4+x)(2-x)} = \frac{1}{2(4+x)} - \frac{3}{2(2-x)}$$

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Partial fractions Exercise C, Question 1

Question:

Express the following as partial fractions:

$$(a) \frac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)}$$

$$(b) \frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)}$$

$$(c) \frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)}$$

Solution:

$$(a) \text{ Let } \frac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+5)} \quad \text{Add fractions}$$

$$\Rightarrow \frac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)} \equiv$$

$$\frac{A(x-2)(x+5) + B(x+1)(x+5) + C(x+1)(x-2)}{(x+1)(x-2)(x+5)}$$

$$\text{So } 2x^2 - 12x - 26 \equiv A(x-2)(x+5) + B(x+1)(x+5) + C(x+1)(x-2)$$

$$\text{Substitute } x = 2 \Rightarrow 8 - 24 - 26 = A \times 0 + B \times 3 \times 7 + C \times 0$$

$$\Rightarrow -42 = 21B$$

$$\Rightarrow B = -2$$

$$\text{Substitute } x = -1 \Rightarrow 2 + 12 - 26 = A \times (-3) \times 4 + B \times 0 + C \times 0$$

$$\Rightarrow -12 = -12A$$

$$\Rightarrow A = 1$$

$$\text{Substitute } x = -5 \Rightarrow 50 + 60 - 26 = A \times 0 + B \times 0 + C \times 28$$

$$\Rightarrow 84 = 28C$$

$$\Rightarrow C = 3$$

$$\text{Hence } \frac{2x^2 - 12x - 26}{(x+1)(x-2)(x+5)} \equiv \frac{1}{(x+1)} - \frac{2}{(x-2)} + \frac{3}{(x+5)}$$

$$(b) \text{ Let } \frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{A}{x} + \frac{B}{(2x+1)} + \frac{C}{(3x-2)} \quad \text{Add fractions}$$

$$\Rightarrow \frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{A(2x+1)(3x-2) + Bx(3x-2) + Cx(2x+1)}{x(2x+1)(3x-2)}$$

$$\text{So } -10x^2 - 8x + 2 \equiv A(2x+1)(3x-2) + Bx(3x-2) + Cx(2x+1)$$

$$\text{Substitute } x = 0 \Rightarrow -0 - 0 + 2 = A \times 1 \times (-2) + B \times 0 + C \times 0$$

$$\Rightarrow 2 = -2A$$

$$\Rightarrow A = -1$$

$$\text{Substitute } x = -\frac{1}{2} \Rightarrow \frac{-10}{4} + 4 + 2 = A \times 0 + B \times \frac{-1}{2} \times$$

$$\frac{-7}{2} + C \times 0$$

$$\Rightarrow \frac{7}{2} = B \times \frac{7}{4}$$

$$\Rightarrow B = 2$$

$$\text{Equate coefficients in } x^2: -10 = 6A + 3B + 2C$$

$$\Rightarrow -10 = -6 + 6 + 2C$$

$$\Rightarrow -10 = 2C$$

$$\Rightarrow -5 = C$$

$$\text{Hence } \frac{-10x^2 - 8x + 2}{x(2x+1)(3x-2)} \equiv \frac{-1}{x} + \frac{2}{(2x+1)} - \frac{5}{(3x-2)}$$

$$(c) \text{ Let } \frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-5)}$$

$$\Rightarrow \frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv$$

$$\frac{A(x+2)(x-5) + B(x+1)(x-5) + C(x+1)(x+2)}{(x+1)(x+2)(x-5)}$$

$$\text{So } -5x^2 - 19x - 32 \equiv A(x+2)(x-5) + B(x+1)(x-5) + C(x+1)(x+2)$$

$$\text{Substitute } x = -1 \Rightarrow -5 + 19 - 32 = A \times 1 \times (-6)$$

$$+ B \times 0 + C \times 0$$

$$\Rightarrow -18 = -6A$$

$$\Rightarrow A = 3$$

$$\text{Substitute } x = 5 \Rightarrow -125 - 95 - 32 = A \times 0 + B \times 0 + C \times 6 \times 7$$

$$\Rightarrow -252 = 42C$$

$$\Rightarrow C = -6$$

$$\text{Substitute } x = -2 \Rightarrow -20 + 38 - 32 = A \times 0 + B \times \begin{pmatrix} -1 \\ -7 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C \times 0$$

$$\Rightarrow -14 = 7B$$

$$\Rightarrow B = -2$$

$$\text{Hence } \frac{-5x^2 - 19x - 32}{(x+1)(x+2)(x-5)} \equiv \frac{3}{(x+1)} - \frac{2}{(x+2)} - \frac{6}{(x-5)}$$

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Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise C, Question 2

Question:

By firstly factorising the denominator, express the following as partial fractions:

$$(a) \frac{6x^2 + 7x - 3}{x^3 - x}$$

$$(b) \frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x}$$

$$(c) \frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6}$$

Solution:

$$(a) x^3 - x \equiv x(x^2 - 1) \equiv x(x + 1)(x - 1)$$

$$\begin{aligned} \text{So } \frac{6x^2 + 7x - 3}{x^3 - x} &\equiv \frac{6x^2 + 7x - 3}{x(x + 1)(x - 1)} \equiv \frac{A}{x} + \frac{B}{(x + 1)} + \frac{C}{(x - 1)} \\ &\equiv \frac{A(x + 1)(x - 1) + Bx(x - 1) + Cx(x + 1)}{x(x + 1)(x - 1)} \end{aligned}$$

$$\text{Setting numerators equal gives } 6x^2 + 7x - 3 \equiv A \begin{pmatrix} x + 1 \\ \end{pmatrix} \begin{pmatrix} x - 1 \\ \end{pmatrix} + Bx \begin{pmatrix} x - 1 \\ \end{pmatrix} + Cx \begin{pmatrix} x + 1 \\ \end{pmatrix}$$

$$\text{Substitute } x = 0 \Rightarrow 0 + 0 - 3 = A \times 1 \times (-1) + B \times 0 + C \times 0$$

$$\Rightarrow -3 = -1A$$

$$\Rightarrow A = 3$$

$$\text{Substitute } x = 1 \Rightarrow 6 + 7 - 3 = A \times 0 + B \times 0 + C \times 1 \times 2$$

$$\Rightarrow 10 = 2C$$

$$\Rightarrow C = 5$$

$$\text{Substitute } x = -1 \Rightarrow 6 - 7 - 3 = A \times 0 + B \times (-1) \times (-2) + C \times 0$$

$$\Rightarrow -4 = 2B$$

$$\Rightarrow B = -2$$

$$\text{Hence } \frac{6x^2 + 7x - 3}{x^3 - x} \equiv \frac{3}{x} - \frac{2}{(x + 1)} + \frac{5}{(x - 1)}$$

$$(b) x^3 + 3x^2 + 2x \equiv x(x^2 + 3x + 2) \equiv x(x + 1)(x + 2)$$

$$\text{So } \frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x} \equiv \frac{5x^2 + 15x + 8}{x(x + 1)(x + 2)} \equiv \frac{A}{x} + \frac{B}{(x + 1)} + \frac{C}{(x + 2)}$$

$$\equiv \frac{A(x+1)(x+2) + Bx(x+2) + Cx(x+1)}{x(x+1)(x+2)}$$

Setting numerators equal gives $5x^2 + 15x + 8 \equiv A \begin{pmatrix} x+1 \end{pmatrix} \begin{pmatrix} x+2 \end{pmatrix} + Bx \begin{pmatrix} x+2 \end{pmatrix} + Cx \begin{pmatrix} x+1 \end{pmatrix}$

Substitute $x = 0 \Rightarrow 0 + 0 + 8 = A \times 1 \times 2 + B \times 0 + C \times 0$

$$\Rightarrow 8 = 2A$$

$$\Rightarrow A = 4$$

Substitute $x = -1 \Rightarrow 5 - 15 + 8 = A \times 0 + B \times (-1) \times 1 + C \times 0$

$$\Rightarrow -2 = -1B$$

$$\Rightarrow B = 2$$

Substitute $x = -2 \Rightarrow 20 - 30 + 8 = A \times 0 + B \times 0 + C \times \begin{pmatrix} -2 \end{pmatrix} \times \begin{pmatrix} -1 \end{pmatrix}$

$$\Rightarrow -2 = 2C$$

$$\Rightarrow C = -1$$

Hence $\frac{5x^2 + 15x + 8}{x^3 + 3x^2 + 2x} \equiv \frac{4}{x} + \frac{2}{(x+1)} - \frac{1}{(x+2)}$

(c) Consider $f(x) = x^3 - 4x^2 + x + 6$

$$f(-1) = -1 - 4 - 1 + 6 = 0$$

Hence $(x+1)$ is a factor

By inspection

$$f(x) = x^3 - 4x^2 + x + 6 = \begin{pmatrix} x+1 \end{pmatrix} \begin{pmatrix} x^2 - 5x + 6 \end{pmatrix} = \begin{pmatrix} x+1 \end{pmatrix} \begin{pmatrix} x-2 \end{pmatrix} \begin{pmatrix} x-3 \end{pmatrix}$$

Note. This last part could have been found by division.

$$\begin{array}{r} x^2 - 5x + 6 \\ x+1 \overline{) x^3 - 4x^2 + x + 6} \\ \underline{x^3 + x^2} \\ -5x^2 + x \\ \underline{-5x^2 - 5x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

Hence $\frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6} \equiv \frac{5x^2 - 15x - 8}{(x+1)(x-2)(x-3)} \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$\equiv \frac{A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)}{(x+1)(x-2)(x-3)}$$

Setting numerators equal gives

$$5x^2 - 15x - 8 \equiv A \left(\frac{x-2}{x-2} \right) \left(\frac{x-3}{x-3} \right) + B \left(\frac{x+1}{x-2} \right) \left(\frac{x-3}{x-3} \right) + C \left(\frac{x+1}{x-2} \right) \left(\frac{x-3}{x-3} \right)$$

$$\text{Substitute } x = 2 \Rightarrow 20 - 30 - 8 = A \times 0 + B \times 3 \times (-1) + C \times 0$$

$$\Rightarrow -18 = -3B$$

$$\Rightarrow B = 6$$

$$\text{Substitute } x = -1 \Rightarrow 5 + 15 - 8 = A \times (-3) \times (-4) + B \times 0 + C \times 0$$

$$\Rightarrow 12 = 12A$$

$$\Rightarrow A = 1$$

$$\text{Substitute } x = 3 \Rightarrow 45 - 45 - 8 = A \times 0 + B \times 0 + C \times 4 \times 1$$

$$\Rightarrow -8 = 4C$$

$$\Rightarrow C = -2$$

$$\text{Hence } \frac{5x^2 - 15x - 8}{x^3 - 4x^2 + x + 6} \equiv \frac{1}{(x+1)} + \frac{6}{(x-2)} - \frac{2}{(x-3)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions Exercise D, Question 1

Question:

Put the following into partial fraction form:

$$\frac{3x^2 + x + 2}{x^2(x + 1)}$$

Solution:

$$\begin{aligned}\text{Let } \frac{3x^2 + x + 2}{x^2(x + 1)} &\equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x + 1)} \\ &\equiv \frac{Ax(x + 1) + B(x + 1) + Cx^2}{x^2(x + 1)}\end{aligned}$$

Set the numerators equal:

$$3x^2 + x + 2 \equiv Ax(x + 1) + B(x + 1) + Cx^2$$

$$\text{Substitute } x = 0 \Rightarrow 0 + 0 + 2 = A \times 0 + B \times 1 + C \times 0$$

$$\Rightarrow 2 = 1B$$

$$\Rightarrow B = 2$$

$$\text{Substitute } x = -1 \Rightarrow 3 - 1 + 2 = A \times 0 + B \times 0 + C \times 1$$

$$\Rightarrow 4 = 1C$$

$$\Rightarrow C = 4$$

$$\text{Equate coefficients in } x^2: \quad 3 = A + C \quad \text{Substitute } C = 4$$

$$\Rightarrow 3 = A + 4$$

$$\Rightarrow A = -1$$

$$\text{Hence } \frac{3x^2 + x + 2}{x^2(x + 1)} \equiv \frac{-1}{x} + \frac{2}{x^2} + \frac{4}{(x + 1)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions Exercise D, Question 2

Question:

Put the following into partial fraction form:

$$\frac{-x^2 - 10x - 5}{(x+1)^2(x-1)}$$

Solution:

$$\begin{aligned}\text{Let } \frac{-x^2 - 10x - 5}{(x+1)^2(x-1)} &\equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)} \\ &\equiv \frac{A(x+1)(x-1) + B(x-1) + C(x+1)^2}{(x+1)^2(x-1)}\end{aligned}$$

Set the numerators equal:

$$-x^2 - 10x - 5 \equiv A(x+1)(x-1) + B(x-1) + C(x+1)^2$$

$$\text{Substitute } x = 1 \Rightarrow -1 - 10 - 5 = A \times 0 + B \times 0 + C \times 4$$

$$\Rightarrow -16 = 4C$$

$$\Rightarrow C = -4$$

$$\text{Substitute } x = -1 \Rightarrow -1 + 10 - 5 = A \times 0 + B \times (-2) + C \times 0$$

$$\Rightarrow 4 = -2B$$

$$\Rightarrow B = -2$$

$$\text{Equate coefficients in } x^2: -1 = A + C \quad \text{Substitute } C = -4$$

$$\Rightarrow -1 = A - 4$$

$$\Rightarrow A = 3$$

$$\text{Hence } \frac{-x^2 - 10x - 5}{(x+1)^2(x-1)} \equiv \frac{3}{(x+1)} - \frac{2}{(x+1)^2} - \frac{4}{(x-1)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions Exercise D, Question 3

Question:

Put the following into partial fraction form:

$$\frac{2x^2 + 2x - 18}{x(x-3)^2}$$

Solution:

$$\begin{aligned}\text{Let } \frac{2x^2 + 2x - 18}{x(x-3)^2} &\equiv \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} \\ &\equiv \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}\end{aligned}$$

Set the numerators equal:

$$2x^2 + 2x - 18 \equiv A(x-3)^2 + Bx(x-3) + Cx$$

$$\text{Substitute } x = 0 \Rightarrow 0 + 0 - 18 = A \times 9 + B \times 0 + C \times 0$$

$$\Rightarrow -18 = 9A$$

$$\Rightarrow A = -2$$

$$\text{Substitute } x = 3 \Rightarrow 18 + 6 - 18 = A \times 0 + B \times 0 + C \times 3$$

$$\Rightarrow 6 = 3C$$

$$\Rightarrow C = 2$$

$$\text{Equate coefficients in } x^2: \quad 2 = A + B \quad \text{Substitute } A = -2$$

$$\Rightarrow 2 = -2 + B$$

$$\Rightarrow B = 4$$

$$\text{Hence } \frac{2x^2 + 2x - 18}{x(x-3)^2} \equiv -\frac{2}{x} + \frac{4}{(x-3)} + \frac{2}{(x-3)^2}$$

Solutionbank

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Partial fractions Exercise D, Question 4

Question:

Put the following into partial fraction form:

$$\frac{7x^2 - 42x + 64}{x(x-4)^2}$$

Solution:

$$\begin{aligned}\text{Let } \frac{7x^2 - 42x + 64}{x(x-4)^2} &\equiv \frac{A}{x} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2} \\ &\equiv \frac{A(x-4)^2 + Bx(x-4) + Cx}{x(x-4)^2}\end{aligned}$$

Set the numerators equal:

$$7x^2 - 42x + 64 \equiv A(x-4)^2 + Bx(x-4) + Cx$$

$$\text{Substitute } x = 0 \Rightarrow 0 - 0 + 64 = A \times 16 + B \times 0 + C \times 0$$

$$\Rightarrow 64 = 16A$$

$$\Rightarrow A = 4$$

$$\text{Substitute } x = 4 \Rightarrow 112 - 168 + 64 = A \times 0 + B \times 0 + C \times 4$$

$$\Rightarrow 8 = 4C$$

$$\Rightarrow C = 2$$

$$\text{Equate coefficients in } x^2: \quad 7 = A + B \quad \text{Substitute } A = 4$$

$$\Rightarrow 7 = 4 + B$$

$$\Rightarrow B = 3$$

$$\text{Hence } \frac{7x^2 - 42x + 64}{x(x-4)^2} \equiv \frac{4}{x} + \frac{3}{(x-4)} + \frac{2}{(x-4)^2}$$

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Partial fractions Exercise D, Question 5

Question:

Put the following into partial fraction form:

$$\frac{5x^2 - 2x - 1}{x^3 - x^2}$$

Solution:

$$x^3 - x^2 \equiv x^2 (x - 1)$$

$$\begin{aligned} \text{So } \frac{5x^2 - 2x - 1}{x^3 - x^2} &\equiv \frac{5x^2 - 2x - 1}{x^2 (x - 1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x - 1)} \\ &\equiv \frac{Ax(x - 1) + B(x - 1) + Cx^2}{x^2 (x - 1)} \end{aligned}$$

Set the numerators equal:

$$5x^2 - 2x - 1 \equiv Ax(x - 1) + B(x - 1) + Cx^2$$

$$\text{Substitute } x = 1 \Rightarrow 5 - 2 - 1 = A \times 0 + B \times 0 + C \times 1$$

$$\Rightarrow 2 = 1C$$

$$\Rightarrow C = 2$$

$$\text{Substitute } x = 0 \Rightarrow 0 - 0 - 1 = A \times 0 + B \times (-1) + C \times 0$$

$$\Rightarrow -1 = -1B$$

$$\Rightarrow B = 1$$

$$\text{Equate coefficients in } x^2: 5 = A + C \quad \text{Substitute } C = 2$$

$$\Rightarrow 5 = A + 2$$

$$\Rightarrow A = 3$$

$$\text{Hence } \frac{5x^2 - 2x - 1}{x^3 - x^2} \equiv \frac{3}{x} + \frac{1}{x^2} + \frac{2}{(x - 1)}$$

Solutionbank

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Partial fractions Exercise D, Question 6

Question:

Put the following into partial fraction form:

$$\frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x}$$

Solution:

$$x^3 - 6x^2 + 9x \equiv x (x^2 - 6x + 9) \equiv x (x - 3)^2$$

$$\begin{aligned} \text{So } \frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x} &\equiv \frac{2x^2 + 2x - 18}{x (x - 3)^2} \equiv \frac{A}{x} + \frac{B}{(x - 3)} + \frac{C}{(x - 3)^2} \\ &\equiv \frac{A (x - 3)^2 + Bx (x - 3) + Cx}{x (x - 3)^2} \end{aligned}$$

Set the numerators equal:

$$2x^2 + 2x - 18 \equiv A (x - 3)^2 + Bx (x - 3) + Cx$$

$$\text{Substitute } x = 0 \Rightarrow 0 + 0 - 18 = A \times 9 + B \times 0 + C \times 0$$

$$\Rightarrow -18 = 9A$$

$$\Rightarrow A = -2$$

$$\text{Substitute } x = 3 \Rightarrow 18 + 6 - 18 = A \times 0 + B \times 0 + C \times 3$$

$$\Rightarrow 6 = 3C$$

$$\Rightarrow C = 2$$

$$\text{Equate coefficients in } x^2: \quad 2 = A + B \quad \text{Substitute } A = -2$$

$$\Rightarrow 2 = -2 + B$$

$$\Rightarrow B = 4$$

$$\text{Hence } \frac{2x^2 + 2x - 18}{x^3 - 6x^2 + 9x} \equiv -\frac{2}{x} + \frac{4}{(x - 3)} + \frac{2}{(x - 3)^2}$$

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Edexcel AS and A Level Modular Mathematics

Partial fractions
Exercise D, Question 7

Question:

Put the following into partial fraction form:

$$\frac{2x}{(x+2)^2}$$

Solution:

$$\text{Let } \frac{2x}{(x+2)^2} \equiv \frac{A}{(x+2)} + \frac{B}{(x+2)^2} \equiv \frac{A(x+2) + B}{(x+2)^2}$$

$$\text{Set the numerators equal: } 2x = A(x+2) + B$$

$$\text{Substitute } x = -2 \Rightarrow -4 = A \times 0 + B \Rightarrow B = -4$$

$$\text{Equate coefficients in } x: 2 = A$$

$$\text{Hence } \frac{2x}{(x+2)^2} \equiv \frac{2}{(x+2)} - \frac{4}{(x+2)^2}$$

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Partial fractions Exercise D, Question 8

Question:

Put the following into partial fraction form:

$$\frac{x^2 + 5x + 7}{(x + 2)^3}$$

Solution:

$$\begin{aligned}\text{Let } \frac{x^2 + 5x + 7}{(x + 2)^3} &\equiv \frac{A}{(x + 2)} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3} \\ &\equiv \frac{A(x + 2)^2 + B(x + 2) + C}{(x + 2)^3}\end{aligned}$$

Set the numerators equal:

$$x^2 + 5x + 7 \equiv A(x + 2)^2 + B(x + 2) + C$$

$$\text{Substitute } x = -2 \Rightarrow 4 - 10 + 7 = A \times 0 + B \times 0 + C \Rightarrow C = 1$$

$$\text{Equate coefficients in } x^2: 1 = A$$

$$\Rightarrow A = 1$$

$$\text{Equate coefficients in } x: 5 = 4A + B \quad \text{Substitute } A = 1$$

$$\Rightarrow 5 = 4 + B$$

$$\Rightarrow B = 1$$

$$\text{Hence } \frac{x^2 + 5x + 7}{(x + 2)^3} \equiv \frac{1}{(x + 2)} + \frac{1}{(x + 2)^2} + \frac{1}{(x + 2)^3}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise E, Question 1

Question:

Express the following improper fractions as a partial fraction:

$$(a) \frac{x^2 + 3x - 2}{(x + 1)(x - 3)}$$

$$(b) \frac{x^2 - 10}{(x - 2)(x + 1)}$$

$$(c) \frac{x^3 - x^2 - x - 3}{x(x - 1)}$$

$$(d) \frac{2x^2 - 1}{(x + 1)^2}$$

Solution:

$$(a) \frac{x^2 + 3x - 2}{(x + 1)(x - 3)} \equiv \frac{x^2 + 3x - 2}{x^2 - 2x - 3}$$

Divide the numerator by the denominator:

$$\begin{array}{r} x^2 - 2x - 3 \overline{) x^2 + 3x - 2} \\ \underline{x^2 - 2x - 3} \\ 5x + 1 \end{array} \leftarrow \text{Remainder}$$

$$\text{Therefore } \frac{x^2 + 3x - 2}{(x + 1)(x - 3)} \equiv 1 + \frac{5x + 1}{(x + 1)(x - 3)}$$

$$\begin{aligned} \text{Let } \frac{5x + 1}{(x + 1)(x - 3)} &\equiv \frac{A}{(x + 1)} + \frac{B}{(x - 3)} && \text{Add fractions} \\ &\equiv \frac{A(x - 3) + B(x + 1)}{(x + 1)(x - 3)} \end{aligned}$$

$$\text{Set the numerators equal: } 5x + 1 \equiv A(x - 3) + B(x + 1)$$

$$\text{Substitute } x = 3 \Rightarrow 5 \times 3 + 1 = A \times 0 + B \times 4$$

$$\Rightarrow 16 = 4B$$

$$\Rightarrow B = 4$$

$$\text{Substitute } x = -1 \Rightarrow 5 \times (-1) + 1 = A \times (-4) + B \times 0$$

$$\Rightarrow -4 = -4A$$

$$\Rightarrow A = 1$$

Hence

$$\frac{x^2 + 3x - 2}{(x+1)(x-3)} \equiv 1 + \frac{5x+1}{(x+1)(x-3)} \equiv 1 + \frac{1}{(x+1)} + \frac{4}{(x-3)}$$

$$(b) \frac{x^2 - 10}{(x-2)(x+1)} \equiv \frac{x^2 - 10}{x^2 - x - 2} \equiv \frac{x^2 + 0x - 10}{x^2 - x - 2}$$

Divide the numerator by the denominator:

$$\begin{array}{r} 1 \\ x^2 - x - 2 \overline{) x^2 + 0x - 10} \\ \underline{x^2 - x - 2} \\ x - 8 \end{array} \leftarrow \text{Remainder}$$

$$\text{Therefore } \frac{x^2 - 10}{(x-2)(x+1)} \equiv 1 + \frac{x-8}{(x-2)(x+1)}$$

$$\begin{aligned} \text{Let } \frac{x-8}{(x-2)(x+1)} &\equiv \frac{A}{(x-2)} + \frac{B}{(x+1)} && \text{Add fractions} \\ &\equiv \frac{A(x+1) + B(x-2)}{(x-2)(x+1)} \end{aligned}$$

$$\text{Set the numerators equal: } x - 8 \equiv A(x+1) + B(x-2)$$

$$\text{Substitute } x = 2 \Rightarrow 2 - 8 = A \times 3 + B \times 0$$

$$\Rightarrow -6 = 3A$$

$$\Rightarrow A = -2$$

$$\text{Substitute } x = -1 \Rightarrow -1 - 8 = A \times 0 + B \times (-3)$$

$$\Rightarrow -9 = -3B$$

$$\Rightarrow B = 3$$

Hence

$$\begin{aligned} \frac{x^2 - 10}{(x-2)(x+1)} &\equiv 1 + \frac{x-8}{(x-2)(x+1)} \equiv 1 + \frac{-2}{(x-2)} + \frac{3}{(x+1)} \\ &\equiv 1 - \frac{2}{(x-2)} + \frac{3}{(x+1)} \end{aligned}$$

$$(c) \frac{x^3 - x^2 - x - 3}{x(x-1)} \equiv \frac{x^3 - x^2 - x - 3}{x^2 - x}$$

Divide the numerator by the denominator:

$$\begin{array}{r} -3x+2 \\ x^2 + 2x - 3 \overline{) x^3 - x^2 - x - 3} \\ \underline{-3x^3 + 4x^2 - 19x + 8} \\ -3x^3 - 6x^2 - 9x \\ \underline{2x^2 + 10x + 8} \\ 2x^2 + 4x - 6 \\ \underline{6x + 14} \end{array} \leftarrow \text{Remainder}$$

$$\text{Therefore } \frac{x^3 - x^2 - x - 3}{x(x-1)} \equiv x + \frac{-x-3}{x(x-1)}$$

$$\begin{aligned}\text{Let } \frac{-x-3}{x(x-1)} &\equiv \frac{A}{x} + \frac{B}{(x-1)} && \text{Add fractions} \\ &\equiv \frac{A(x-1) + Bx}{x(x-1)}\end{aligned}$$

Set the numerators equal: $-x-3 \equiv A(x-1) + Bx$

$$\text{Substitute } x = 1 \Rightarrow -1-3 = A \times 0 + B \times 1$$

$$\Rightarrow -4 = 1B$$

$$\Rightarrow B = -4$$

$$\text{Substitute } x = 0 \Rightarrow -3 = A \times (-1) + B \times 0$$

$$\Rightarrow A = 3$$

$$\text{Hence } \frac{x^3 - x^2 - x - 3}{x(x-1)} \equiv x + \frac{-x-3}{x(x-1)} \equiv x + \frac{3}{x} - \frac{4}{(x-1)}$$

$$(d) \frac{2x^2 - 1}{(x+1)^2} \equiv \frac{2x^2 - 1}{(x+1)(x+1)} \equiv \frac{2x^2 - 1}{x^2 + 2x + 1} \equiv \frac{2x^2 + 0x - 1}{x^2 + 2x + 1}$$

Divide the numerator by the denominator:

$$\begin{array}{r} 2 \\ x^2 + 2x + 1 \overline{) 2x^2 + 0x - 1} \\ \underline{2x^2 + 4x + 2} \\ -4x - 3 \quad \leftarrow \text{Remainder} \end{array}$$

$$\text{Therefore } \frac{2x^2 - 1}{(x+1)^2} \equiv 2 + \frac{-4x-3}{(x+1)^2}$$

$$\begin{aligned}\text{Let } \frac{-4x-3}{(x+1)^2} &\equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2} && \text{Add fractions} \\ &\equiv \frac{A(x+1) + B}{(x+1)^2}\end{aligned}$$

Set the numerators equal: $-4x-3 \equiv A(x+1) + B$

$$\text{Substitute } x = -1 \Rightarrow -4 \times (-1) - 3 = A \times 0 + B$$

$$\Rightarrow 1 = B$$

$$\Rightarrow B = 1$$

Equate coefficients in x : $-4 = A$

$$\Rightarrow A = -4$$

$$\text{Hence } \frac{2x^2 - 1}{(x+1)^2} \equiv 2 + \frac{-4x-3}{(x+1)^2} \equiv 2 - \frac{4}{(x+1)} + \frac{1}{(x+1)^2}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise E, Question 2

Question:

By factorising the denominator, express the following as partial fraction:

(a) $\frac{4x^2 + 17x - 11}{x^2 + 3x - 4}$

(b) $\frac{x^4 - 4x^3 + 9x^2 - 17x + 12}{x^3 - 4x^2 + 4x}$

Solution:

(a) Divide the numerator by the denominator:

$$\begin{array}{r} 4 \\ x^2 + 3x - 4 \overline{) 4x^2 + 17x - 11} \\ \underline{4x^2 + 12x - 16} \\ 5x + 5 \end{array} \leftarrow \text{Remainder}$$

$$\begin{aligned} \text{Therefore } \frac{4x^2 + 17x - 11}{x^2 + 3x - 4} &\equiv 4 + \frac{5x + 5}{x^2 + 3x - 4} && \text{Factorise denominator} \\ &\equiv 4 + \frac{5x + 5}{(x + 4)(x - 1)} \end{aligned}$$

$$\begin{aligned} \text{Let } \frac{5x + 5}{(x + 4)(x - 1)} &\equiv \frac{A}{(x + 4)} + \frac{B}{(x - 1)} && \text{Add fractions} \\ &\equiv \frac{A(x - 1) + B(x + 4)}{(x + 4)(x - 1)} \end{aligned}$$

$$\text{Set the numerators equal: } 5x + 5 \equiv A(x - 1) + B(x + 4)$$

$$\text{Substitute } x = 1 \Rightarrow 5 \times 1 + 5 = A \times 0 + B \times 5$$

$$\Rightarrow 10 = 5B$$

$$\Rightarrow B = 2$$

$$\text{Substitute } x = -4 \Rightarrow 5 \times (-4) + 5 = A \times (-5) + B \times 0$$

$$\Rightarrow -15 = -5A$$

$$\Rightarrow A = 3$$

Hence

$$\frac{4x^2 + 17x - 11}{x^2 + 3x - 4} \equiv 4 + \frac{5x + 5}{(x + 4)(x - 1)} \equiv 4 + \frac{3}{(x + 4)} + \frac{2}{(x - 1)}$$

(b) Divide the numerator by the denominator:

$$\begin{array}{r}
 4x^2 + 4x + 0 \overline{) x^4 - 4x^3 + 9x^2 - 17x + 12} \\
 \underline{x^4 - 4x^3 + 4x^2 + 0x} \\
 5x^2 - 17x + 12 \quad \leftarrow \text{Remainder}
 \end{array}$$

Therefore $\frac{x^4 - 4x^3 + 9x^2 - 17x + 12}{x^3 - 4x^2 + 4x}$

$$\equiv x + \frac{5x^2 - 17x + 12}{x^3 - 4x^2 + 4x} \quad \text{Take out a factor of } x \text{ in the denominator}$$

$$\equiv x + \frac{5x^2 - 17x + 12}{x(x^2 - 4x + 4)} \quad \text{Factorise the denominator fully}$$

$$\equiv x + \frac{5x^2 - 17x + 12}{x(x-2)^2}$$

Let $\frac{5x^2 - 17x + 12}{x(x-2)^2} \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ Add fractions

$$\equiv \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2}$$

Set the numerators equal: $5x^2 - 17x + 12 \equiv A(x-2)^2 + Bx(x-2) + Cx$

Substitute $x = 0 \Rightarrow 0 - 0 + 12 = A \times 4 + B \times 0 + C \times 0$

$$\Rightarrow 12 = 4A$$

$$\Rightarrow A = 3$$

Substitute $x = 2 \Rightarrow 20 - 34 + 12 = A \times 0 + B \times 0 + C \times 2$

$$\Rightarrow -2 = 2C$$

$$\Rightarrow C = -1$$

Equate coefficients in x^2 : $5 = A + B$

$$\Rightarrow 5 = 3 + B$$

$$\Rightarrow B = 2$$

Hence

$$\begin{aligned}
 \frac{x^4 - 4x^3 + 9x^2 - 17x + 12}{x^3 - 4x^2 + 4x} &\equiv x + \frac{5x^2 - 17x + 12}{x(x-2)^2} \\
 &\equiv x + \frac{3}{x} + \frac{2}{(x-2)} - \frac{1}{(x-2)^2}
 \end{aligned}$$

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Edexcel AS and A Level Modular Mathematics

Partial fractions Exercise E, Question 3

Question:

Show that $\frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3}$ can be expressed in the form

$A + Bx + \frac{C}{(x-1)} + \frac{D}{(x+3)}$, where A, B, C and D are constants to be found.

Solution:

Divide the numerator by the denominator:

$$\begin{array}{r}
 \overline{-3x + 2} \\
 x^2 + 2x - 3 \overline{) -3x^3 - 4x^2 + 19x + 8} \\
 \underline{-3x^3 - 6x^2 + 9x} \\
 2x^2 + 10x + 8 \\
 \underline{2x^2 + 4x - 6} \\
 6x + 14 \quad \leftarrow \text{Remainder}
 \end{array}$$

Therefore $\frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3} \equiv -3x + 2 + \frac{6x + 14}{x^2 + 2x - 3}$ Factorise denominator

$$\equiv -3x + 2 + \frac{6x + 14}{(x-1)(x+3)}$$

$$\text{Let } \frac{6x + 14}{(x-1)(x+3)} \equiv \frac{C}{(x-1)} + \frac{D}{(x+3)} \equiv \frac{C(x+3) + D(x-1)}{(x-1)(x+3)}$$

Set the numerators equal: $6x + 14 \equiv C(x+3) + D(x-1)$

$$\text{Substitute } x = 1 \Rightarrow 6 + 14 = C \times 4 + D \times 0$$

$$\Rightarrow 20 = 4C$$

$$\Rightarrow 5 = C$$

$$\text{Substitute } x = -3 \Rightarrow 6 \times (-3) + 14 = C \times 0 + D \times (-4)$$

$$\Rightarrow -4 = -4D$$

$$\Rightarrow D = 1$$

Hence

$$\begin{aligned}
 \frac{-3x^3 - 4x^2 + 19x + 8}{x^2 + 2x - 3} &\equiv -3x + 2 + \frac{6x + 14}{(x-1)(x+3)} \\
 &\equiv -3x + 2 + \frac{5}{(x-1)} + \frac{1}{(x+3)}
 \end{aligned}$$

So $A = 2, B = -3, C = 5$ and $D = 1$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise F, Question 1

Question:

Express the following as a partial fraction:

(a) $\frac{x-3}{x(x-1)}$

(b) $\frac{7x^2+2x-2}{x^2(x+1)}$

(c) $\frac{-15x+21}{(x-2)(x+1)(x-5)}$

(d) $\frac{x^2+1}{x(x-2)}$

Solution:

(a) Let $\frac{x-3}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{(x-1)}$ Add the fractions

$$\equiv \frac{A(x-1) + Bx}{x(x-1)}$$

Set the numerators equal: $x-3 \equiv A(x-1) + Bx$

Substitute $x = 1 \Rightarrow 1-3 = A \times 0 + B \times 1$

$$\Rightarrow B = -2$$

Substitute $x = 0 \Rightarrow 0-3 = A \times (-1) + B \times 0$

$$\Rightarrow -3 = -1A$$

$$\Rightarrow A = 3$$

Hence $\frac{x-3}{x(x-1)} \equiv \frac{3}{x} - \frac{2}{(x-1)}$

(b) Let $\frac{7x^2+2x-2}{x^2(x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ Add the fractions

$$\equiv \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$$

Set the numerators equal:

$$7x^2 + 2x - 2 \equiv Ax(x+1) + B(x+1) + Cx^2$$

Substitute $x = 0 \Rightarrow 0 + 0 - 2 = A \times 0 + B \times 1 + C \times 0$

$$\Rightarrow -2 = 1B$$

$$\Rightarrow B = -2$$

Substitute $x = -1 \Rightarrow 7 - 2 - 2 = A \times 0 + B \times 0 + C \times 1$

$$\Rightarrow 3 = 1C$$

$$\Rightarrow C = 3$$

Equate coefficients in x^2 : $7 = A + C$ Substitute $C = 3$

$$\Rightarrow 7 = A + 3$$

$$\Rightarrow A = 4$$

$$\text{Hence } \frac{7x^2 + 2x - 2}{x^2(x+1)} \equiv \frac{4}{x} - \frac{2}{x^2} + \frac{3}{(x+1)}$$

$$\begin{aligned} \text{(c) Let } \frac{-15x + 21}{(x-2)(x+1)(x-5)} &\equiv \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x-5)} && \text{Add the fractions} \\ &\equiv \frac{A(x+1)(x-5) + B(x-2)(x-5) + C(x-2)(x+1)}{(x-2)(x+1)(x-5)} \end{aligned}$$

Set the numerators equal:

$$-15x + 21 \equiv A(x+1)(x-5) + B(x-2)(x-5) + C(x-2)(x+1)$$

$$\text{Substitute } x = -1 \Rightarrow 15 + 21 = A \times 0 + B \times (-3) \times (-6) + C \times 0$$

$$\Rightarrow 36 = 18B$$

$$\Rightarrow B = 2$$

$$\text{Substitute } x = 5 \Rightarrow -75 + 21 = A \times 0 + B \times 0 + C \times 3 \times 6$$

$$\Rightarrow -54 = 18C$$

$$\Rightarrow C = -3$$

$$\text{Substitute } x = 2 \Rightarrow -30 + 21 = A \times 3 \times (-3) + B \times 0 + C \times 0$$

$$\Rightarrow -9 = -9A$$

$$\Rightarrow A = 1$$

$$\text{Hence } \frac{-15x + 21}{(x-2)(x+1)(x-5)} \equiv \frac{1}{(x-2)} + \frac{2}{(x+1)} - \frac{3}{(x-5)}$$

$$\text{(d) } \frac{x^2 + 1}{x(x-2)} \equiv \frac{x^2 + 1}{x^2 - 2x} \equiv \frac{x^2 + 0x + 1}{x^2 - 2x + 0}$$

Divide the numerator by the denominator:

$$\begin{array}{r} \overline{1} \\ x^2 - 2x + 0 \overline{) x^2 + 0x + 1} \\ \underline{x^2 - 2x + 0} \\ 2x + 1 \leftarrow \text{Remainder} \end{array}$$

$$\text{Therefore } \frac{x^2 + 1}{x(x-2)} \equiv 1 + \frac{2x + 1}{x(x-2)}$$

$$\text{Let } \frac{2x + 1}{x(x-2)} \equiv \frac{A}{x} + \frac{B}{(x-2)} \equiv \frac{A(x-2) + Bx}{x(x-2)}$$

Set the numerators equal: $2x + 1 \equiv A(x-2) + Bx$

$$\text{Substitute } x = 0 \Rightarrow 1 = A \times (-2) + B \times 0$$

$$\Rightarrow 1 = -2A$$

$$\Rightarrow A = -\frac{1}{2}$$

$$\text{Substitute } x = 2 \Rightarrow 2 \times 2 + 1 = A \times 0 + B \times 2$$

$$\Rightarrow 5 = 2B$$

$$\Rightarrow B = \frac{5}{2}$$

Hence

$$\frac{x^2 + 1}{x(x - 2)} \equiv 1 + \frac{2x + 1}{x(x - 2)} \equiv 1 + \frac{-\frac{1}{2}}{x} + \frac{\frac{5}{2}}{(x - 2)}$$

$$\equiv 1 - \frac{1}{2x} + \frac{5}{2(x - 2)}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise F, Question 2

Question:

Write the following algebraic fractions as a partial fraction:

(a) $\frac{3x+1}{x^2+2x+1}$

(b) $\frac{2x^2+2x-8}{x^2+2x-3}$

(c) $\frac{3x^2+12x+8}{(x+2)^3}$

(d) $\frac{x^4}{x^2-2x+1}$

Solution:

(a) $\frac{3x+1}{x^2+2x+1} \equiv \frac{3x+1}{(x+1)^2}$ Repeated factor in denominator

Let $\frac{3x+1}{(x+1)^2} \equiv \frac{A}{(x+1)} + \frac{B}{(x+1)^2} \equiv \frac{A(x+1)+B}{(x+1)^2}$

Set the numerators equal: $3x+1 \equiv A(x+1) + B$

Substitute $x = -1 \Rightarrow -3+1 = A \times 0 + B$

$\Rightarrow B = -2$

Equate coefficients of x : $3 = A$

$\Rightarrow A = 3$

Hence $\frac{3x+1}{x^2+2x+1} \equiv \frac{3x+1}{(x+1)^2} \equiv \frac{3}{(x+1)} - \frac{2}{(x+1)^2}$

(b) $\frac{2x^2+2x-8}{x^2+2x-3}$ is an **improper fraction**

Dividing gives

$$\begin{array}{r} 2 \\ x^2+2x-3 \overline{) 2x^2+2x-8} \\ \underline{2x^2+4x-6} \\ -2x-2 \leftarrow \text{Remainder} \end{array}$$

Therefore $\frac{2x^2+2x-8}{x^2+2x-3} \equiv 2 + \frac{-2x-2}{x^2+2x-3}$ Factorise the denominator

$$\equiv 2 + \frac{-2x-2}{(x+3)(x-1)}$$

$$\text{Let } \frac{-2x-2}{(x+3)(x-1)} \equiv \frac{A}{(x+3)} + \frac{B}{(x-1)} \equiv \frac{A(x-1) + B(x+3)}{(x+3)(x-1)}$$

Set the numerators equal: $-2x-2 \equiv A(x-1) + B(x+3)$

$$\text{Substitute } x = 1 \Rightarrow -2-2 = A \times 0 + B \times 4$$

$$\Rightarrow -4 = 4B$$

$$\Rightarrow B = -1$$

$$\text{Substitute } x = -3 \Rightarrow 6-2 = A \times (-4) + B \times 0$$

$$\Rightarrow 4 = -4A$$

$$\Rightarrow A = -1$$

Hence

$$\begin{aligned} \frac{2x^2+2x-8}{x^2+2x-3} &\equiv 2 + \frac{-2x-2}{(x+3)(x-1)} \\ &\equiv 2 - \frac{1}{(x+3)} - \frac{1}{(x-1)} \end{aligned}$$

$$\begin{aligned} \text{(c) Let } \frac{3x^2+12x+8}{(x+2)^3} &\equiv \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \\ &\equiv \frac{A(x+2)^2 + B(x+2) + C}{(x+2)^3} \end{aligned}$$

Set the numerators equal:

$$3x^2 + 12x + 8 \equiv A(x+2)^2 + B(x+2) + C$$

$$\text{Substitute } x = -2 \Rightarrow 12 - 24 + 8 = A \times 0 + B \times 0 + C$$

$$\Rightarrow C = -4$$

$$\text{Equate coefficients in } x^2: 3 = A$$

$$\Rightarrow A = 3$$

$$\text{Equate coefficients in } x: 12 = 4A + B \quad \text{Substitute } A = 3$$

$$\Rightarrow 12 = 12 + B$$

$$\Rightarrow B = 0$$

$$\text{Hence } \frac{3x^2+12x+8}{(x+2)^3} \equiv \frac{3}{(x+2)} - \frac{4}{(x+2)^3}$$

$$\text{(d) } \frac{x^4}{x^2-2x+1} \equiv \frac{x^4+0x^3+0x^2+0x+0}{x^2-2x+1}$$

Divide the numerator by the denominator:

$$\begin{array}{r}
 x^2 + 2x + 3 \\
 x^2 - 2x + 1 \overline{) x^4 + 0x^3 + 0x^2 + 0x + 0} \\
 \underline{x^4 - 2x^3 + x^2} \\
 2x^3 - x^2 + 0x \\
 \underline{2x^3 - 4x^2 + 2x} \\
 3x^2 - 2x + 0 \\
 \underline{3x^2 - 6x + 3} \\
 4x - 3 \quad \leftarrow \text{Remainder}
 \end{array}$$

Therefore

$$\begin{aligned}
 \frac{x^4}{x^2 - 2x + 1} &\equiv x^2 + 2x + 3 + \frac{4x - 3}{x^2 - 2x + 1} && \text{Factorise the denominator} \\
 &\equiv x^2 + 2x + 3 + \frac{4x - 3}{(x - 1)^2}
 \end{aligned}$$

$$\text{Let } \frac{4x - 3}{(x - 1)^2} \equiv \frac{A}{(x - 1)} + \frac{B}{(x - 1)^2} \equiv \frac{A(x - 1) + B}{(x - 1)^2}$$

$$\text{Set the numerators equal: } 4x - 3 \equiv A(x - 1) + B$$

$$\text{Substitute } x = 1 \Rightarrow 4 - 3 = B \Rightarrow B = 1$$

$$\text{Equate coefficients in } x: 4 = A$$

Hence

$$\frac{x^4}{x^2 - 2x + 1} \equiv x^2 + 2x + 3 + \frac{4x - 3}{(x - 1)^2} \equiv x^2 + 2x + 3 + \frac{4}{(x - 1)} + \frac{1}{(x - 1)^2}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Partial fractions

Exercise F, Question 3

Question:

Given that $f(x) = 2x^3 + 9x^2 + 10x + 3$:

(a) Show that -3 is a root of $f(x)$.

(b) Express $\frac{10}{f(x)}$ as partial fractions.

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Solution:

$$\begin{aligned} \text{(a) } f(-3) &= 2 \times (-27) + 9 \times 9 + 10 \times (-3) \\ &+ 3 = -54 + 81 - 30 + 3 = 0 \end{aligned}$$

Therefore -3 is a root $\Rightarrow (x + 3)$ is a factor

$$\begin{aligned} \text{(b) } f(x) &= 2x^3 + 9x^2 + 10x + 3 \quad (x + 3) \text{ is a factor} \\ &= (x + 3)(2x^2 + 3x + 1) \quad \text{By inspection} \\ &= (x + 3)(2x + 1)(x + 1) \end{aligned}$$

$$\begin{aligned} \frac{10}{f(x)} &\equiv \frac{10}{(x + 3)(2x + 1)(x + 1)} \equiv \frac{A}{(x + 3)} + \frac{B}{(2x + 1)} + \frac{C}{(x + 1)} \\ &\equiv \frac{A(2x + 1)(x + 1) + B(x + 3)(x + 1) + C(x + 3)(2x + 1)}{(x + 3)(2x + 1)(x + 1)} \end{aligned}$$

Set the numerators equal:

$$10 \equiv A(2x + 1)(x + 1) + B(x + 3)(x + 1) + C(x + 3)(2x + 1)$$

$$\text{Substitute } x = -1 \Rightarrow 10 = A \times 0 + B \times 0 + C \times 2 \times (-1)$$

$$\Rightarrow 10 = -2C$$

$$\Rightarrow C = -5$$

$$\text{Substitute } x = -3 \Rightarrow 10 = A \times (-5) \times (-2) + B \times 0 + C \times 0$$

$$\Rightarrow 10 = 10A$$

$$\Rightarrow A = 1$$

$$\text{Substitute } x = -\frac{1}{2} \Rightarrow 10 = A \times 0 + B \times \left(2 \times \frac{1}{2}\right) \times \left(\frac{1}{2}\right) + C \times 0$$

$$\Rightarrow 10 = 1.25B$$

$$\Rightarrow B = 8$$

$$\text{Hence } \frac{10}{f(x)} \equiv \frac{1}{(x+3)} + \frac{8}{(2x+1)} - \frac{5}{(x+1)}$$