

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Simplify:

(a) $\frac{4x + 4}{x + 1}$

(b) $\frac{2x - 1}{6x - 3}$

(c) $\frac{x + 4}{x + 2}$

(d) $\frac{x + \frac{1}{2}}{4x + 2}$

(e) $\frac{4x + 2y}{6x + 3y}$

(f) $\frac{a + 3}{a + 6}$

(g) $\frac{5p - 5q}{10p - 10q}$

(h) $\frac{\frac{1}{2}a + b}{2a + 4b}$

(i) $\frac{x^2}{x^2 + 3x}$

(j) $\frac{x^2 - 3x}{x^2 - 9}$

(k) $\frac{x^2 + 5x + 4}{x^2 + 8x + 16}$

$$(l) \frac{x^3 - 2x^2}{x^2 - 4}$$

$$(m) \frac{x^2 - 4}{x^2 + 4}$$

$$(n) \frac{x + 2}{x^2 + 5x + 6}$$

$$(o) \frac{2x^2 - 5x - 3}{2x^2 - 7x - 4}$$

$$(p) \frac{\frac{1}{2}x^2 + x - 4}{\frac{1}{4}x^2 + \frac{3}{2}x + 2}$$

$$(q) \frac{3x^2 - x - 2}{\frac{1}{2}x + \frac{1}{3}}$$

$$(r) \frac{x^2 - 5x - 6}{\frac{1}{3}x - 2}$$

Solution:

(a)

$$\frac{4x + 4}{x + 1} = \frac{4x \times \cancel{(x+1)}}{1x \times \cancel{(x+1)}} = \frac{4}{1} = 4$$

(b)

$$\frac{2x - 1}{6x - 3} = \frac{1 \times \cancel{(2x-1)}}{3 \times \cancel{(2x-1)}} = \frac{1}{3}$$

(c) $\frac{x + 4}{x + 2}$ will not simplify any further

(d)

$$\frac{x + 12}{4x + 2} = \frac{x + 12 \times 2}{4x + 2 \times 2} = \frac{2x + 1}{8x + 4} = \frac{1 \times \cancel{(2x+1)}}{4 \times \cancel{(2x+1)}} = \frac{1}{4}$$

(e)

$$\frac{4x+2y}{6x+3y} = \frac{2\cancel{(2x+y)}}{3\cancel{(2x+y)}} = \frac{2}{3}$$

(f) $\frac{a+3}{a+6}$ will not simplify any further

(g)

$$\frac{5p-5q}{10p-10q} = \frac{5\cancel{(p-q)}}{10\cancel{(p-q)}} = \frac{5^1}{10_2} = \frac{1}{2}$$

(h)

$$\frac{12a+b \times 2}{2a+4b \times 2} = \frac{a+2b}{4a+8b} = \frac{1 \times \cancel{(a+2b)}}{4 \times \cancel{(a+2b)}} = \frac{1}{4}$$

(i)

$$\frac{x^2}{x^2+3x} = \frac{\cancel{x} \times x}{\cancel{x}(x+3)} = \frac{x}{x+3}$$

(j)

$$\frac{x^2-3x}{x^2-9} = \frac{x\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \frac{x}{x+3}$$

(k)

$$\frac{x^2+5x+4}{x^2+8x+16} = \frac{(x+1)\cancel{(x+4)}}{(x+4)\cancel{(x+4)}} = \frac{x+1}{x+4}$$

(l)

$$\frac{x^3-2x^2}{x^2-4} = \frac{x^2\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \frac{x^2}{x+2}$$

(m) $\frac{x^2-4}{x^2+4}$ will not simplify any further. The denominator doesn't factorise.

(n)

$$\frac{x+2}{x^2+5x+6} = \frac{1 \times \cancel{(x+2)}}{(x+3)\cancel{(x+2)}} = \frac{1}{x+3}$$

(o)

$$\frac{2x^2-5x-3}{2x^2-7x-4} = \frac{\cancel{(2x+1)}(x-3)}{\cancel{(2x+1)}(x-4)} = \frac{x-3}{x-4}$$

(p)

$$\frac{12x^2 + x - 4}{14x^2 - 32x + 2} \times 4 = \frac{2x^2 + 4x - 16}{x^2 + 6x + 8} = \frac{2(x-2)(\cancel{x+4})}{(x+2)(\cancel{x+4})} = \frac{2(x-2)}{x+2}$$

(q)

$$\frac{3x^2 - x - 2}{12x + 13} \times 6 = \frac{6(3x^2 - x - 2)}{3x + 2} = \frac{6(\cancel{3x+2})(x-1)}{1 \times (\cancel{3x+2})} = 6(x-1)$$

(r)

$$\frac{x^2 - 5x - 6}{13x - 2} \times 3 = \frac{3(x^2 - 5x - 6)}{x - 6} = \frac{3(x+1)(\cancel{x-6})}{1 \times (\cancel{x-6})} = 3(x+1)$$

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Exercise B, Question 1

Question:

Simplify:

(a) $\frac{a}{d} \times \frac{a}{c}$

(b) $\frac{a^2}{c} \times \frac{c}{a}$

(c) $\frac{2}{x} \times \frac{x}{4}$

(d) $\frac{3}{x} \div \frac{6}{x}$

(e) $\frac{4}{xy} \div \frac{x}{y}$

(f) $\frac{2r^2}{5} \div \frac{4}{r^3}$

(g) $\left(x + 2 \right) \times \frac{1}{x^2 - 4}$

(h) $\frac{1}{a^2 + 6a + 9} \times \frac{a^2 - 9}{2}$

(i) $\frac{x^2 - 3x}{y^2 + y} \times \frac{y + 1}{x}$

(j) $\frac{y}{y + 3} \div \frac{y^2}{y^2 + 4y + 3}$

(k) $\frac{x^2}{3} \div \frac{2x^3 - 6x^2}{x^2 - 3x}$

(l) $\frac{4x^2 - 25}{4x - 10} \div \frac{2x + 5}{8}$

$$(m) \frac{x+3}{x^2+10x+25} \times \frac{x^2+5x}{x^2+3x}$$

$$(n) \frac{3y^2+4y-4}{10} \div \frac{3y+6}{15}$$

$$(o) \frac{x^2+2xy+y^2}{2} \times \frac{4}{(x-y)^2}$$

Solution:

$$(a) \frac{a}{d} \times \frac{a}{c} = \frac{a \times a}{d \times c} = \frac{a^2}{cd}$$

(b)

$$\frac{\cancel{a^2}^1 \times \cancel{a^1}_1}{\cancel{a^1}_1 \times \cancel{a^1}_1} = \frac{a \times 1}{1 \times 1} = a$$

(c)

$$\frac{\cancel{2}^1}{\cancel{2}_1} \times \frac{\cancel{1}^1}{\cancel{2}_2} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$$

(d)

$$\frac{3}{x} \div \frac{6}{x} = \frac{\cancel{3}^1}{\cancel{3}_1} \times \frac{\cancel{x}^1}{\cancel{6}_2} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$$

(e)

$$\frac{4}{xy} \div \frac{x}{y} = \frac{4}{x \cancel{y}_1} \times \frac{\cancel{y}^1}{x} = \frac{4 \times 1}{x \times x} = \frac{4}{x^2}$$

(f)

$$\frac{2r^2}{5} \div \frac{4}{r^3} = \frac{\cancel{2}^1 \cancel{r^2}}{5} \times \frac{r^3}{\cancel{4}_2} = \frac{r^5}{10}$$

(g)

$$(x+2) \times \frac{1}{x^2-4} = \frac{x+2^1}{1} = \frac{1}{(\cancel{x+2})_1 (x-2)} = \frac{1 \times 1}{1 \times (x-2)} = \frac{1}{x-2}$$

(h)

$$\frac{1}{a^2+6a+9} \times \frac{a^2-9}{2} = \frac{1}{(\cancel{a+3})(a+3)} \times \frac{(\cancel{a+3})(a-3)}{2} = \frac{a-3}{2(a+3)}$$

(i)

$$\frac{x^2 - 3x}{y^2 + y} \times \frac{y+1}{x} = \frac{\cancel{x^1}(x-3)}{y(\cancel{y+1})_1} \times \frac{(\cancel{y+1})^1}{\cancel{x}_1} = \frac{x-3}{y}$$

(j)

$$\frac{y}{y+3} \div \frac{y^2}{y^2+4y+3} = \frac{y}{y+3} \times \frac{y^2+4y+3}{y^2} = \frac{\cancel{y}}{\cancel{y+3}} \times \frac{(y+1)(\cancel{y+3})}{y^2} = \frac{y+1}{y}$$

(k)

$$\frac{x^2}{3} \div \frac{2x^3-6x^2}{x^2-3x} = \frac{x^2}{3} \times \frac{x^2-3x}{2x^3-6x^2} = \frac{\cancel{x^2}^1}{3} \times \frac{x(\cancel{x-3})^1}{2\cancel{x^2}(x-3)_1} = \frac{1 \times x}{3 \times 2} = \frac{x}{6}$$

(l)

$$\frac{4x^2-25}{4x-10} \div \frac{2x+5}{8} = \frac{4x^2-25}{4x-10} \times \frac{8}{(2x+5)} = \frac{(\cancel{2x+5})^1(\cancel{2x-5})^1}{2(\cancel{2x-5})_1} \times \frac{8}{(\cancel{2x+5})_1} = \frac{1 \times 8}{2 \times 1} = 4$$

(m)

$$\frac{x+3}{x^2+10x+25} \times \frac{x^2+5x}{x^2+3x} = \frac{\cancel{x+3}^1}{(\cancel{x+5})_1(x+5)} \times \frac{\cancel{x^1}(x+5)^1}{\cancel{x}_1(\cancel{x+3})_1} = \frac{1}{x+5}$$

(n)

$$\frac{3y^2+4y-4}{10} \div \frac{3y+6}{15} = \frac{3y^2+4y-4}{10} \times \frac{15}{3y+6} = \frac{(3y-2)(\cancel{y+2})^1}{\cancel{10}_2} \times \frac{\cancel{15}^3}{\cancel{3}(y+2)_1} = \frac{3y-2}{2}$$

(o)

$$\frac{x^2+2xy+y^2}{2} \times \frac{4}{(x-y)^2} = \frac{(x+y)^2}{\cancel{2}_1} \times \frac{\cancel{4}^2}{(x-y)^2} = \frac{2(x+y)^2}{(x-y)^2}$$

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Exercise C, Question 1

Question:

Simplify:

(a) $\frac{1}{p} + \frac{1}{q}$

(b) $\frac{a}{b} - 1$

(c) $\frac{1}{2x} + \frac{1}{x}$

(d) $\frac{3}{x^2} - \frac{1}{x}$

(e) $\frac{3}{4x} + \frac{1}{8x}$

(f) $\frac{x}{y} + \frac{y}{x}$

(g) $\frac{1}{x+2} - \frac{1}{x+1}$

(h) $\frac{2}{x+3} - \frac{1}{x-2}$

(i) $\frac{1}{3} (x+2) - \frac{1}{2} (x+3)$

(j) $\frac{3x}{(x+4)^2} - \frac{1}{(x+4)}$

(k) $\frac{1}{2(x+3)} + \frac{1}{3(x-1)}$

(l) $\frac{2}{x^2+2x+1} + \frac{1}{x+1}$

$$(m) \frac{3}{x^2 + 3x + 2} - \frac{2}{x^2 + 4x + 4}$$

$$(n) \frac{2}{a^2 + 6a + 9} - \frac{3}{a^2 + 4a + 3}$$

$$(o) \frac{2}{y^2 - x^2} + \frac{3}{y - x}$$

$$(p) \frac{x + 2}{x^2 - x - 12} - \frac{x + 1}{x^2 + 5x + 6}$$

$$(q) \frac{3x + 1}{(x + 2)^3} - \frac{2}{(x + 2)^2} + \frac{4}{(x + 2)}$$

Solution:

$$(a) \frac{1}{p} + \frac{1}{q} = \frac{q}{pq} + \frac{p}{pq} = \frac{q + p}{pq}$$

$$(b) \frac{a}{b} - 1 = \frac{a}{b} - \frac{1}{1} = \frac{a}{b} - \frac{b}{b} = \frac{a - b}{b}$$

$$(c) \frac{1}{2x} + \frac{1}{x} = \frac{1}{2x} + \frac{2}{2x} = \frac{1 + 2}{2x} = \frac{3}{2x}$$

$$(d) \frac{3}{x^2} - \frac{1}{x} = \frac{3}{x^2} - \frac{x}{x^2} = \frac{3 - x}{x^2}$$

$$(e) \frac{3}{4x} + \frac{1}{8x} = \frac{6}{8x} + \frac{1}{8x} = \frac{7}{8x}$$

$$(f) \frac{x}{y} + \frac{y}{x} = \frac{x^2}{xy} + \frac{y^2}{xy} = \frac{x^2 + y^2}{xy}$$

$$(g) \frac{1}{(x + 2)} - \frac{1}{(x + 1)} = \frac{x + 1}{(x + 2)(x + 1)} - \frac{x + 2}{(x + 2)(x + 1)} =$$

$$\frac{(x + 1) - (x + 2)}{(x + 2)(x + 1)} = \frac{-1}{(x + 2)(x + 1)}$$

$$\begin{aligned}
 \text{(h)} \quad \frac{2}{(x+3)} - \frac{1}{(x-2)} &= \frac{2(x-2)}{(x+3)(x-2)} - \frac{(x+3)}{(x+3)(x-2)} = \\
 &= \frac{x-7}{(x+3)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i)} \quad \frac{1}{3}(x+2) - \frac{1}{2}(x+3) &= \frac{x+2}{3} - \frac{x+3}{2} = \frac{2(x+2)}{6} - \frac{3(x+3)}{6} = \\
 &= \frac{-x-5}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(j)} \quad \frac{3x}{(x+4)^2} - \frac{1}{(x+4)} &= \frac{3x}{(x+4)^2} - \frac{x+4}{(x+4)^2} = \frac{3x - (x+4)}{(x+4)^2} = \frac{2x-4}{(x+4)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(k)} \quad \frac{1}{2(x+3)} + \frac{1}{3(x-1)} &= \frac{3(x-1)}{6(x+3)(x-1)} + \frac{2(x+3)}{6(x+3)(x-1)} \\
 &= \frac{3(x-1) + 2(x+3)}{6(x+3)(x-1)} \\
 &= \frac{5x+3}{6(x+3)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(l)} \quad \frac{2}{x^2+2x+1} + \frac{1}{x+1} &= \frac{2}{(x+1)^2} + \frac{1}{(x+1)} \\
 &= \frac{2}{(x+1)^2} + \frac{x+1}{(x+1)^2} \\
 &= \frac{2+x+1}{(x+1)^2} \\
 &= \frac{x+3}{(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(m)} \quad \frac{3}{x^2+3x+2} - \frac{2}{x^2+4x+4} &= \frac{3}{(x+1)(x+2)} - \frac{2}{(x+2)^2} \\
 &= \frac{3(x+2)}{(x+1)(x+2)^2} - \frac{2(x+1)}{(x+1)(x+2)^2}
 \end{aligned}$$

$$= \frac{3(x+2) - 2(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{x+4}{(x+1)(x+2)^2}$$

$$(n) \frac{2}{a^2 + 6a + 9} - \frac{3}{a^2 + 4a + 3}$$

$$= \frac{2}{(a+3)^2} - \frac{3}{(a+1)(a+3)}$$

$$= \frac{2(a+1)}{(a+1)(a+3)^2} - \frac{3(a+3)}{(a+1)(a+3)^2}$$

$$= \frac{2(a+1) - 3(a+3)}{(a+1)(a+3)^2}$$

$$= \frac{-a-7}{(a+1)(a+3)^2}$$

$$(o) \frac{2}{y^2 - x^2} + \frac{3}{y - x}$$

$$= \frac{2}{(y+x)(y-x)} + \frac{3}{(y-x)}$$

$$= \frac{2}{(y+x)(y-x)} + \frac{3(y+x)}{(y+x)(y-x)}$$

$$= \frac{2+3(y+x)}{(y+x)(y-x)}$$

$$= \frac{2+3y+3x}{(y+x)(y-x)}$$

$$(p) \frac{x+2}{x^2 - x - 12} - \frac{x+1}{x^2 + 5x + 6}$$

$$= \frac{x+2}{(x-4)(x+3)} - \frac{x+1}{(x+3)(x+2)}$$

$$= \frac{(x+2)(x+2)}{(x+2)(x+3)(x-4)} - \frac{(x+1)(x-4)}{(x+2)(x+3)(x-4)}$$

$$= \frac{(x^2 + 4x + 4) - (x^2 - 3x - 4)}{(x+2)(x+3)(x-4)}$$

$$= \frac{7x+8}{(x+2)(x+3)(x-4)}$$

$$\begin{aligned} \text{(q)} \quad & \frac{3x+1}{(x+2)^3} - \frac{2}{(x+2)^2} + \frac{4}{(x+2)} \\ &= \frac{3x+1}{(x+2)^3} - \frac{2(x+2)}{(x+2)^3} + \frac{4(x+2)^2}{(x+2)^3} \\ &= \frac{(3x+1) - (2x+4) + 4(x^2+4x+4)}{(x+2)^3} \\ &= \frac{4x^2+17x+13}{(x+2)^3} \end{aligned}$$

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Exercise D, Question 1

Question:

Express the following improper fractions in ‘mixed’ number form by: (i) using long division, (ii) using the remainder theorem

(a) $\frac{x^3 + 2x^2 + 3x - 4}{x - 1}$

(b) $\frac{2x^3 + 3x^2 - 4x + 5}{x + 3}$

(c) $\frac{x^3 - 8}{x - 2}$

(d) $\frac{2x^2 + 4x + 5}{x^2 - 1}$

(e) $\frac{8x^3 + 2x^2 + 5}{2x^2 + 2}$

(f) $\frac{4x^3 - 5x^2 + 3x - 14}{x^2 + 2x - 1}$

(g) $\frac{x^4 + 3x^2 - 4}{x^2 + 1}$

(h) $\frac{x^4 - 1}{x + 1}$

(i) $\frac{2x^4 + 3x^3 - 2x^2 + 4x - 6}{x^2 + x - 2}$

Solution:

(a) (i)

$$\begin{array}{r}
 x^2 + 3x + 6 \\
 x-1 \overline{) x^3 + 2x^2 + 3x - 4} \\
 \underline{x^3 - x^2} \\
 3x^2 + 3x \\
 \underline{3x^2 - 3x} \\
 6x - 4 \\
 \underline{6x - 6} \\
 2
 \end{array}$$

(ii) Let $x^3 + 2x^2 + 3x - 4 \equiv (Ax^2 + Bx + C)(x - 1) + R$

Let $x = 1$

$$1 + 2 + 3 - 4 = (A + B + C) \times 0 + R$$

$$\Rightarrow 2 = R$$

Equate terms in $x^3 \Rightarrow 1 = A$

Equate terms in x^2

$$\Rightarrow 2 = -A + B \quad (\text{substitute } A = 1)$$

$$\Rightarrow 2 = -1 + B$$

$$\Rightarrow B = 3$$

Equate constant terms

$$\Rightarrow -4 = -C + R \quad (\text{substitute } R = 2)$$

$$\Rightarrow -4 = -C + 2$$

$$\Rightarrow C = 6$$

Hence $\frac{x^3 + 2x^2 + 3x - 4}{x - 1} \equiv x^2 + 3x + 6 + \frac{2}{x - 1}$

(b) (i)

$$\begin{array}{r}
 2x^2 - 3x + 5 \\
 x+3 \overline{) 2x^3 + 3x^2 - 4x + 5} \\
 \underline{2x^3 + 6x^2} \\
 -3x^2 - 4x \\
 \underline{-3x^2 - 9x} \\
 5x + 5 \\
 \underline{5x + 15} \\
 -10
 \end{array}$$

(ii) Let $2x^3 + 3x^2 - 4x + 5 \equiv (Ax^2 + Bx + C)(x + 3) + R$

Let $x = -3$

$$2 \times -27 + 3 \times 9 + 12 + 5 = (9A - 3B + C) \times 0 + R$$

$$\Rightarrow -10 = R$$

Equate terms in $x^3 \Rightarrow 2 = A$

Equate terms in x^2

$$\Rightarrow 3 = B + 3A \quad (\text{substitute } A = 2)$$

$$\Rightarrow 3 = B + 6$$

$$\Rightarrow B = -3$$

Equate constant terms

$$\Rightarrow 5 = 3C + R \quad (\text{substitute } R = -10)$$

$$\Rightarrow 5 = 3C - 10$$

$$\Rightarrow 3C = 15$$

$$\Rightarrow C = 5$$

Hence $\frac{2x^3 + 3x^2 - 4x + 5}{x + 3} \equiv 2x^2 - 3x + 5 - \frac{10}{x + 3}$

(c) (i)

$$\begin{array}{r} x^2 + 2x + 4 \\ x - 2 \overline{) x^3 + 0x^2 + 0x - 8} \\ \underline{x^3 - 2x^2} \\ 2x^2 + 0x \\ \underline{2x^2 - 4x} \\ 4x - 8 \\ \underline{4x - 8} \\ 0 \end{array}$$

(ii) Let $x^3 - 8 \equiv (Ax^2 + Bx + C)(x - 2) + R$

Let $x = 2$

$$8 - 8 = (4A + 2B + C) \times 0 + R$$

$$\Rightarrow 0 = R$$

Equate terms in $x^3 \Rightarrow 1 = A$

Equate terms in x^2

$$\Rightarrow 0 = B - 2A \quad (\text{substitute } A = 1)$$

$$\Rightarrow 0 = B - 2$$

$$\Rightarrow B = 2$$

Equate constant terms

$$\Rightarrow -8 = -2C + R \quad (\text{substitute } R = 0)$$

$$\Rightarrow -2C = -8$$

$$\Rightarrow C = 4$$

Hence $\frac{x^3 - 8}{x - 2} \equiv x^2 + 2x + 4$

There is no remainder. So $(x - 2)$ is a factor.

(d) (i)

$$\begin{array}{r}
 2 \\
 x^2 + 0x - 1 \overline{) 2x^2 + 4x + 5} \\
 \underline{2x^2 + 0x - 2} \\
 4x + 7
 \end{array}$$

$4x + 7$ is 'less than' $(x^2 - 1)$ so it is the remainder.

(ii) Let

$$2x^2 + 4x + 5 \equiv A(x^2 - 1) + \frac{Bx + C}{1}$$

If the divisor is quadratic then the remainder can be linear.

$$\text{Equate terms in } x^2 \Rightarrow 2 = A$$

$$\text{Equate terms in } x \Rightarrow 4 = B$$

Equate constant terms

$$\Rightarrow 5 = -A + C \quad (\text{substitute } A = 2)$$

$$\Rightarrow 5 = -2 + C$$

$$\Rightarrow C = 7$$

$$\text{Hence } \frac{2x^2 + 4x + 5}{x^2 - 1} \equiv 2 + \frac{4x + 7}{x^2 - 1}$$

(e) (i)

$$\begin{array}{r}
 4x + 1 \\
 2x^2 + 0x + 2 \overline{) 8x^3 + 2x^2 + 0x + 5} \\
 \underline{8x^3 + 0x^2 + 8x} \\
 2x^2 - 8x + 5 \\
 \underline{2x^2 + 0x + 2} \\
 -8x + 3
 \end{array}$$

$$(ii) \text{ Let } 8x^3 + 2x^2 + 5 \equiv (Ax + B)(2x^2 + 2) + Cx + D$$

Equate terms in x^3

$$\Rightarrow 8 = 2A$$

$$\Rightarrow A = 4$$

Equate terms in x^2

$$\Rightarrow 2 = 2B$$

$$\Rightarrow B = 1$$

Equate terms in x

$$\Rightarrow 0 = 2A + C \quad (\text{substitute } A = 4)$$

$$\Rightarrow 0 = 8 + C$$

$$\Rightarrow C = -8$$

Equate constant terms

$$\Rightarrow 5 = 2B + D \quad (\text{substitute } B = 1)$$

$$\Rightarrow 5 = 2 + D$$

$$\Rightarrow D = 3$$

Hence
$$\frac{8x^3 + 2x^2 + 5}{2x^2 + 2} \equiv 4x + 1 + \frac{-8x + 3}{2x^2 + 2}$$

(f) (i)

$$\begin{array}{r} 4x - 13 \\ x^2 + 2x - 1 \overline{) 4x^3 - 5x^2 + 3x - 14} \\ \underline{4x^3 + 8x^2 - 4x} \\ -13x^2 + 7x - 14 \\ \underline{-13x^2 - 26x + 13} \\ 33x - 27 \end{array}$$

(ii) Let $4x^3 - 5x^2 + 3x - 14 \equiv (Ax + B)(x^2 + 2x - 1) + Cx + D$

Equate terms in $x^3 \Rightarrow 4 = A$

Equate terms in x^2

$$\Rightarrow -5 = B + 2A \quad (\text{substitute } A = 4)$$

$$\Rightarrow -5 = B + 8$$

$$\Rightarrow B = -13$$

Equate terms in x

$$\Rightarrow 3 = -A + 2B + C \quad (\text{substitute } A = 4, B = -13)$$

$$\Rightarrow 3 = -4 + (-26) + C$$

$$\Rightarrow C = 33$$

Equate constant terms

$$\Rightarrow -14 = -B + D \quad (\text{substitute } B = -13)$$

$$\Rightarrow -14 = 13 + D$$

$$\Rightarrow D = -27$$

Hence
$$\frac{4x^3 - 5x^2 + 3x - 14}{x^2 + 2x - 1} \equiv 4x - 13 + \frac{33x - 27}{x^2 + 2x - 1}$$

(g) (i)

$$\begin{array}{r}
 x^2 + 2 \\
 x^2 + 0x + 1 \overline{) x^4 + 0x^3 + 3x^2 + 0x - 4} \\
 \underline{x^4 + 0x^3 + x^2} \\
 2x^2 + 0x - 4 \\
 \underline{2x^2 + 0x + 2} \\
 -6
 \end{array}$$

(ii) Let $x^4 + 3x^2 - 4 \equiv (Ax^2 + Bx + C)(x^2 + 1) + Dx + E$

Equate terms in $x^4 \Rightarrow 1 = A$

Equate terms in $x^3 \Rightarrow 0 = B$

Equate terms in x^2

$$\Rightarrow 3 = A + C \quad (\text{substitute } A = 1)$$

$$\Rightarrow 3 = 1 + C$$

$$\Rightarrow C = 2$$

Equate terms in x

$$\Rightarrow 0 = B + D \quad (\text{substitute } B = 0)$$

$$\Rightarrow 0 = 0 + D$$

$$\Rightarrow D = 0$$

Equate constant terms

$$\Rightarrow -4 = C + E \quad (\text{substitute } C = 2)$$

$$\Rightarrow -4 = 2 + E$$

$$\Rightarrow E = -6$$

Hence $\frac{x^4 + 3x^2 - 4}{x^2 + 1} \equiv x^2 + 2 - \frac{6}{x^2 + 1}$

(h) (i)

$$\begin{array}{r}
 x^3 - x^2 + x - 1 \\
 x + 1 \overline{) x^4 + 0x^3 + 0x^2 + 0x - 1} \\
 \underline{x^4 + x^3} \\
 -x^3 + 0x^2 \\
 \underline{-x^3 - x^2} \\
 x^2 + 0x \\
 \underline{x^2 + x} \\
 -x - 1 \\
 \underline{-x - 1} \\
 0
 \end{array}$$

There is no remainder so $(x + 1)$ is a factor of $x^4 - 1$.

(ii) Let $x^4 - 1 \equiv (Ax^3 + Bx^2 + Cx + D)(x + 1) + E$

Let $x = -1$

$$\begin{aligned} (-1)^4 - 1 &= (-A + B - C + D) \times 0 + E \\ \Rightarrow E &= 0 \end{aligned}$$

Equate terms in $x^4 \Rightarrow 1 = A$

Equate terms in x^3

$$\Rightarrow 0 = A + B \quad (\text{substitute } A = 1)$$

$$\Rightarrow 0 = 1 + B$$

$$\Rightarrow B = -1$$

Equate terms in x^2

$$\Rightarrow 0 = B + C \quad (\text{substitute } B = -1)$$

$$\Rightarrow 0 = -1 + C$$

$$\Rightarrow C = 1$$

Equate terms in x

$$\Rightarrow 0 = D + C \quad (\text{substitute } C = 1)$$

$$\Rightarrow 0 = D + 1$$

$$\Rightarrow D = -1$$

Hence $\frac{x^4 - 1}{x + 1} \equiv x^3 - x^2 + x - 1$

(i) (i)

$$\begin{array}{r} 2x^2 + x + 1 \\ x^2 + x - 2 \overline{) 2x^4 + 3x^3 - 2x^2 + 4x - 6} \\ \underline{2x^4 + 2x^3 - 4x^2} \\ x^3 + 2x^2 + 4x \\ \underline{x^3 + x^2 - 2x} \\ x^2 + 6x - 6 \\ \underline{x^2 + x - 2} \\ 5x - 4 \end{array}$$

(ii) Let $2x^4 + 3x^3 - 2x^2 + 4x - 6 \equiv (Ax^2 + Bx + C)(x^2 + x - 2) + Dx + E$

Equate terms in $x^4 \Rightarrow 2 = A$

Equate terms in x^3

$$\Rightarrow 3 = A + B \quad (\text{substitute } A = 2)$$

$$\Rightarrow 3 = 2 + B$$

$$\Rightarrow B = 1$$

Equate terms in x^2

$$\Rightarrow -2 = -2A + B + C \quad (\text{substitute } A = 2, B = 1)$$

$$\Rightarrow -2 = -4 + 1 + C$$

$$\Rightarrow C = 1$$

Equate terms in x

$$\Rightarrow 4 = C - 2B + D \quad (\text{substitute } C = 1, B = 1)$$

$$\Rightarrow 4 = 1 - 2 + D$$

$$\Rightarrow D = 5$$

Equate constant terms

$$\Rightarrow -6 = -2C + E \quad (\text{substitute } C = 1)$$

$$\Rightarrow -6 = -2 + E$$

$$\Rightarrow E = -4$$

Hence
$$\frac{2x^4 + 3x^3 - 2x^2 + 4x - 6}{x^2 + x - 2} \equiv 2x^2 + x + 1 + \frac{5x - 4}{x^2 + x - 2}$$

Solutionbank

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Exercise D, Question 2

Question:

Find the value of the constants A , B , C , D and E in the following identity:

$$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + Bx + C)(x^2 - 3) + Dx + E$$

Solution:

$$3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (Ax^2 + Bx + C)(x^2 - 3) + Dx + E$$

$$\text{Equate terms in } x^4 \Rightarrow 3 = A$$

$$\text{Equate terms in } x^3 \Rightarrow -4 = B$$

$$\text{Equate terms in } x^2$$

$$\Rightarrow -8 = -3A + C \quad (\text{substitute } A = 3)$$

$$\Rightarrow -8 = -9 + C$$

$$\Rightarrow C = 1$$

$$\text{Equate terms in } x$$

$$\Rightarrow 16 = -3B + D \quad (\text{substitute } B = -4)$$

$$\Rightarrow 16 = 12 + D$$

$$\Rightarrow D = 4$$

$$\text{Equate constant terms}$$

$$\Rightarrow -2 = -3C + E \quad (\text{substitute } C = 1)$$

$$\Rightarrow -2 = -3 + E$$

$$\Rightarrow E = 1$$

$$\text{Hence } 3x^4 - 4x^3 - 8x^2 + 16x - 2 \equiv (3x^2 - 4x + 1)(x^2 - 3) + 4x + 1$$

A good idea in equalities is to check with an easy value of x because it should be true for all values of x .

$$\text{Substitute } x = 1 \text{ into LHS} \Rightarrow 3 - 4 - 8 + 16 - 2 = 5$$

$$\text{Substitute } x = 1 \text{ into RHS} \Rightarrow \left(\begin{matrix} 3 & -4 & +1 \end{matrix} \right) \times \left(\begin{matrix} 1 & -3 \end{matrix} \right)$$

$$+ 4 + 1 = 0 \times -2 + 4 + 1 = 5 \checkmark$$

Solutionbank

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Exercise E, Question 1

Question:

Simplify the following fractions:

(a) $\frac{ab}{c} \times \frac{c^2}{a^2}$

(b) $\frac{x^2 + 2x + 1}{4x + 4}$

(c) $\frac{x^2 + x}{2} \div \frac{x + 1}{4}$

(d) $\frac{x + \frac{1}{x} - 2}{x - 1}$

(e) $\frac{a + 4}{a + 8}$

(f) $\frac{b^2 + 4b - 5}{b^2 + 2b - 3}$

Solution:

(a)

$$\frac{ab}{c} \times \frac{c^2}{a^2} = \frac{b \times c}{a} = \frac{bc}{a}$$

(b)

$$\frac{x^2 + 2x + 1}{4x + 4} = \frac{(x+1)(\cancel{x+1})}{4(\cancel{x+1})} = \frac{x+1}{4}$$

(c)

$$\frac{x^2 + x}{2} \div \frac{x+1}{4} = \frac{x^2 + x}{2} \times \frac{4}{x+1} = \frac{x(\cancel{x+1})^1}{\cancel{2}_1} \times \frac{\cancel{4}^2}{(\cancel{x+1})_1} = 2x$$

(d)

$$\frac{x+1x-2}{x-1} \times x = \frac{x^2+1-2x}{x(x-1)} = \frac{(x-1)(\cancel{x-1})^1}{x(\cancel{x-1})_1} = \frac{x-1}{x}$$

(e) $\frac{a+4}{a+8}$ doesn't simplify as there are no common factors.

(f)

$$\frac{b^2+4b-5}{b^2+2b-3} = \frac{(b+5)(\cancel{b-1})^1}{(b+3)(\cancel{b-1})_1} = \frac{b+5}{b+3}$$

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Exercise E, Question 2

Question:

Simplify:

(a) $\frac{x}{4} + \frac{x}{3}$

(b) $\frac{4}{y} - \frac{3}{2y}$

(c) $\frac{x+1}{2} - \frac{x-2}{3}$

(d) $\frac{x^2 - 5x - 6}{x - 1}$

(e) $\frac{x^3 + 7x - 1}{x + 2}$

(f) $\frac{x^4 + 3}{x^2 + 1}$

Solution:

(a) $\frac{x}{4} + \frac{x}{3} = \frac{3x}{12} + \frac{4x}{12} = \frac{7x}{12}$

(b) $\frac{4}{y} - \frac{3}{2y} = \frac{8}{2y} - \frac{3}{2y} = \frac{5}{2y}$

(c) $\frac{x+1}{2} - \frac{x-2}{3} = \frac{3(x+1)}{6} - \frac{2(x-2)}{6} = \frac{3(x+1) - 2(x-2)}{6} = \frac{x+7}{6}$

(d) $\frac{x^2 - 5x - 6}{x - 1} = \frac{(x-6)(x+1)}{(x-1)}$ No common factors so divide.

$$\begin{array}{r}
 x-4 \\
 x-1 \overline{) x^2 - 5x - 6} \\
 \underline{x^2 - 1x} \\
 -4x - 6 \\
 \underline{-4x + 4} \\
 -10
 \end{array}$$

Hence $\frac{x^2 - 5x - 6}{x - 1} = x - 4 - \frac{10}{x - 1}$

(e) $\frac{x^3 + 7x - 1}{x + 2}$

$$\begin{array}{r}
 x^2 - 2x + 11 \\
 x + 2 \overline{) x^3 + 0x^2 + 7x - 1} \\
 \underline{x^3 + 2x^2} \\
 -2x^2 + 7x \\
 \underline{-2x^2 - 4x} \\
 11x - 1 \\
 \underline{11x + 22} \\
 -23
 \end{array}$$

Hence $\frac{x^3 + 7x - 1}{x + 2} = x^2 - 2x + 11 - \frac{23}{x + 2}$

(f) $\frac{x^4 + 3}{x^2 + 1}$

$$\begin{array}{r}
 x^2 - 1 \\
 x^2 + 0x + 1 \overline{) x^4 + 0x^3 + 0x^2 + 0x + 3} \\
 \underline{x^4 + 0x^3 + 1x^2} \\
 -1x^2 + 0x + 3 \\
 \underline{-1x^2 + 0x - 1} \\
 4
 \end{array}$$

Hence $\frac{x^4 + 3}{x^2 + 1} = x^2 - 1 + \frac{4}{x^2 + 1}$

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Exercise E, Question 3

Question:

Find the value of the constants A , B , C and D in the following identity:

$$x^3 - 6x^2 + 11x + 2 \equiv (x - 2)(Ax^2 + Bx + C) + D$$

Solution:

$$x^3 - 6x^2 + 11x + 2 \equiv (x - 2)(Ax^2 + Bx + C) + D$$

Let $x = 2$

$$8 - 24 + 22 + 2 = 0 \times (4A + 2B + C) + D$$

$$\Rightarrow D = 8$$

$$\text{Equate coefficients in } x^3 \Rightarrow 1 = A$$

$$\text{Equate coefficients in } x^2$$

$$\Rightarrow -6 = -2A + B \quad (\text{substitute } A = 1)$$

$$\Rightarrow -6 = -2 + B$$

$$\Rightarrow B = -4$$

$$\text{Equate coefficients in } x$$

$$\Rightarrow 11 = C - 2B \quad (\text{substitute } B = -4)$$

$$\Rightarrow 11 = C + 8$$

$$\Rightarrow C = 3$$

$$\text{Hence } x^3 - 6x^2 + 11x + 2 = (x - 2)(x^2 - 4x + 3) + 8$$

Check. Equate constant terms: $2 = -2 \times 3 + 8 \checkmark$

Solutionbank

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Exercise E, Question 4

Question:

$$f(x) = x + \frac{3}{x-1} - \frac{12}{x^2 + 2x - 3} \quad \{ x \in \mathbb{R}, x > 1 \}$$

$$\text{Show that } f(x) = \frac{x^2 + 3x + 3}{x + 3}$$

[E]

Solution:

$$\begin{aligned} f(x) &= x + \frac{3}{x-1} - \frac{12}{x^2 + 2x - 3} \\ &= \frac{x}{1} + \frac{3}{x-1} - \frac{12}{(x+3)(x-1)} \\ &= \frac{x(x+3)(x-1)}{(x+3)(x-1)} + \frac{3(x+3)}{(x+3)(x-1)} - \frac{12}{(x+3)(x-1)} \\ &= \frac{x(x+3)(x-1) + 3(x+3) - 12}{(x+3)(x-1)} \\ &= \frac{x(x^2 + 2x - 3) + 3x + 9 - 12}{(x+3)(x-1)} \\ &= \frac{x^3 + 2x^2 - 3x + 3x + 9 - 12}{(x+3)(x-1)} \\ &= \frac{x^3 + 2x^2 - 3}{(x+3)(x-1)} \quad [\text{Factorise numerator. } (x-1) \text{ is a factor as } f(1) \\ &= 0.] \\ &= \frac{(x-1)(x^2 + 3x + 3)}{(x+3)(x-1)} \quad (\text{cancel common factors}) \\ &= \frac{x^2 + 3x + 3}{x+3} \end{aligned}$$

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Exercise E, Question 5

Question:

Show that $\frac{x^4 + 2}{x^2 - 1} \equiv x^2 + B + \frac{C}{x^2 - 1}$ for constants B and C , which should be found.

Solution:

We need to find B and C such that

$$\frac{x^4 + 2}{x^2 - 1} \equiv x^2 + B + \frac{C}{x^2 - 1}$$

Multiply both sides by $(x^2 - 1)$:

$$x^4 + 2 \equiv (x^2 + B)(x^2 - 1) + C$$

$$x^4 + 2 \equiv x^4 + Bx^2 - x^2 - B + C$$

Compare terms in x^2

$$\Rightarrow 0 = B - 1$$

$$\Rightarrow B = 1$$

Compare constant terms

$$\Rightarrow 2 = -B + C \quad (\text{substitute } B = 1)$$

$$\Rightarrow 2 = -1 + C$$

$$\Rightarrow C = 3$$

$$\text{Hence } x^4 + 2 \equiv (x^2 + 1)(x^2 - 1) + 3$$

$$\text{So } \frac{x^4 + 2}{x^2 - 1} = x^2 + 1 + \frac{3}{x^2 - 1}$$

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Exercise E, Question 6

Question:

Show that $\frac{4x^3 - 6x^2 + 8x - 5}{2x + 1}$ can be put in the form $Ax^2 + Bx + C + \frac{D}{2x + 1}$. Find the values of the constants A , B , C and D .

Solution:

$$\begin{array}{r}
 2x^2 - 4x + 6 \\
 2x + 1 \overline{) 4x^3 - 6x^2 + 8x - 5} \\
 \underline{4x^3 + 2x^2} \\
 -8x^2 + 8x \\
 \underline{-8x^2 - 4x} \\
 12x - 5 \\
 \underline{12x + 6} \\
 -11
 \end{array}$$

Hence $\frac{4x^3 - 6x^2 + 8x - 5}{2x + 1} = 2x^2 - 4x + 6 - \frac{11}{2x + 1}$

So $A = 2$, $B = -4$, $C = 6$ and $D = -11$