# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise A, Question 1**

### Question:

Draw mapping diagrams and graphs for the following operations:

(a) ‘subtract 5’ on the set { 10 , 5 , 0 , − 5 , $x$ {

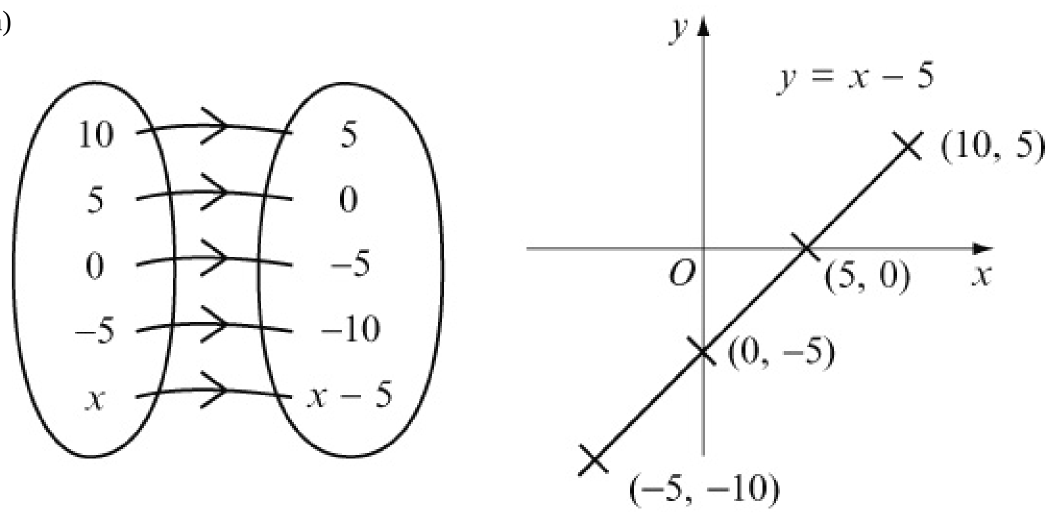
(b) ‘double and add 3’ on the set { − 2 , 2 , 4 , 6 , $x$ {

(c) ‘square and then subtract 1’ on the set { − 3 , − 1 , 0 , 1 , 3 , $x$ {

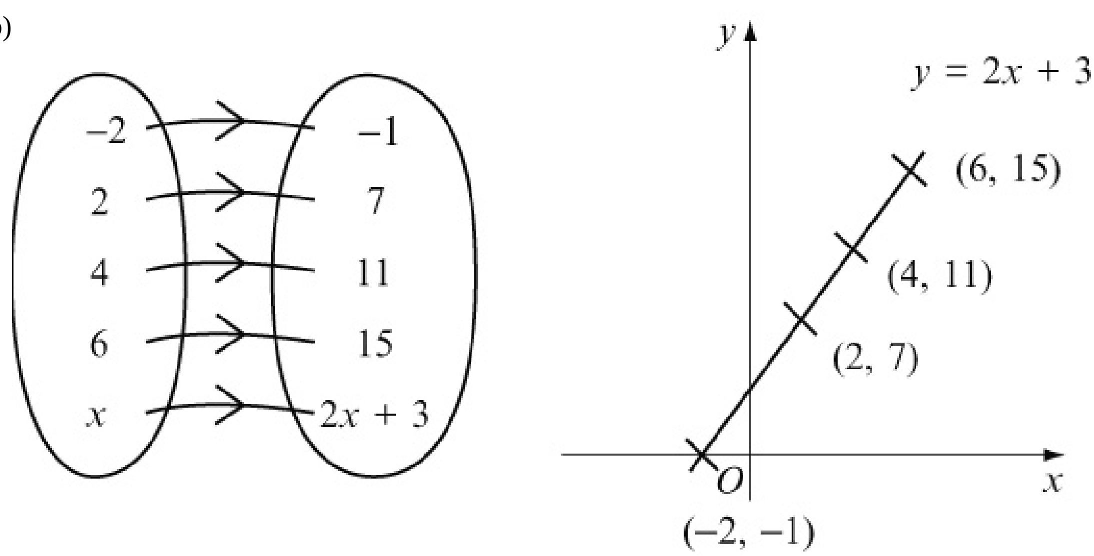
(d) ‘the positive square root’ on the set { − 4 , 0 , 1 , 4 , 9 , $x$ { .

### Solution:

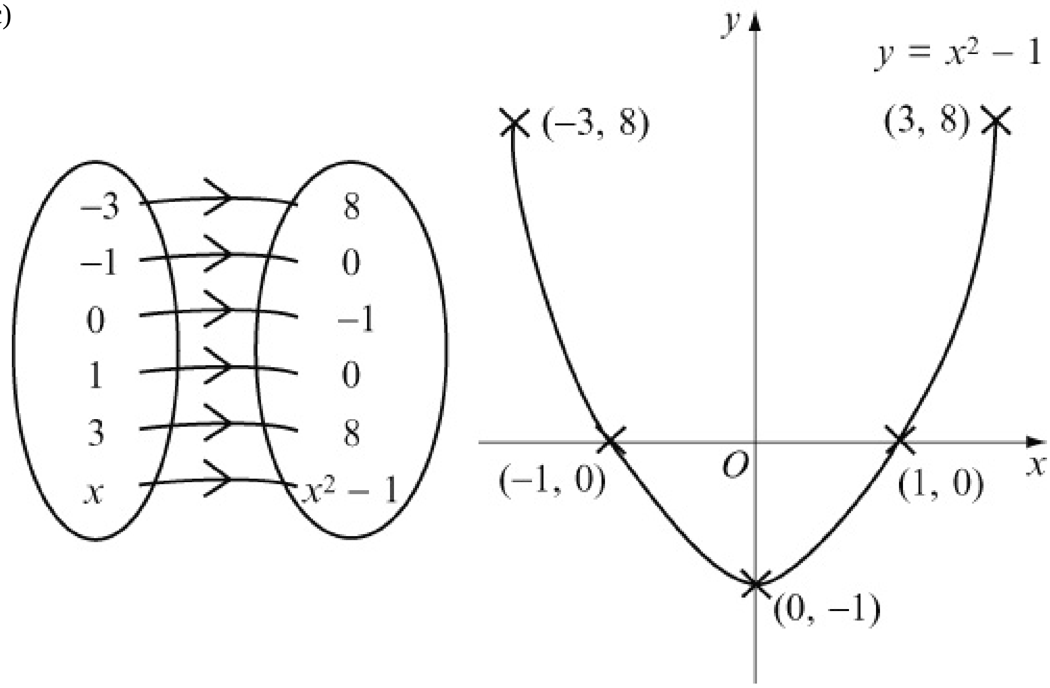
(a)



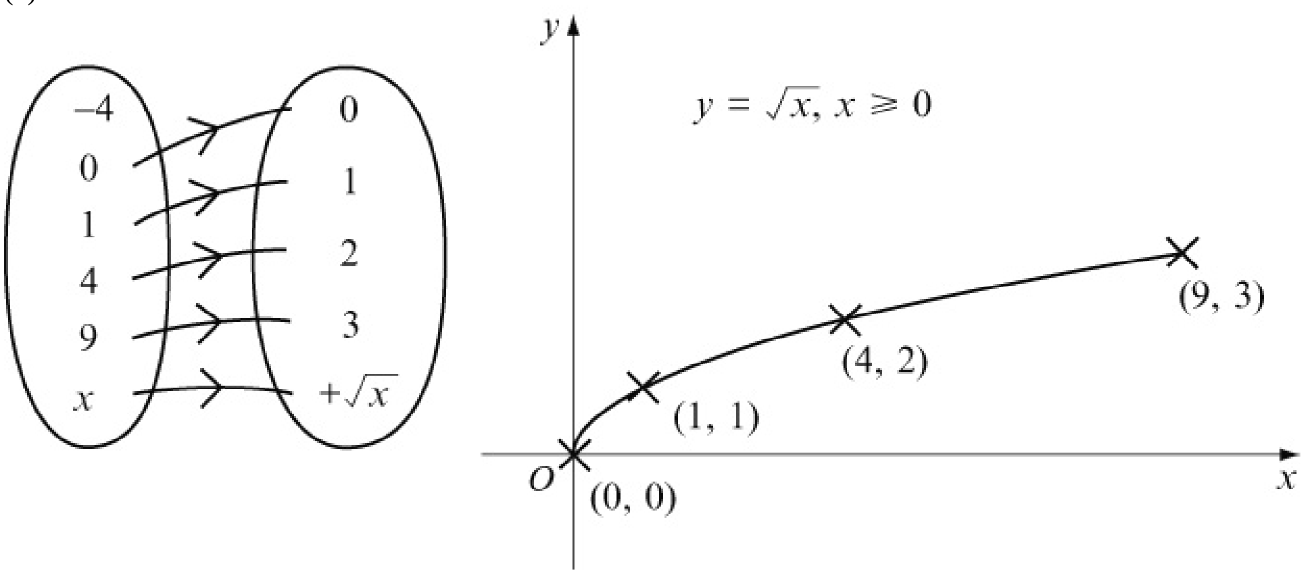

(b)



(c)




(d)




**Note**: You cannot take the square root of a negative number.

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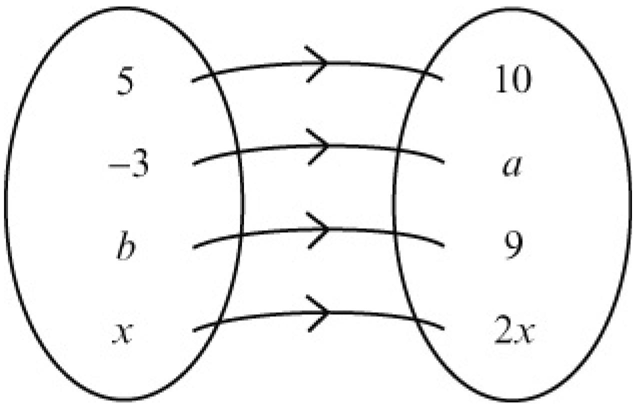
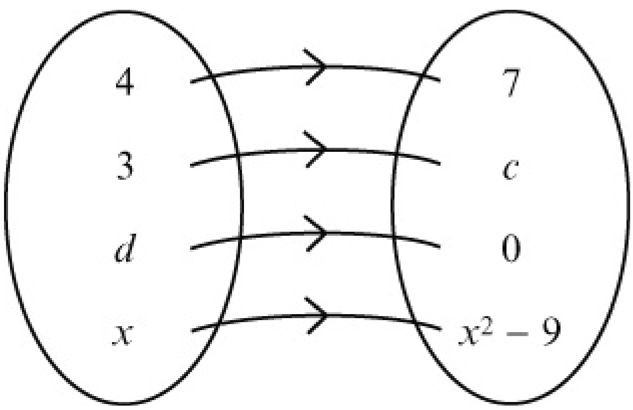
## Edexcel AS and A Level Modular Mathematics

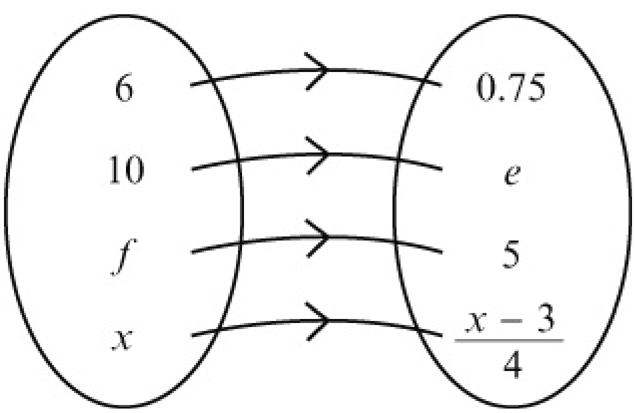
**Exercise A, Question 2**

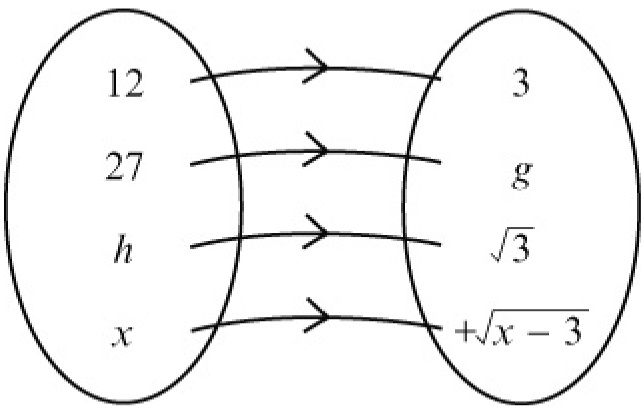
### Question:

Find the missing numbers ***a*** to ***h*** in the following mapping diagrams:






**Solution:**

$x \rightarrow 2x$ is 'doubling'
$-3 \rightarrow a$ so $a = -6$

$b \rightarrow 9$ so $b \times 2 = 9 \Rightarrow b = 4\frac{1}{2}$

$x \rightarrow x^2 - 9$ is 'squaring then subtracting 9'
$3 \rightarrow c$ so $c = 3^2 - 9 = 0$
$d \rightarrow 0$ so $d^2 - 9 = 0 \Rightarrow d^2 = 9 \Rightarrow d = \pm 3$

$x \rightarrow \frac{x-3}{4}$ is 'subtract 3, then divide by 4'

$10 \rightarrow e$ so $e = (10 - 3) \div 4 = 1.75$

$f \rightarrow 5$ so $\frac{f-3}{4} = 5 \Rightarrow f = 23$

$x \rightarrow +\sqrt{x-3}$ is 'subtract 3, then take the positive square root'
$27 \rightarrow g$ so $g = +\sqrt{27-3} = +\sqrt{24} = +2\sqrt{6}$
$h \rightarrow +\sqrt{3}$ so $\sqrt{h-3} = \sqrt{3} \Rightarrow h - 3 = 3 \Rightarrow h = 6$

So $a = -6$, $b = 4\frac{1}{2}$, $c = 0$, $d = \pm 3$, $e = 1.75$, $f = 23$, $g = 2\sqrt{6}$, $h = 6$

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**Exercise B, Question 1**

### Question:

Find:

(a) f(3) where $f(x) = 5x + 1$

(b) $g(-2)$ where $g(x) = 3x^2 - 2$

(c) h(0) where $h : x \rightarrow 3^x$

(d) $j(-2)$ where $j : x \rightarrow 2^{-x}$

### Solution:

(a) $f(x) = 5x + 1$
Substitute $x = 3 \Rightarrow f(3) = 5 \times 3 + 1 = 16$

(b) $g(x) = 3x^2 - 2$
Substitute $x = -2 \Rightarrow g(-2) = 3 \times (-2)^2 - 2 = 3 \times 4 - 2 = 10$

(c) $h(x) = 3^x$
Substitute $x = 0 \Rightarrow h(0) = 3^0 = 1$

(d) $j(x) = 2^{-x}$
Substitute $x = -2 \Rightarrow j(-2) = 2^{-(-2)} = 2^2 = 4$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 2**

### Question:

Calculate the value(s) of $a$, $b$, $c$ and $d$ given that:

(a) $p(a) = 16$ where $p(x) = 3x - 2$

(b) $q(b) = 17$ where $q(x) = x^2 - 3$

(c) $r(c) = 34$ where $r(x) = 2(2^x) + 2$

(d) $s(d) = 0$ where $s(x) = x^2 + x - 6$

### Solution:

(a) $p(x) = 3x - 2$
Substitute $x = a$ and $p(a) = 16$ then
$16 = 3a - 2$
$18 = 3a$
$a = 6$

(b) $q(x) = x^2 - 3$
Substitute $x = b$ and $q(b) = 17$ then
$17 = b^2 - 3$
$20 = b^2$
$b = \pm\sqrt{20}$
$b = \pm 2\sqrt{5}$

(c) $r(x) = 2 \times 2^x + 2$
Substitute $x = c$ and $r(c) = 34$ then
$34 = 2 \times 2^c + 2$
$32 = 2 \times 2^c$
$16 = 2^c$
$c = 4$

(d) $s(x) = x^2 + x - 6$
Substitute $x = d$ and $s(d) = 0$ then
$0 = d^2 + d - 6$
$0 = (d + 3)(d - 2)$
$d = 2, -3$

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## Edexcel AS and A Level Modular Mathematics

**Exercise B, Question 3**

### Question:

For the following functions
(i) sketch the graph of the function
(ii) state the range
(iii) describe if the function is one-to-one or many-to-one.

(a) $m(x) = 3x + 2$

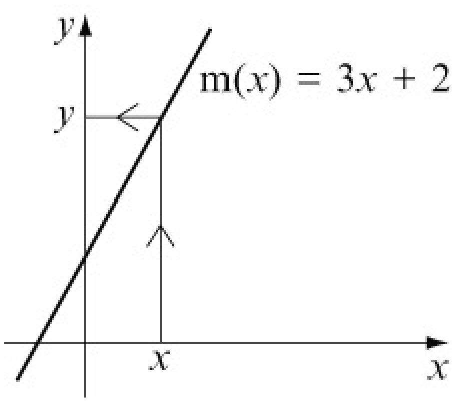
(b) $n(x) = x^2 + 5$

(c) $p(x) = \sin(x)$

(d) $q(x) = x^3$

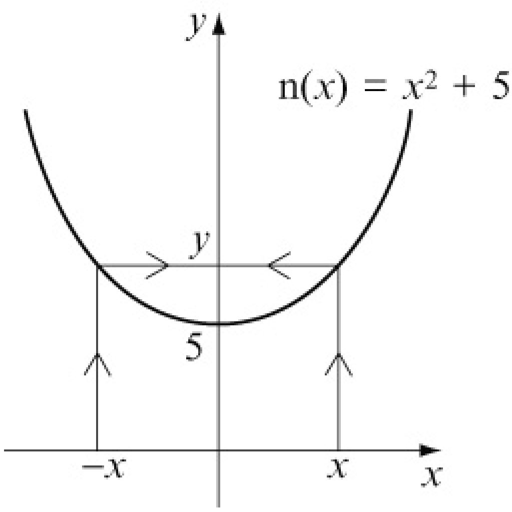
### Solution:

(a) $m(x) = 3x + 2$
(i)



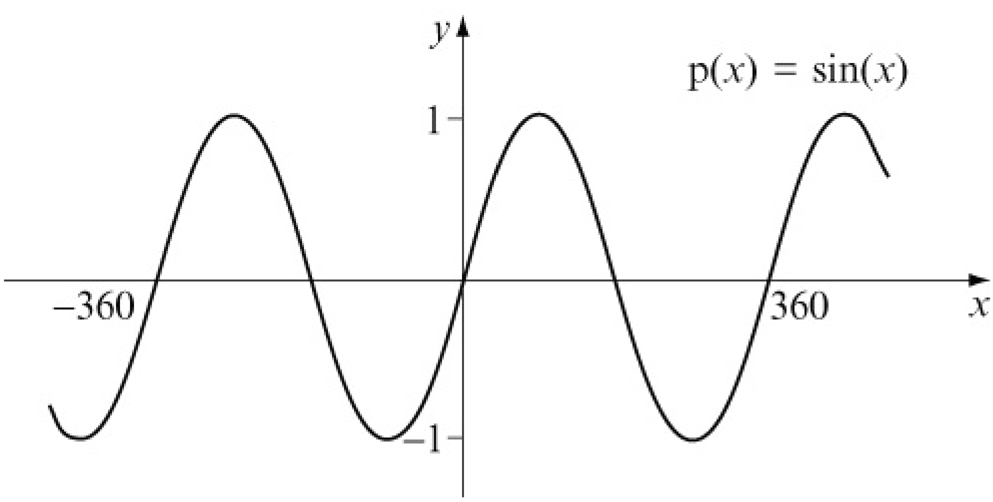

(ii) Range of $m(x)$ is $-\infty < m(x) < \infty$
or $m(x) \in \mathbb{R}$ (all of the real numbers)
(iii) Function is one-to-one

(b) $n(x) = x^2 + 5$
(i)



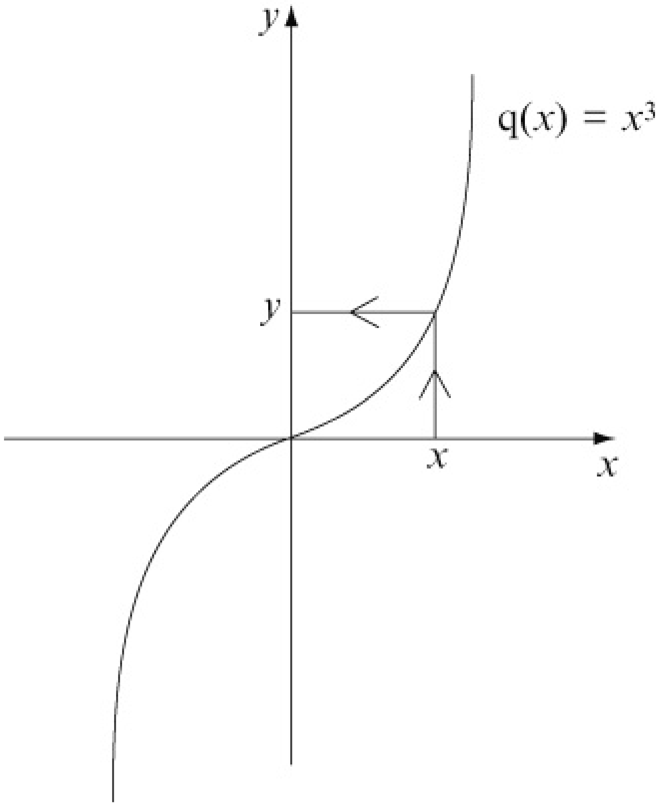

(ii) Range of $n(x)$ is $n(x) \geq 5$
(iii) Function is many-to-one

(c) $p(x) = \sin(x)$
(i)




(ii) Range of $p(x)$ is $-1 \leq p(x) \leq 1$
(iii) Function is many-to-one

(d) $q(x) = x^3$
(i)




(ii) Range of $q(x)$ is $-\infty < q(x) < \infty$ or $q(x) \in \mathbb{R}$

(iii) Function is one-to-one

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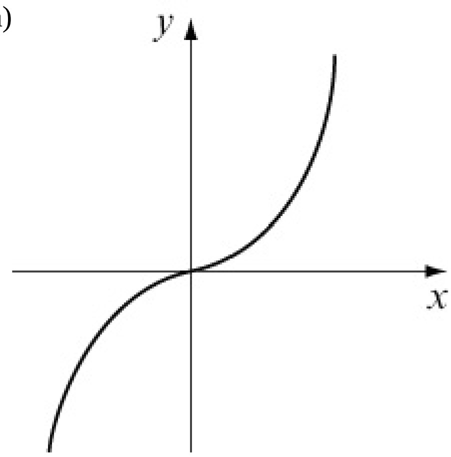
**Exercise B, Question 4**

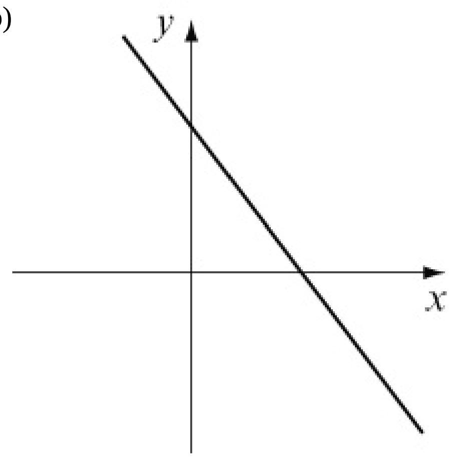
### Question:

State whether or not the following graphs represent functions. Give reasons for your answers and describe the type of function.

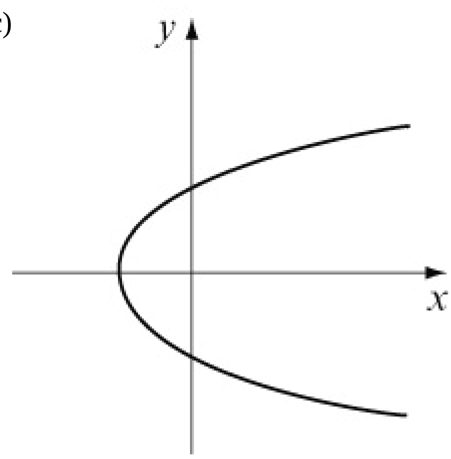
(a)



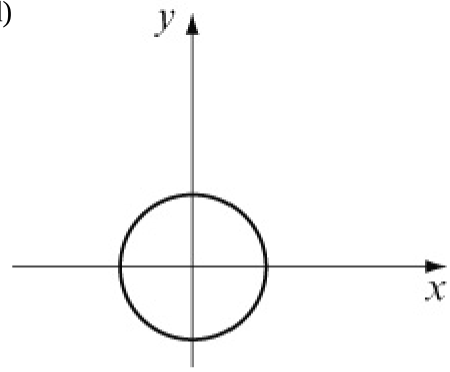

(b)



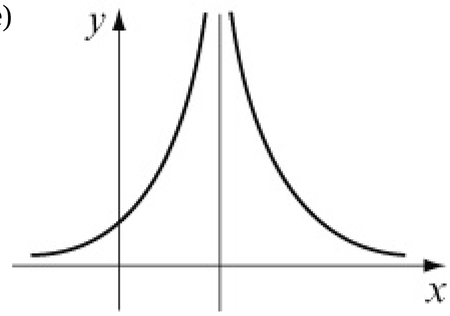

(c)



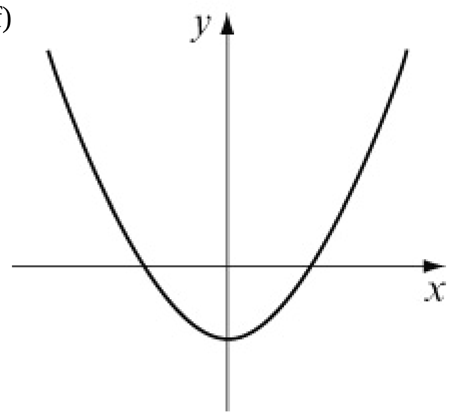
(d)




(e)

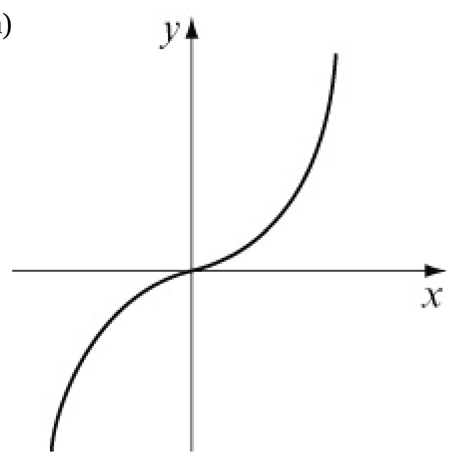



(f)



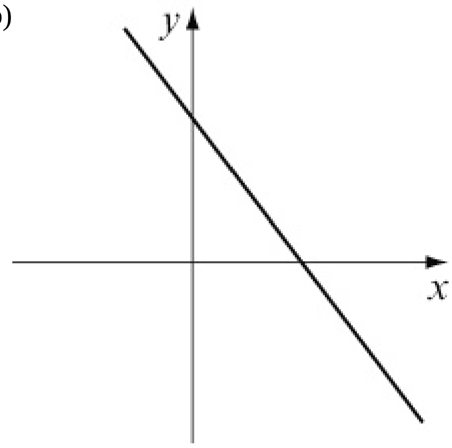

**Solution:**

(a)



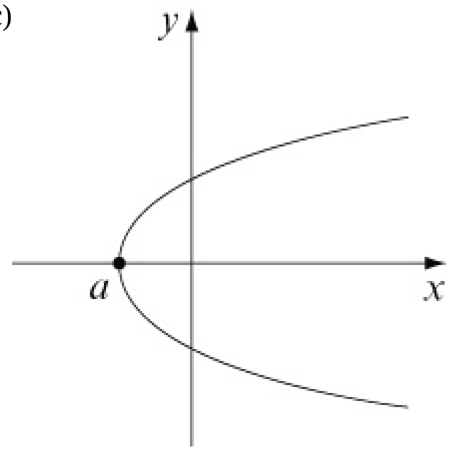

One-to-one function

(b)



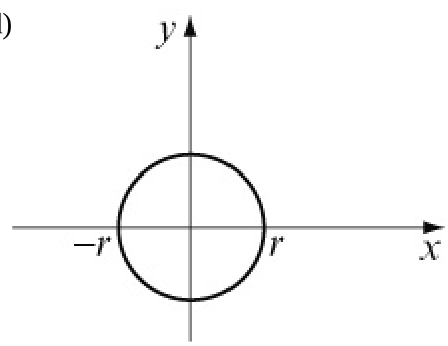

One-to-one function

(c)



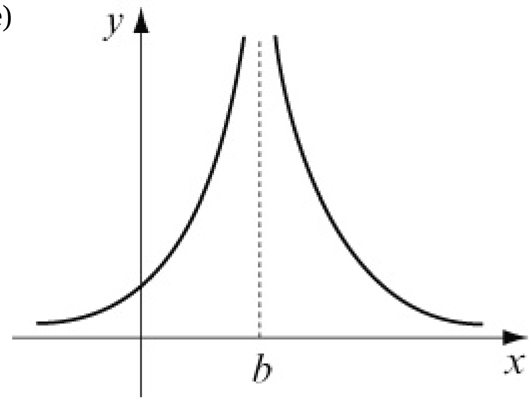

Not a function.
The values left of $x = a$ do not get mapped anywhere.
The values right of $x = a$ get mapped to two values of $y$.

(d)



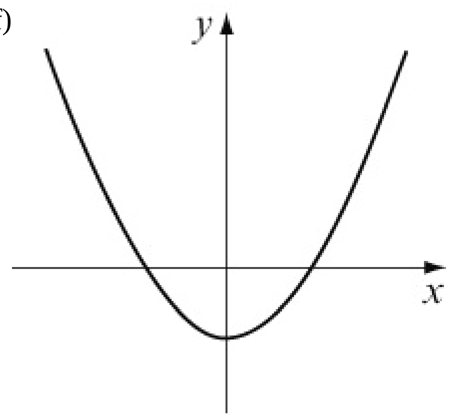

Not a function. Similar to part (c).
Values of $x$ between $-r$ and $+r$ get mapped to two values of $y$.
Values outside this don't get mapped anywhere.

(e)




Not a function. The value $x = b$ doesn't get mapped anywhere.

(f)




Many-to-one function. Two values of $x$ get mapped to the same value of $y$.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 1**

### Question:

The functions below are defined for the discrete domains.
(i) Represent each function on a mapping diagram, writing down the elements in the range.
(ii) State if the function is one-to-one or many-to-one.

(a) $f(x) = 2x + 1$ for the domain $\{ x = 1, 2, 3, 4, 5 \{$ .

(b) $g(x) = + \sqrt{x}$ for the domain $\{ x = 1, 4, 9, 16, 25, 36 \{$ .

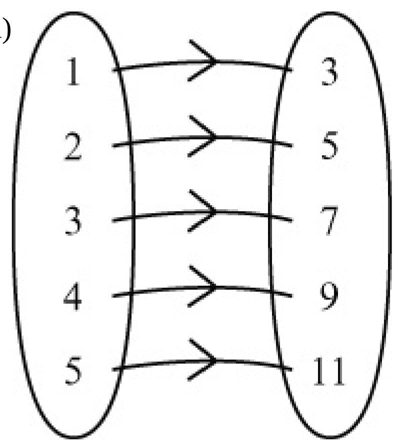
(c) $h(x) = x^2$ for the domain $\{ x = -2, -1, 0, 1, 2 \{$ .

(d) $j(x) = \frac{2}{x}$ for the domain $\{ x = 1, 2, 3, 4, 5 \{$ .

### Solution:

(a) $f(x) = 2x + 1$ 'Double and add 1'

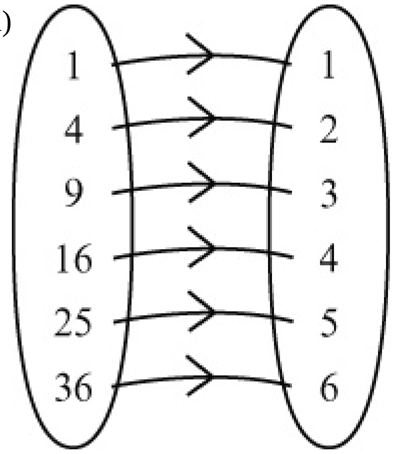
(i)




(ii) One-to-one function

(b) $g(x) = + \sqrt{x}$ 'The positive square root'

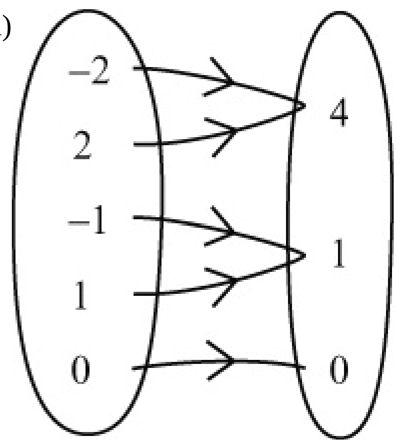
(i)




(ii) One-to-one function

(c) $h(x) = x^2$ 'Square the numbers in the domain'

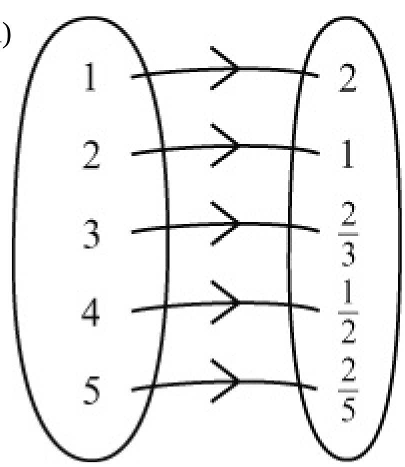
(i)




(ii) Many-to-one function

(d) $j(x) = \frac{2}{x}$ '2 divided by numbers in the domain'

(i)




(ii) One-to-one function

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 2**

### Question:

The functions below are defined for continuous domains.
(i) Represent each function on a graph.
(ii) State the range of the function.
(iii) State if the function is one-to-one or many-to-one.

(a) $m(x) = 3x + 2$ for the domain $\{x > 0\}$.

(b) $n(x) = x^2 + 5$ for the domain $\{x \geq 2\}$.

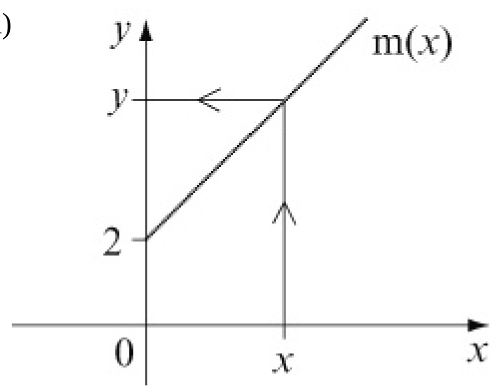
(c) $p(x) = 2\sin x$ for the domain $\{0 \leq x \leq 180\}$.

(d) $q(x) = +\sqrt{x+2}$ for the domain $\{x \geq -2\}$.

### Solution:

(a) $m(x) = 3x + 2$ for $x > 0$

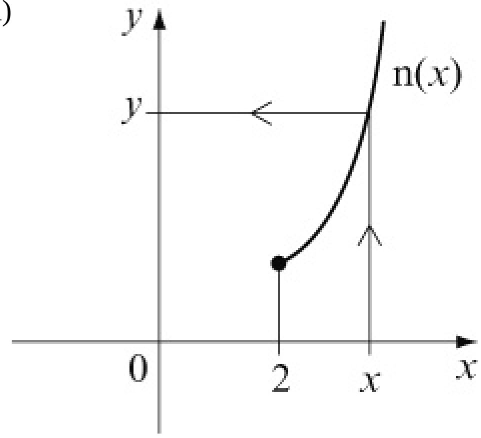
(i)




$3x + 2$ is a linear function of gradient 3 passing through 2 on the $y$ axis.
(ii) $x = 0$ does not exist in the domain
So range is $m(x) > 3 \times 0 + 2 \Rightarrow m(x) > 2$
(iii) $m(x)$ is a one-to-one function

(b) $n(x) = x^2 + 5$ for $x \geq 2$

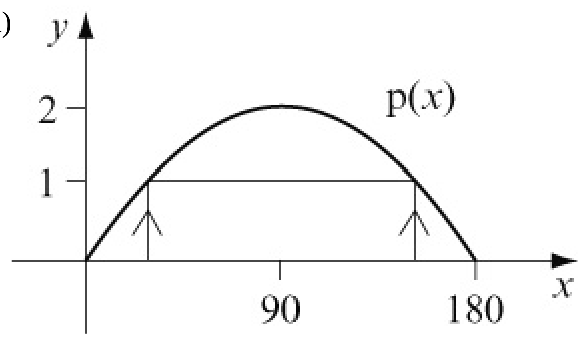
(i)




$x^2 + 5$ is a parabola with minimum point at $(0, 5)$ .
The domain however is only values bigger than or equal to 2.
(ii) $x = 2$ exists in the domain
So range is $n(x) \geq 2^2 + 5 \Rightarrow n(x) \geq 9$
(iii) $n(x)$ is a one-to-one function

(c) $p(x) = 2\sin x$ for $0 \leq x \leq 180$

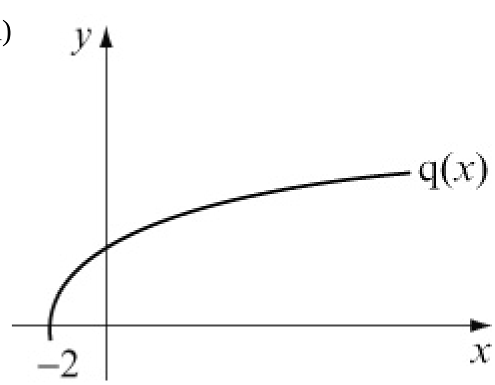
(i)




$2\sin x$ has the same shape as $\sin x$ except that it has been stretched by a factor of 2 parallel to the $y$ axis.
(ii) Range of p($x$) is $0 \leq p(x) \leq 2$
(iii) The function is many-to-one

(d) $q(x) = +\sqrt{x+2}$ for $x \geq -2$

(i)



$\sqrt{x+2}$ is the $\sqrt{x}$ graph translated 2 units to the left.
(ii) The range of q($x$) is $q(x) \geq 0$
(iii) The function is one-to-one

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 3**

### Question:

The mappings f($x$) and g($x$) are defined by

$$f(x) = \begin{cases} 4 - x & x < 4 \\ x^2 + 9 & x \geq 4 \end{cases}$$

$$g(x) = \begin{cases} 4 - x & x < 4 \\ x^2 + 9 & x > 4 \end{cases}$$

Explain why f($x$) is a function and g($x$) is not.
Sketch the function f($x$) and find

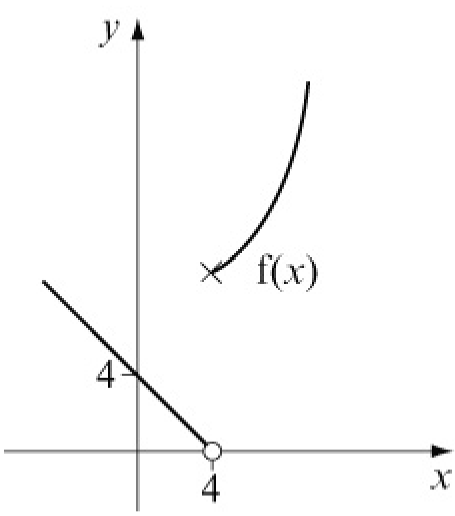
(a) f(3)

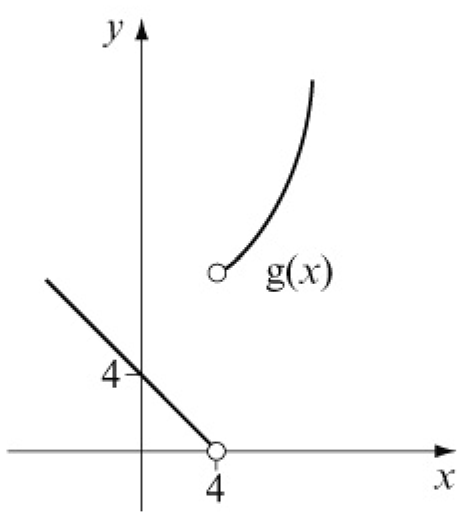
(b) f(10)

(c) the value(s) of $a$ such that $f(a) = 90$.

### Solution:

$4 - x$ is a linear function of gradient $-1$ passing through 4 on the $y$ axis.
$x^2 + 9$ is a $\cup$ -shaped quadratic
At $x = 4$ $4 - x = 0$ and $x^2 + 9 = 25$



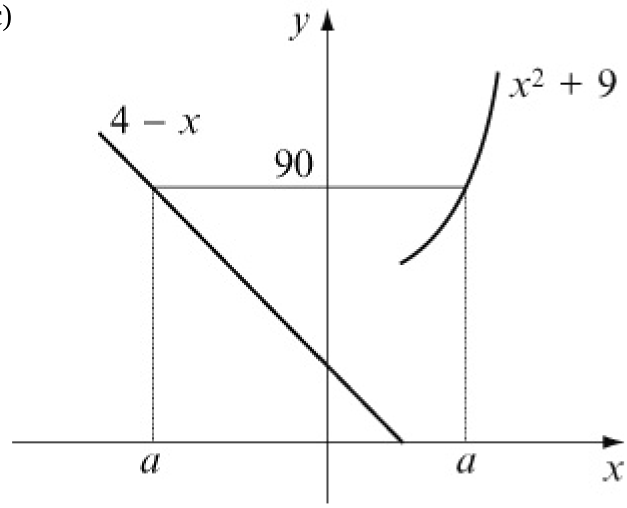




g($x$) is not a function because the element 4 of the domain does not get mapped anywhere.
In f($x$) it gets mapped to 25.

(a) $f(3) = 4 - 3 = 1$ (Use $4 - x$ as $3 < 4$)

(b) $f(10) = 10^2 + 9 = 109$ (Use $x^2 + 9$ as $10 > 4$)

(c)




The negative value of $a$ is where $4 - a = 90 \Rightarrow a = -86$

The positive value of $a$ is where

$a^2 + 9 = 90$

$a^2 = 81$

$a = \pm 9$

$a = 9$

The values of $a$ are $-86$ and 9.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 4**

### Question:

The function s($x$) is defined by

$$s(x) = \begin{cases} x^2 - 6 & x < 0 \\ 10 - x & x \geq 0 \end{cases}$$

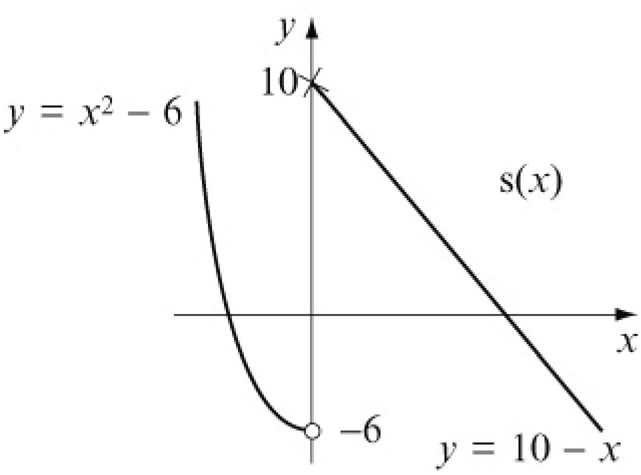
(a) Sketch s($x$).

(b) Find the value(s) of $a$ such that $s(a) = 43$.

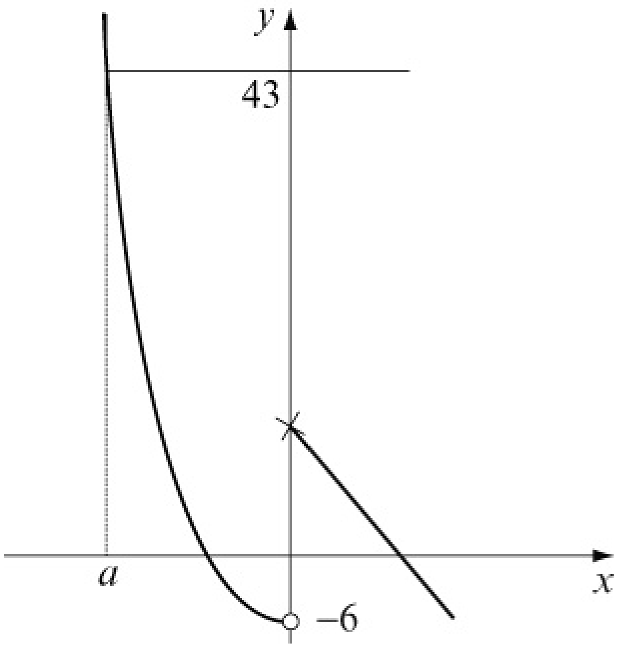
(c) Find the values of the domain that get mapped to themselves in the range.

### Solution:

(a) $x^2 - 6$ is a $\cup$ -shaped quadratic with a minimum value of $(0, -6)$.
$10 - x$ is a linear function with gradient $-1$ passing through 10 on the $y$ axis.




(b) There is only one value of $a$ such that $s(a) = 43$ (see graph).




$s(a) = 43$

$a^2 - 6 = 43$

$a^2 = 49$

$a = \pm 7$

Value is negative so $a = -7$

(c) If value gets mapped to itself then $s(b) = b$

For $10 - x$ part

$10 - b = b$

$\Rightarrow \quad 10 = 2b$

$\Rightarrow \quad b = 5$

**Check.** $s(5) = 10 - 5 = 5$✓

For $x^2 - 6$ part

$b^2 - 6 = b$

$\Rightarrow \quad b^2 - b - 6 = 0$

$\Rightarrow \quad (b - 3)(b + 2) = 0$

$\Rightarrow \quad b = 3, -2$

$b$ must be negative

$\Rightarrow \quad b = -2$

**Check.** $s(-2) = (-2)^2 - 6 = 4 - 6 = -2$✓

Values that get mapped to themselves are $-2$ and 5.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 5**

### Question:

The function g($x$) is defined by $g(x) = cx + d$ where $c$ and $d$ are constants to be found. Given $g(3) = 10$ and $g(8) = 12$ find the values of $c$ and $d$.

### Solution:

$g(x) = cx + d$

$g(3) = 10 \Rightarrow c \times 3 + d = 10$

$g(8) = 12 \Rightarrow c \times 8 + d = 12$

$3c + d = 10$ ①

$8c + d = 12$ ②

② − ①: $5c = 2$ $(\div 5)$

$\Rightarrow c = 0.4$

Substitute $c = 0.4$ into ①:

$3 \times 0.4 + d = 10$

$1.2 + d = 10$

$d = 8.8$

Hence $g(x) = 0.4x + 8.8$

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## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 6**

### Question:

The function f(*x*) is defined by $f(x) = ax^3 + bx - 5$ where *a* and *b* are constants to be found. Given that $f(1) = -4$ and $f(2) = 9$, find the values of the constants *a* and *b*.

### Solution:

$f(x) = ax^3 + bx - 5$

$f(1) = -4 \Rightarrow a \times 1^3 + b \times 1 - 5 = -4$

$\Rightarrow a + b - 5 = -4$

$\Rightarrow a + b = 1$ ①

$f(2) = 9 \Rightarrow a \times 2^3 + b \times 2 - 5 = 9$

$\Rightarrow 8a + 2b - 5 = 9$

$\Rightarrow 8a + 2b = 14$

$\Rightarrow 4a + b = 7$ ②

② − ①: $3a = 6$

$\Rightarrow a = 2$

Substitute $a = 2$ in ①:

$2 + b = 1$

$b = -1$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise C, Question 7**

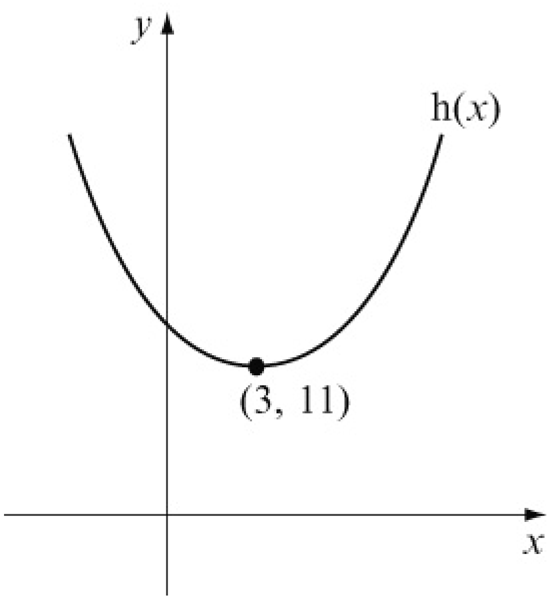
### Question:

The function h($x$) is defined by $h(x) = x^2 - 6x + 20 \{ x \geq a \}$. Given that h($x$) is a one-to-one function find the smallest possible value of the constant $a$.

### Solution:

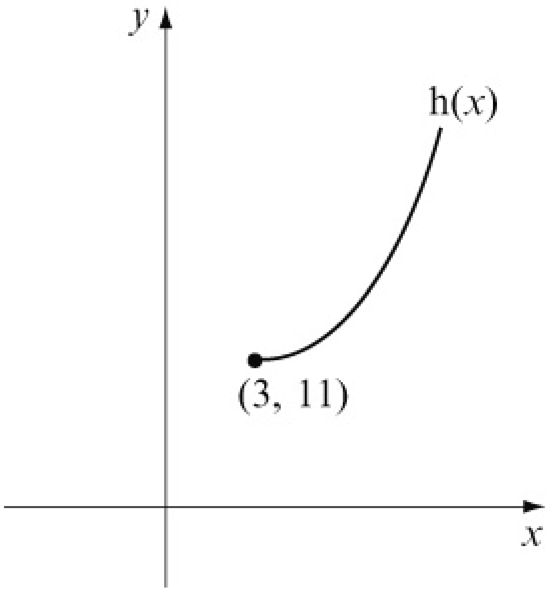
$h(x) = x^2 - 6x + 20 = (x - 3)^2 - 9 + 20 = (x - 3)^2 + 11$

This is a $\cup$ -shaped quadratic with minimum point at (3, 11).




This is a many-to-one function.

For h($x$) to be one-to-one, $x \geq 3$




Hence smallest value of $a$ is 3.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 1**

### Question:

Given the functions $f(x) = 4x + 1$, $g(x) = x^2 - 4$ and $h(x) = \frac{1}{x}$, find expressions for the functions:

(a) $fg(x)$

(b) $gf(x)$

(c) $gh(x)$

(d) $fh(x)$

(e) $f^2(x)$

### Solution:

(a) $fg(x) = f(x^2 - 4) = 4(x^2 - 4) + 1 = 4x^2 - 15$

(b) $gf(x) = g(4x + 1) = (4x + 1)^2 - 4 = 16x^2 + 8x - 3$

(c) $gh(x) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 - 4 = \frac{1}{x^2} - 4$

(d) $fh(x) = f\left(\frac{1}{x}\right) = 4 \times \left(\frac{1}{x}\right) + 1 = \frac{4}{x} + 1$

(e) $f^2(x) = ff(x) = f(4x + 1) = 4(4x + 1) + 1 = 16x + 5$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 2**

### Question:

For the following functions f($x$) and g($x$), find the composite functions fg($x$) and gf($x$). In each case find a suitable domain and the corresponding range when

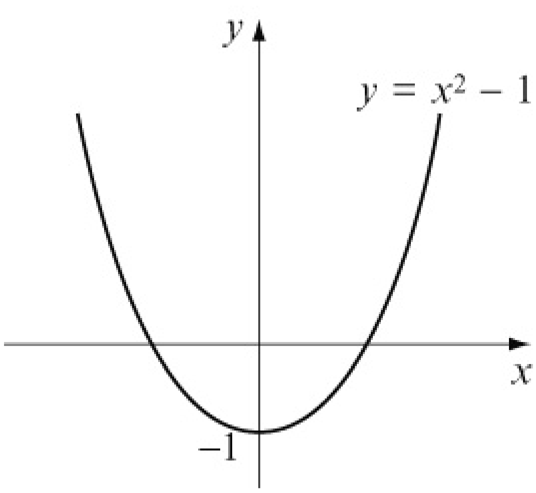
(a) $f(x) = x - 1$, $g(x) = x^2$

(b) $f(x) = x - 3$, $g(x) = + \sqrt{x}$

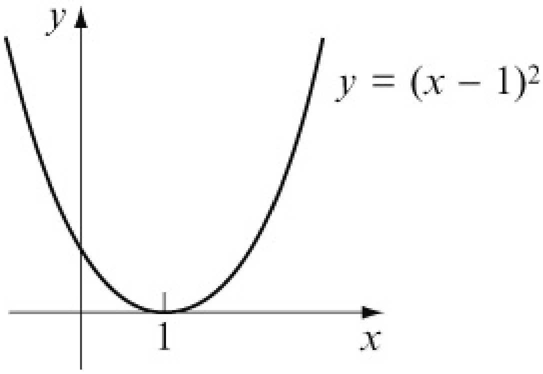
(c) $f(x) = 2^x$, $g(x) = x + 3$

### Solution:

(a) $f(x) = x - 1$, $g(x) = x^2$
$fg(x) = f(x^2) = x^2 - 1$
Domain $x \in \mathbb{R}$
Range $fg(x) \geq -1$




$gf(x) = g(x - 1) = (x - 1)^2$
Domain $x \in \mathbb{R}$
Range $gf(x) \geq 0$



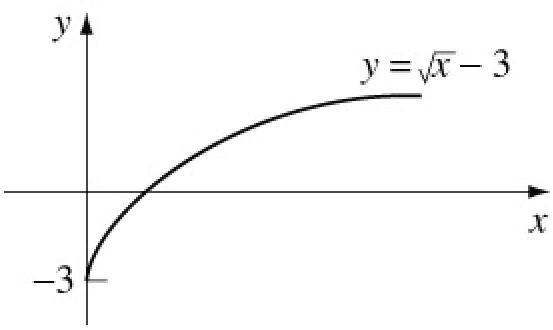

(b) $f(x) = x - 3$, $g(x) = +\sqrt{x}$

$fg(x) = f(+\sqrt{x}) = \sqrt{x} - 3$

Domain $x \geq 0$

(It will not be defined for negative numbers)

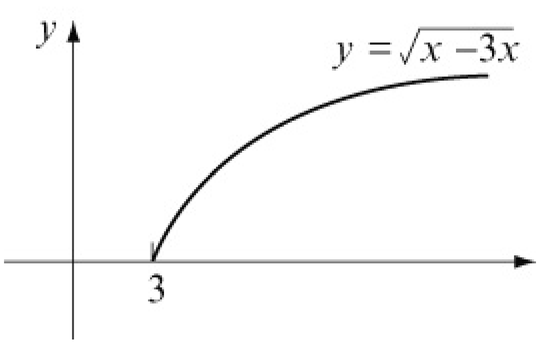
Range $fg(x) \geq -3$




$gf(x) = g(x - 3) = \sqrt{x - 3}$

Domain $x \geq 3$

Range $gf(x) \geq 0$

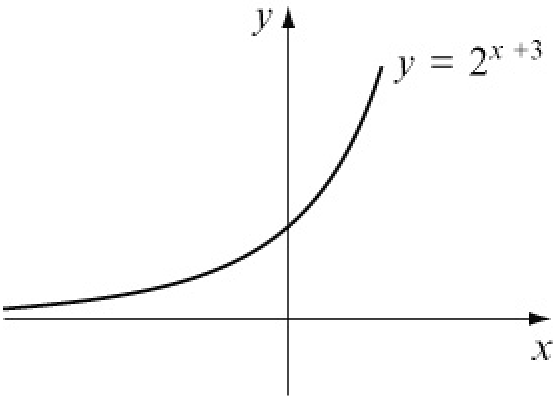



(c) $f(x) = 2^x$, $g(x) = x + 3$

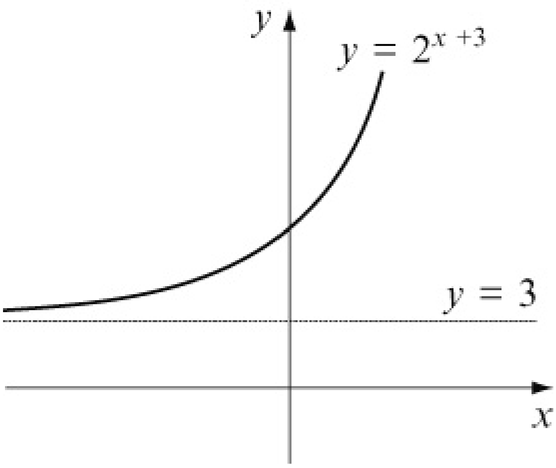
$fg(x) = f(x + 3) = 2^{x+3}$

Domain $x \in \mathbb{R}$

Range $fg(x) > 0$




$gf(x) = g(2^x) = 2^x + 3$
Domain $x \in \mathbb{R}$
Range $gf(x) > 3$



# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 3**

### Question:

If $f(x) = 3x - 2$ and $g(x) = x^2$, find the number(s) $a$ such that $fg(a) = gf(a)$.

### Solution:

$f(x) = 3x - 2$, $g(x) = x^2$

$fg(x) = f(x^2) = 3x^2 - 2$

$gf(x) = g(3x - 2) = (3x - 2)^2$

If $fg(a) = gf(a)$

$3a^2 - 2 = (3a - 2)^2$

$3a^2 - 2 = 9a^2 - 12a + 4$

$0 = 6a^2 - 12a + 6$

$0 = a^2 - 2a + 1$

$0 = (a - 1)^2$

Hence $a = 1$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 4**

### Question:

Given that $s(x) = \frac{1}{x-2}$ and $t(x) = 3x + 4$ find the number $m$ such that $ts(m) = 16$.

### Solution:

$s(x) = \frac{1}{x-2}$, $t(x) = 3x + 4$

$ts(x) = t\left(\frac{1}{x-2}\right) = 3 \times \left(\frac{1}{x-2}\right) + 4 = \frac{3}{x-2} + 4$

If $ts(m) = 16$

$\frac{3}{m-2} + 4 = 16 \quad (-4)$

$\frac{3}{m-2} = 12 \quad [\times (m-2)]$

$3 = 12(m-2) \quad (\div 12)$

$\frac{3}{12} = m - 2$

$0.25 = m - 2$

$m = 2.25$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 5**

### Question:

The functions l($x$), m($x$), n($x$) and p($x$) are defined by $l(x) = 2x + 1$, $m(x) = x^2 - 1$, $n(x) = \frac{1}{x+5}$ and $p(x) = x^3$. Find in terms of l, m, n and p the functions:

(a) $4x + 3$

(b) $4x^2 + 4x$

(c) $\frac{1}{x^2 + 4}$

(d) $\frac{2}{x+5} + 1$

(e) $(x^2 - 1)^3$

(f) $2x^2 - 1$

(g) $x^{27}$

### Solution:

(a) $4x + 3 = 2(2x + 1) + 1 = 2\,l(x) + 1 = ll(x)$ [or $l^2(x)$]

(b) $4x^2 + 4x = (2x + 1)^2 - 1 = [l(x)]^2 - 1 = ml(x)$

(c) $\frac{1}{x^2 + 4} = \frac{1}{(x^2 - 1) + 5} = \frac{1}{m(x) + 5} = nm(x)$

(d) $\frac{2}{x+5} + 1 = 2 \times \frac{1}{x+5} + 1 = 2\,n(x) + 1 = ln(x)$

(e) $(x^2 - 1)^3 = [m(x)]^3 = pm(x)$

(f) $2x^2 - 1 = 2(x^2 - 1) + 1 = 2\,m(x) + 1 = lm(x)$

(g) $x^{27} = [ ( x^3 ) ^3 ] ^3 = \{ [ p ( x ) ] ^3 \{ ^3 = [ pp ( x ) ] ^3 = ppp ( x )$
$= p^3 ( x )$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 6**

### Question:

If $m(x) = 2x + 3$ and $n(x) = \frac{x-3}{2}$, prove that $mn(x) = x$.

### Solution:

$m(x) = 2x + 3$, $n(x) = \frac{x-3}{2}$

$$mn(x) = m\left(\frac{x-3}{2}\right) = \cancel{2}\left(\frac{x-3}{\cancel{2}}\right) + 3 = x - 3 + 3 = x$$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 7**

### Question:

If $s(x) = \frac{3}{x+1}$ and $t(x) = \frac{3-x}{x}$, prove that $st(x) = x$.

### Solution:

$s(x) = \frac{3}{x+1}$, $t(x) = \frac{3-x}{x}$

$$\begin{aligned} st(x) &= s\left(\frac{3-x}{x}\right) \\ &= \frac{3}{\frac{3-x}{x}+1} \begin{matrix}\times x \\ \times x\end{matrix} \\ &= \frac{3x}{3-x+x} \\ &= \frac{\cancel{3}x}{\cancel{3}} \\ &= x \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise D, Question 8**

### Question:

If $f(x) = \frac{1}{x+1}$, prove that $f^2(x) = \frac{x+1}{x+2}$. Hence find an expression for $f^3(x)$.

### Solution:

$f(x) = \frac{1}{x+1}$

$$\begin{aligned} ff(x) &= f\left(\frac{1}{x+1}\right) \\ &= \frac{1}{\frac{1}{x+1}+1} \begin{matrix}\times(x+1)\\ \times(x+1)\end{matrix} \\ &= \frac{x+1}{1+x+1} \\ &= \frac{x+1}{x+2} \end{aligned}$$

$$\begin{aligned} f^3(x) = f[f^2(x)] &= f\left(\frac{x+1}{x+2}\right) \\ &= \frac{1}{\frac{x+1}{x+2}+1} \begin{matrix}\times(x+2)\\ \times(x+2)\end{matrix} \\ &= \frac{x+2}{x+1+x+2} \\ &= \frac{x+2}{2x+3} \end{aligned}$$

# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 1**

### Question:

For the following functions f($x$), sketch the graphs of f($x$) and $f^{-1}(x)$ on the same set of axes. Determine also the equation of $f^{-1}(x)$ .

(a) $f(x) = 2x + 3 \quad \{ x \in \mathbb{R} \{$

(b) $f(x) = \frac{x}{2} \quad \left\{ x \in \mathbb{R} \right\}$

(c) $f(x) = \frac{1}{x} \quad \left\{ x \in \mathbb{R}, \quad x \neq 0 \right\}$

(d) $f(x) = 4 - x \quad \{ x \in \mathbb{R} \{$

(e) $f(x) = x^2 + 2 \quad \{ x \in \mathbb{R}, \quad x \geq 0 \{$

(f) $f(x) = x^3 \quad \{ x \in \mathbb{R} \{$

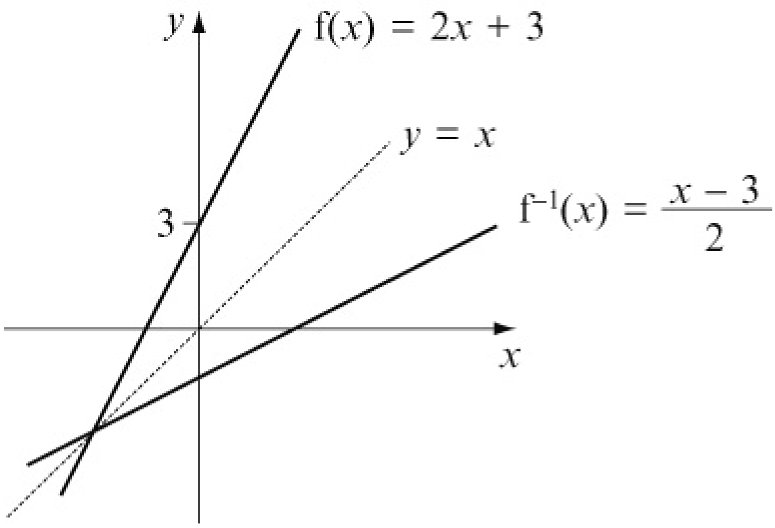
### Solution:

(a) If $y = 2x + 3$

$y - 3 = 2x$

$\frac{y-3}{2} = x$

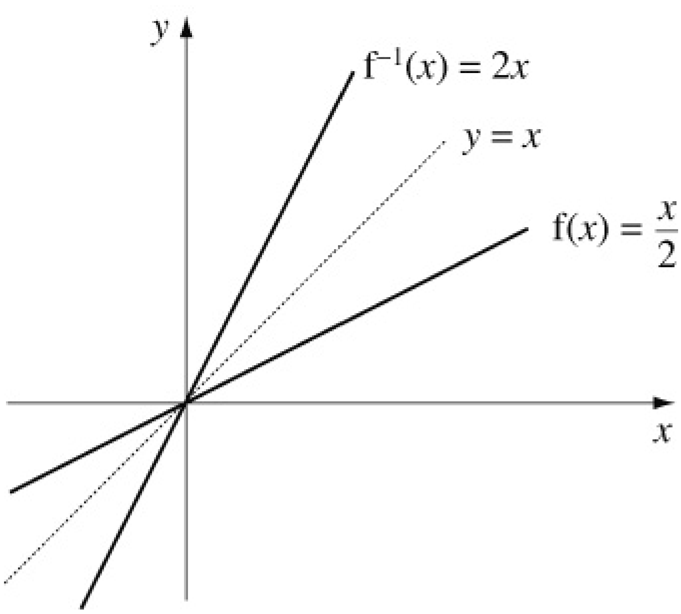
Hence $f^{-1}(x) = \frac{x-3}{2}$




(b) If $y = \frac{x}{2}$

$2y = x$

Hence $f^{-1}(x) = 2x$



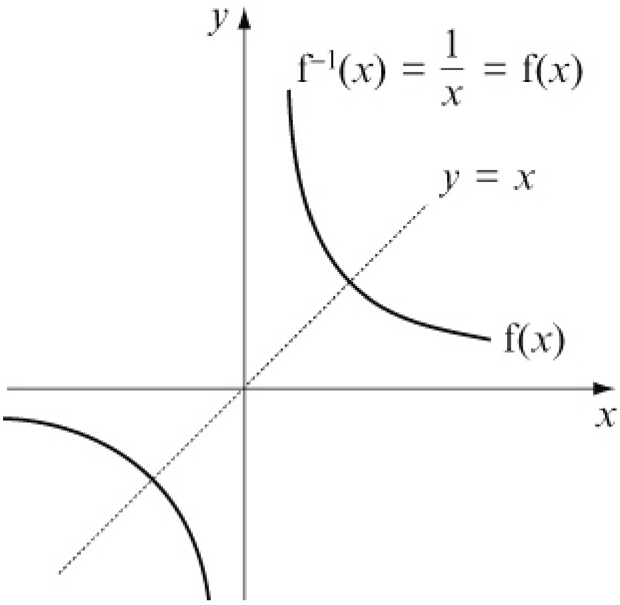

(c) If $y = \frac{1}{x}$

$yx = 1$

$x = \frac{1}{y}$

Hence $f^{-1}(x) = \frac{1}{x}$

Note that the inverse to the function is identical to the function.



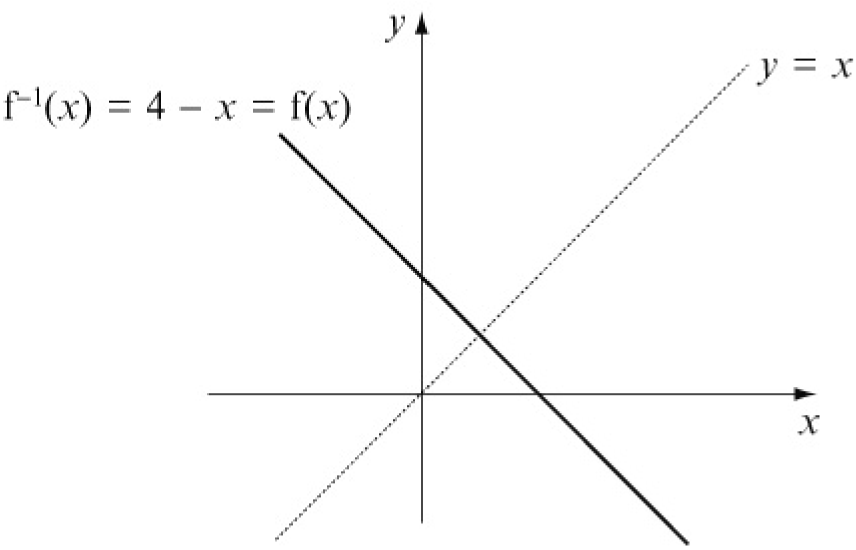

(d) If $y = 4 - x$

$x + y = 4$

$x = 4 - y$

Hence $f^{-1}(x) = 4 - x$

Note that the inverse to the function is identical to the function.

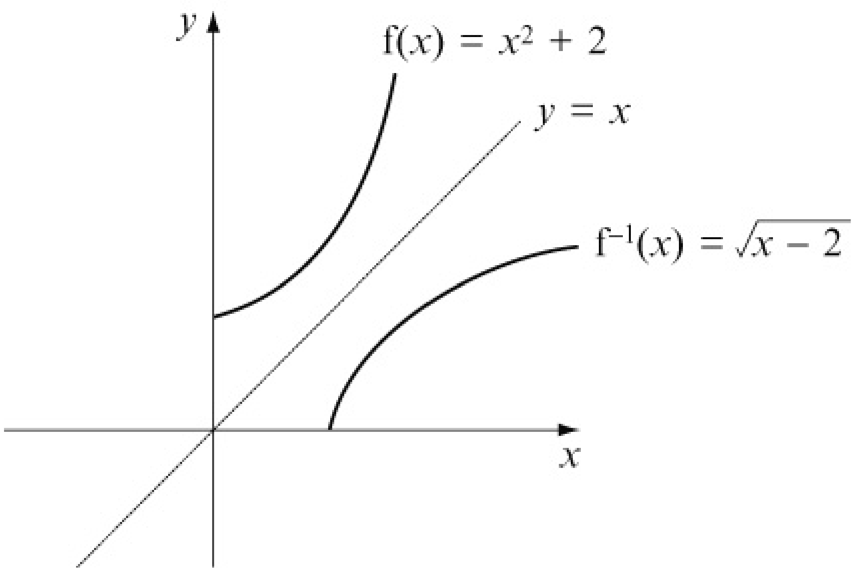



(e) If $y = x^2 + 2$

$y - 2 = x^2$

$\sqrt{y - 2} = x$

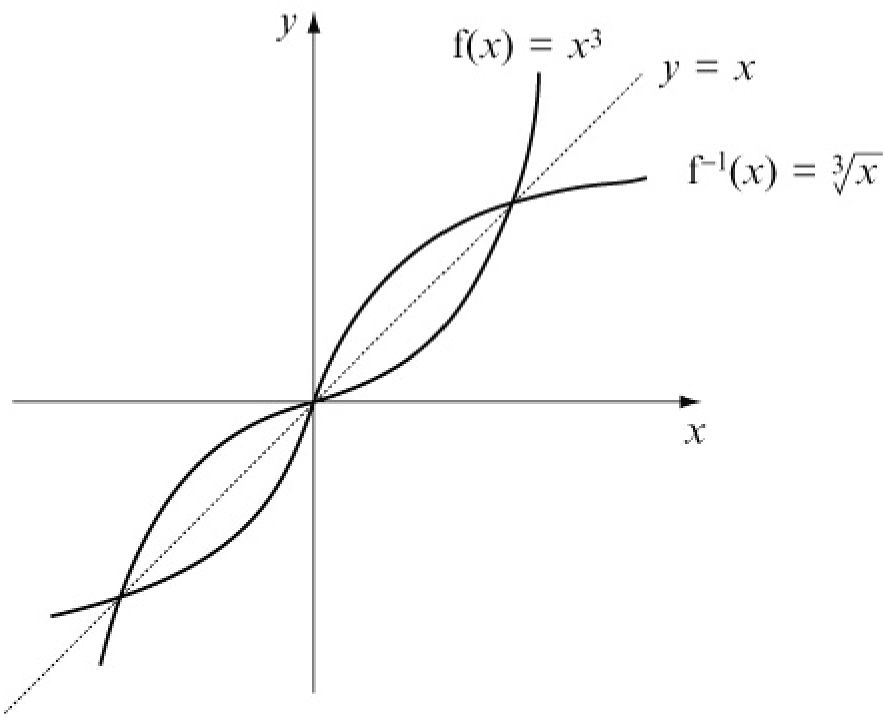
Hence $f^{-1}(x) = \sqrt{x - 2}$




(f) If $y = x^3$

$\sqrt[3]{y} = x$

Hence $f^{-1}(x) = \sqrt[3]{x}$



# Solutionbank

## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 2**

### Question:

Determine which of the functions in Question 1 are self inverses. (That is to say the function and its inverse are identical.)

### Solution:

Look back at Question 1.

1(c) $f(x) = \frac{1}{x}$ and

1(d) $f(x) = 4 - x$

are both identical to their inverses.

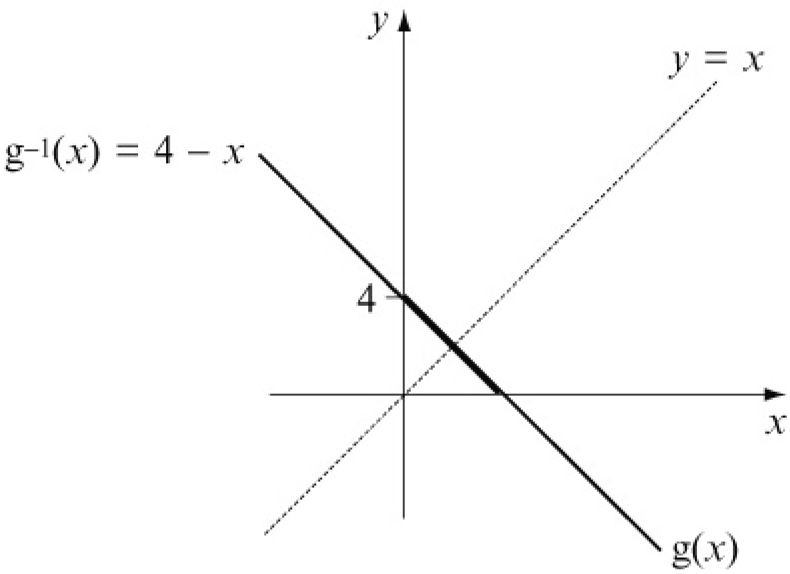
# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 3**

### Question:

Explain why the function $g(x) = 4 - x \ \{x \in \mathbb{R}, \ x > 0\}$ is not identical to its inverse.

### Solution:




$g(x) = 4 - x$
has domain $x > 0$
and range $g(x) < 4$
Hence $g^{-1}(x) = 4 - x$
has domain $x < 4$
and range $g^{-1}(x) > 0$
Although g($x$) and $g^{-1}(x)$ have identical equations they act on different numbers and so are not identical. See graph.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 4**

### Question:

For the following functions g($x$), sketch the graphs of g($x$) and $g^{-1}(x)$ on the same set of axes. Determine the equation of $g^{-1}(x)$, taking care with its domain.

(a) $g(x) = \frac{1}{x}$ $\left\{ x \in \mathbb{R}, x \geq 3 \right\}$

(b) $g(x) = 2x - 1$ $\{ x \in \mathbb{R}, x \geq 0 \{$

(c) $g(x) = \frac{3}{x-2}$ $\left\{ x \in \mathbb{R}, x > 2 \right\}$

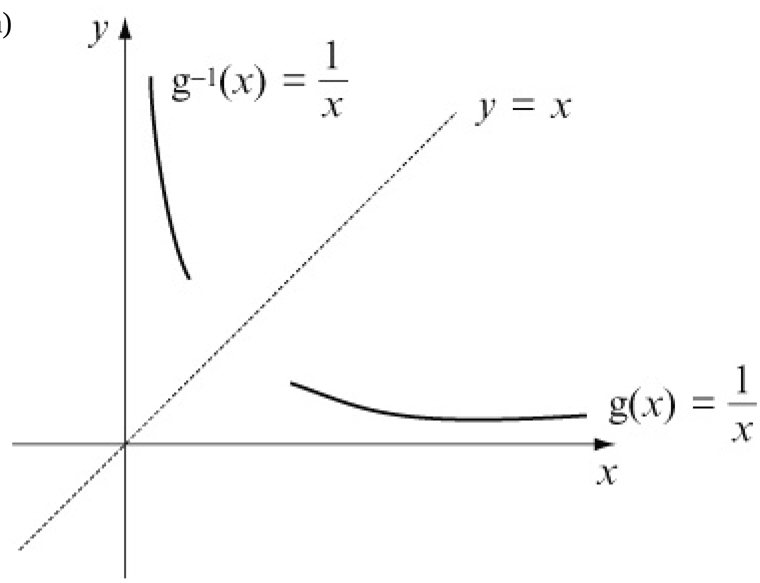
(d) $g(x) = \sqrt{x-3}$ $\{ x \in \mathbb{R}, x \geq 7 \{$

(e) $g(x) = x^2 + 2$ $\{ x \in \mathbb{R}, x > 4 \{$

(f) $g(x) = x^3 - 8$ $\{ x \in \mathbb{R}, x \leq 2 \{$

### Solution:

(a)



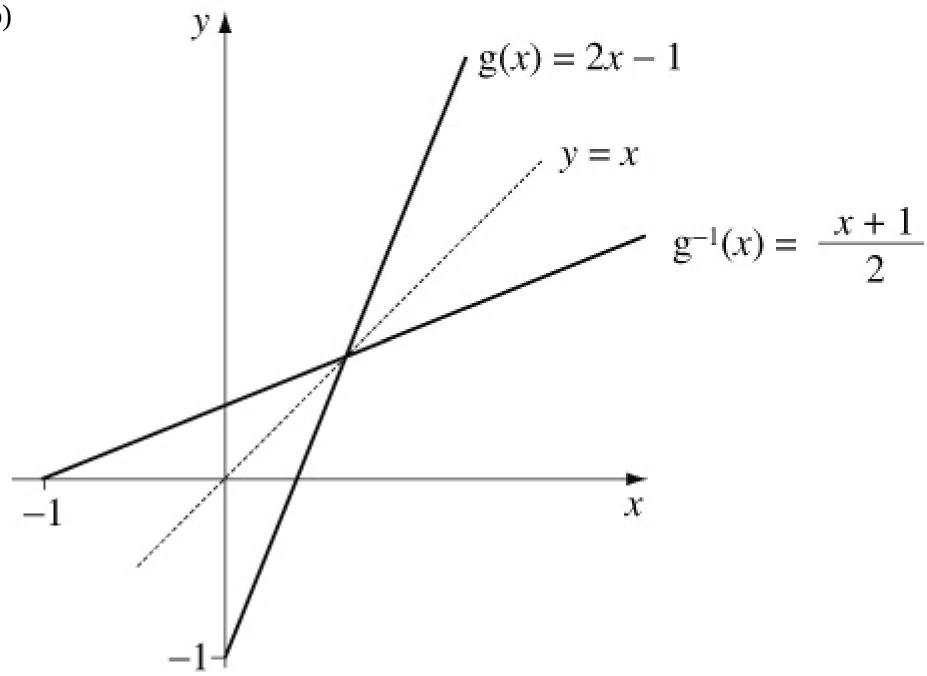
$$g(x) = \frac{1}{x} \left\{ x \in \mathbb{R},\ x \geq 3 \right\}$$

has range $g(x) \in \mathbb{R},\ 0 < g(x) \leq \frac{1}{3}$

Changing the subject of the formula gives

$$g^{-1}(x) = \frac{1}{x} \left\{ x \in \mathbb{R},\ 0 < x \leq \frac{1}{3} \right\}$$

(b)



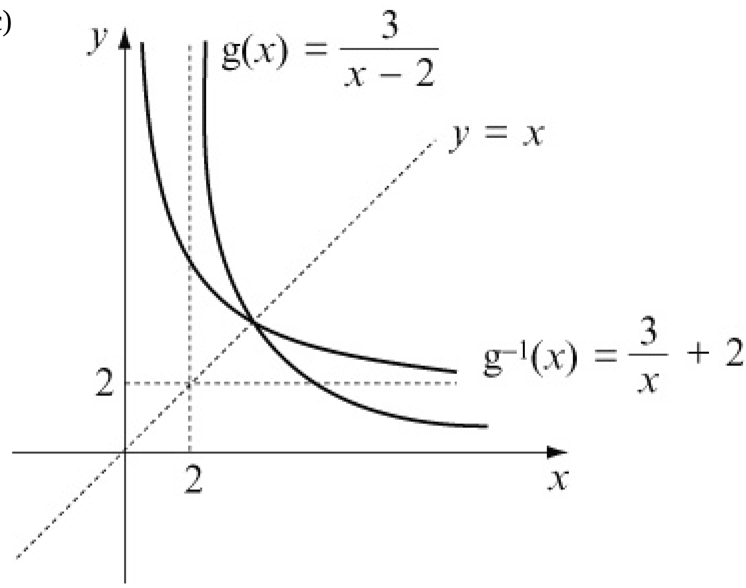

$g(x) = 2x - 1 \ \{ x \in \mathbb{R},\ x \geq 0 \{$

has range $g(x) \in \mathbb{R},\ g(x) \geq -1$

Changing the subject of the formula gives

$$g^{-1}(x) = \frac{x+1}{2} \left\{ x \in \mathbb{R},\ x \geq -1 \right\}$$

(c)




$g(x) = \frac{3}{x-2} \left\{ x \in \mathbb{R}, \quad x > 2 \right\}$

has range $g(x) \in \mathbb{R}, \quad g(x) > 0$

Changing the subject of the formula gives

$y = \frac{3}{x-2}$

$y(x-2) = 3$

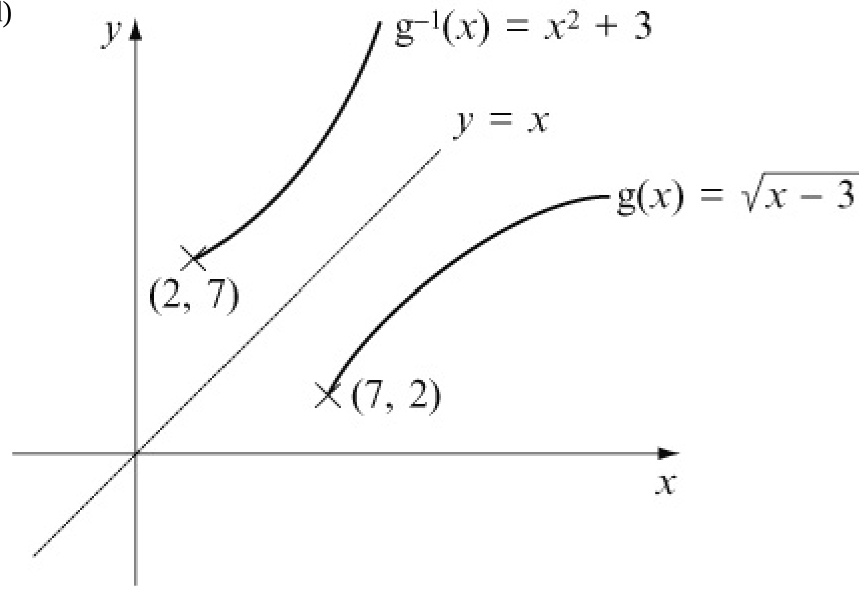
$x - 2 = \frac{3}{y}$

$x = \frac{3}{y} + 2 \qquad \left( \text{or} \quad \frac{3+2y}{y} \right)$

Hence $g^{-1}(x) = \frac{3}{x} + 2 \qquad \left( \text{or} \quad \frac{3+2x}{x} \right)$

$\{ x \in \mathbb{R}, \quad x > 0 \{$

(d)




$g(x) = \sqrt{x-3}$ { $x \in \mathbb{R}$, $x \geq 7$ {

has range $g(x) \in \mathbb{R}$, $g(x) \geq 2$

Changing the subject of the formula gives

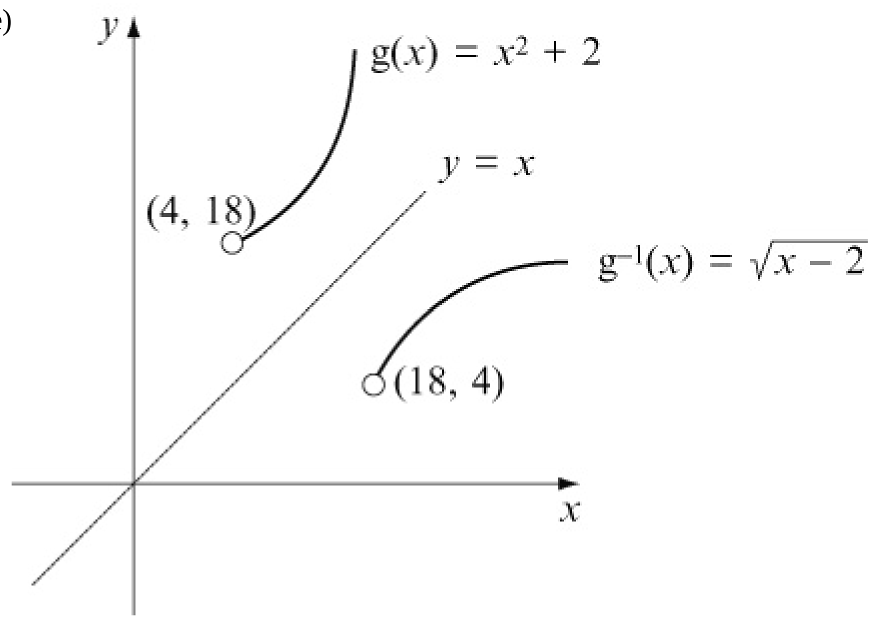
$y = \sqrt{x-3}$

$y^2 = x - 3$

$x = y^2 + 3$

Hence $g^{-1}(x) = x^2 + 3$ with domain $x \in \mathbb{R}$, $x \geq 2$

(e)



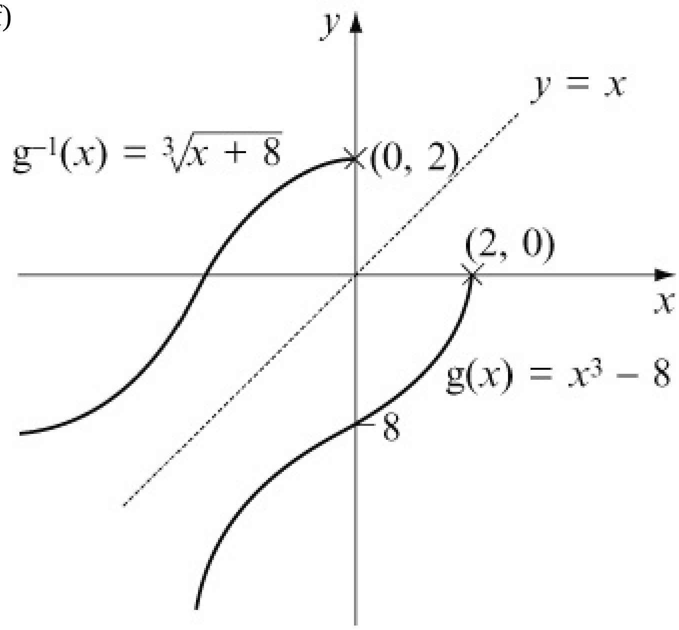

$g(x) = x^2 + 2$ { $x \in \mathbb{R}$, $x > 4$ {

has range $g(x) \in \mathbb{R}$, $g(x) > 18$

Changing the subject of the formula gives

$g^{-1}(x) = \sqrt{x-2}$ with domain $x \in \mathbb{R}$, $x > 18$

(f)




$g(x) = x^3 - 8$ { $x \in \mathbb{R}$, $x \leq 2$ {

has range $g(x) \in \mathbb{R}$, $g(x) \leq 0$

Changing the subject of the formula gives

$y = x^3 - 8$

$y + 8 = x^3$

$\sqrt[3]{y+8} = x$

Hence $g^{-1}(x) = \sqrt[3]{x+8}$ with domain $x \in \mathbb{R}$, $x \leq 0$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

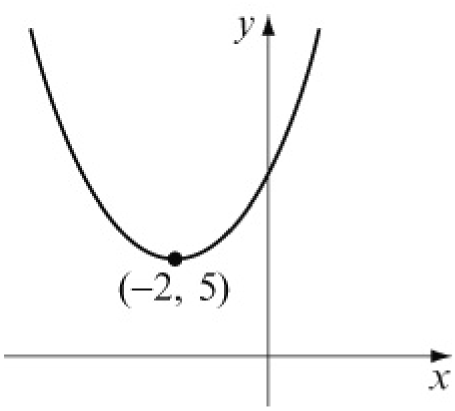
**Exercise E, Question 5**

### Question:

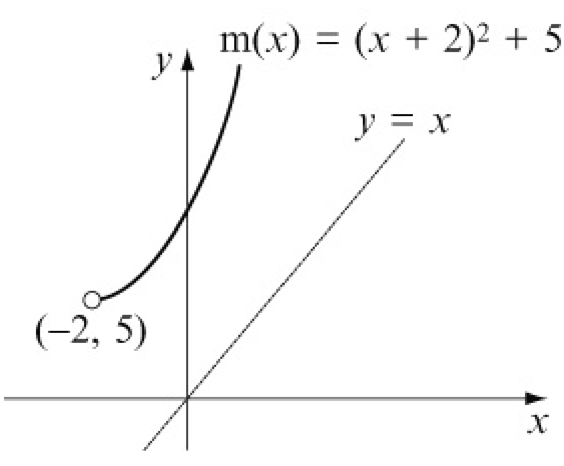
The function m($x$) is defined by $\mathrm{m}(x) = x^2 + 4x + 9 \quad \{ x \in \mathbb{R},\ x > a \{$ for some constant $a$. If $\mathrm{m}^{-1}(x)$ exists, state the least value of $a$ and hence determine the equation of $\mathrm{m}^{-1}(x)$. State its domain.

### Solution:

$\mathrm{m}(x) = x^2 + 4x + 9 \quad \{ x \in \mathbb{R},\ x > a \{$ .
Let $y = x^2 + 4x + 9$
$y = (x + 2)^2 - 4 + 9$
$y = (x + 2)^2 + 5$
This has a minimum value of $(-2, 5)$ .




For m($x$) to have an inverse it must be one-to-one.
Hence the least value of $a$ is $-2$.



m($x$) would have a range of $\text{m}(x) \in \mathbb{R}, \text{m}(x) > 5$

Changing the subject of the formula gives

$y = (x+2)^2 + 5$

$y - 5 = (x+2)^2$

$\sqrt{y-5} = x + 2$

$\sqrt{y-5} - 2 = x$

Hence $\text{m}^{-1}(x) = \sqrt{x-5} - 2$ with domain $x \in \mathbb{R}, x > 5$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 6**

### Question:

Determine $t^{-1}(x)$ if the function t($x$) is defined by $t(x) = x^2 - 6x + 5$ $\{x \in \mathbb{R}, x \geq 5\}$.

### Solution:

$t(x) = x^2 - 6x + 5$ $\{x \in \mathbb{R}, x \geq 5\}$

Let $y = x^2 - 6x + 5$ (complete the square)

$y = (x-3)^2 - 9 + 5$

$y = (x-3)^2 - 4$

This has a minimum point at $(3, -4)$.

**Note.** Since $x \geq 5$ is the domain, t($x$) is a one-to-one function.

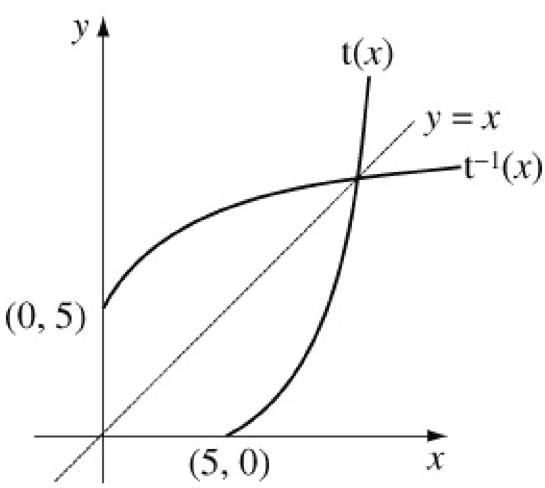
Change the subject of the formula to find $t^{-1}(x)$:

$y = (x-3)^2 - 4$

$y + 4 = (x-3)^2$

$\sqrt{y+4} = x - 3$

$\sqrt{y+4} + 3 = x$




$t(x) = x^2 - 6x + 5$ $\{x \in \mathbb{R}, x \geq 5\}$

has range $t(x) \in \mathbb{R}, t(x) \geq 0$

So $t^{-1}(x) = \sqrt{x+4} + 3$ and has domain $x \in \mathbb{R}, x \geq 0$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 7**

### Question:

The function h($x$) is defined by $h(x) = \frac{2x+1}{x-2} \left\{ x \in \mathbb{R}, \ x \neq 2 \right\}$.

(a) What happens to the function as $x$ approaches 2?

(b) Find $h^{-1}(3)$.

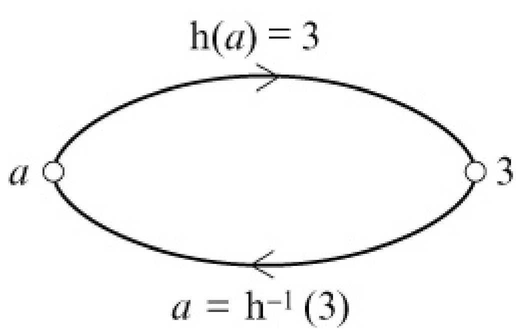
(c) Find $h^{-1}(x)$, stating clearly its domain.

(d) Find the elements of the domain that get mapped to themselves by the function.

### Solution:

(a) As $x \to 2$ $h(x) \to \frac{5}{0}$ and hence $h(x) \to \infty$

(b) To find $h^{-1}(3)$ we can find what element of the domain gets mapped to 3.




So $h(a) = 3$

$\frac{2a+1}{a-2} = 3$

$2a + 1 = 3a - 6$

$7 = a$

So $h^{-1}(3) = 7$

(c) Let $y = \frac{2x+1}{x-2}$ and find $x$ as a function of $y$.

$y(x-2) = 2x+1$
$yx - 2y = 2x + 1$
$yx - 2x = 2y + 1$
$x(y-2) = 2y+1$
$x = \frac{2y+1}{y-2}$

So $h^{-1}(x) = \frac{2x+1}{x-2} \left\{ x \in \mathbb{R}, \ x \neq 2 \right\}$

Hence the inverse function has exactly the same equation as the function. **But** the elements don't get mapped to themselves, see part (b).

(d) For elements to get mapped to themselves
$h(b) = b$
$\frac{2b+1}{b-2} = b$
$2b + 1 = b(b-2)$
$2b + 1 = b^2 - 2b$
$0 = b^2 - 4b - 1$
$b = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

The elements $2 + \sqrt{5}$ and $2 - \sqrt{5}$ get mapped to themselves by the function.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise E, Question 8**

### Question:

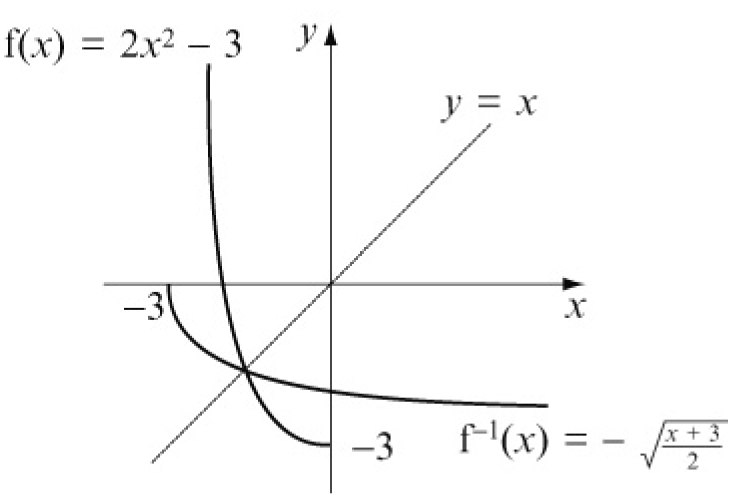
The function $f(x)$ is defined by $f(x) = 2x^2 - 3 \quad \{ x \in \mathbb{R}, \ x < 0 \{$ .
Determine

(a) $f^{-1}(x)$ clearly stating its domain

(b) the values of $a$ for which $f(a) = f^{-1}(a)$ .

### Solution:

(a) Let $y = 2x^2 - 3$
$y + 3 = 2x^2$
$\frac{y+3}{2} = x^2$
$\sqrt{\frac{y+3}{2}} = x$
The domain of $f^{-1}(x)$ is the range of $f(x)$ .
$f(x) = 2x^2 - 3 \quad \{ x \in \mathbb{R}, \ x < 0 \{$
has range $f(x) > -3$
Hence $f^{-1}(x)$ must be the **negative** square root
$f^{-1}(x) = -\sqrt{\frac{x+3}{2}}$ has domain $x \in \mathbb{R}, \ x > -3$




(b) If $f(a) = f^{-1}(a)$ then $a$ is negative (see graph).
Solve $f(a) = a$

$2a^2 - 3 = a$

$2a^2 - a - 3 = 0$

$(2a - 3)(a + 1) = 0$

$a = \frac{3}{2}, -1$

Therefore $a = -1$

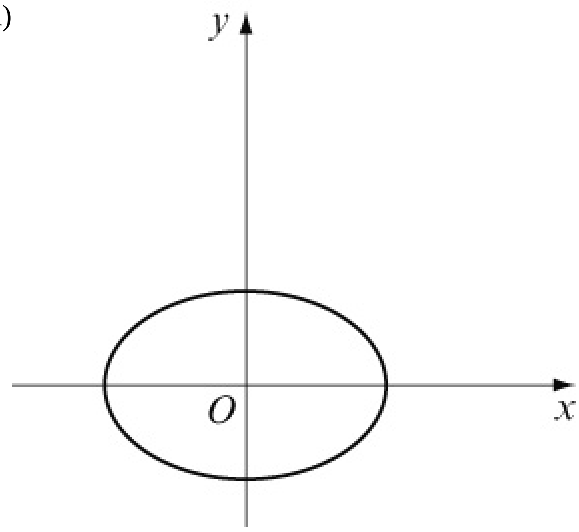
# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 1**

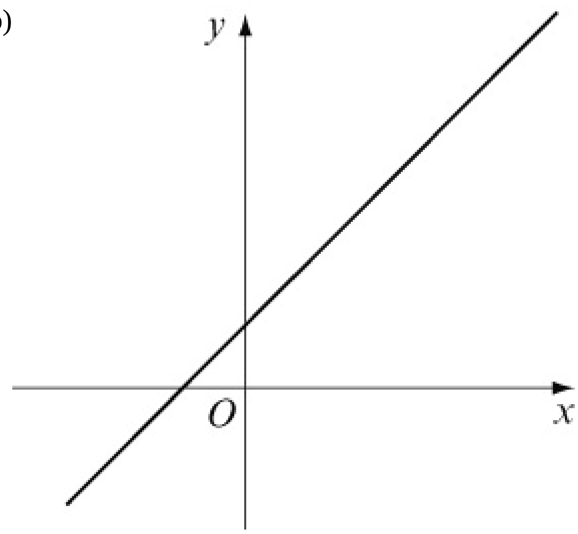
### Question:

Categorise the following as
(i) not a function
(ii) a one-to-one function
(iii) a many-to-one function.

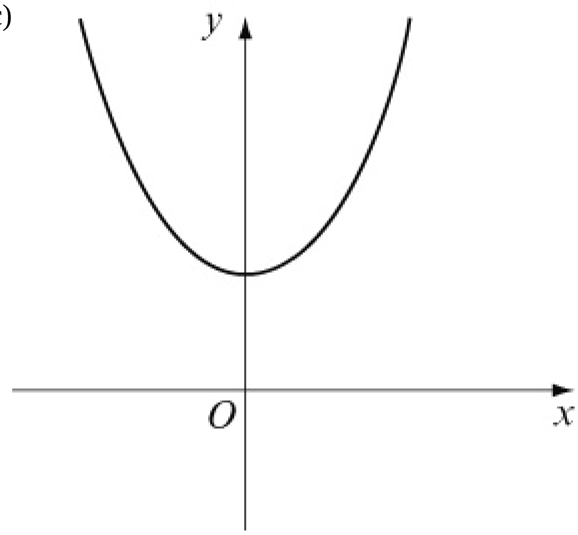
(a)



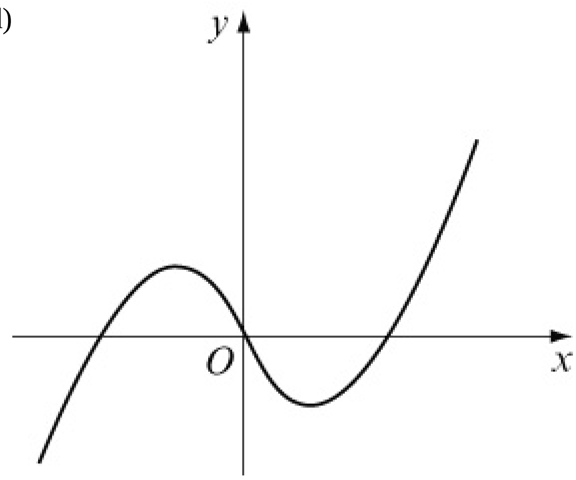

(b)



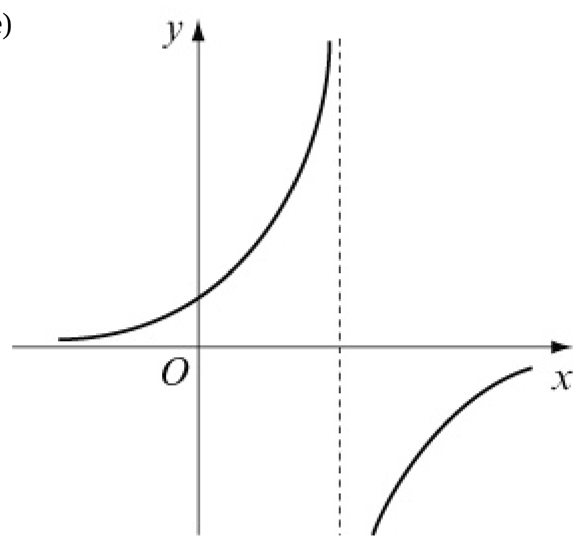
(c)



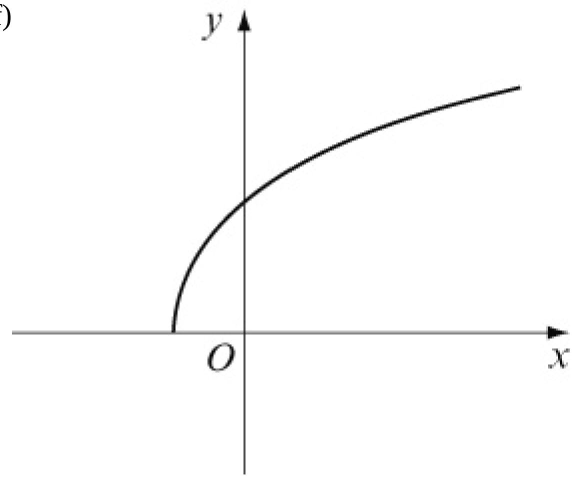

(d)




(e)

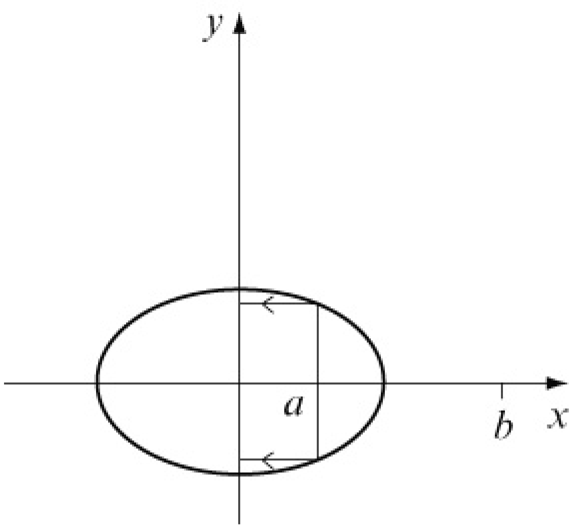


(f)



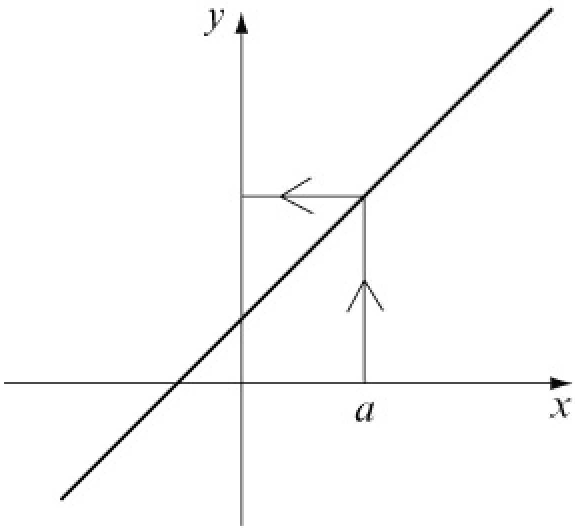

**Solution:**

(a) not a function

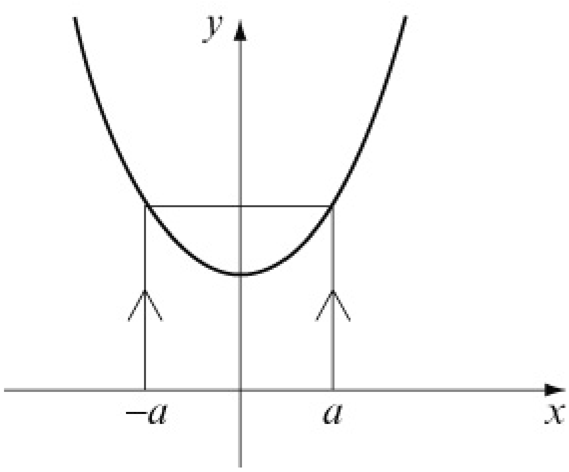



*x* value *a* gets mapped to two values of *y*.
*x* value *b* gets mapped to no values

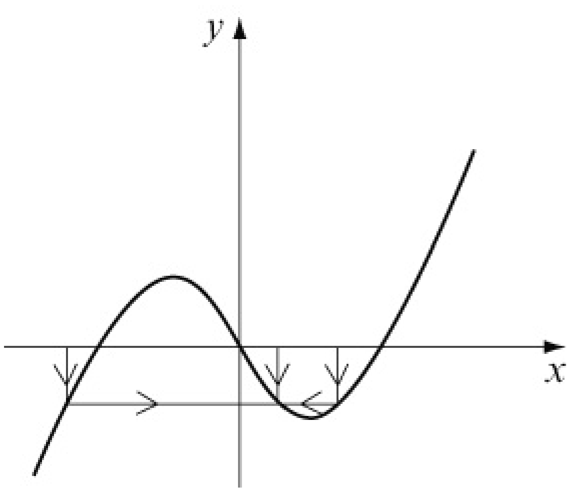
(b) one-to-one function



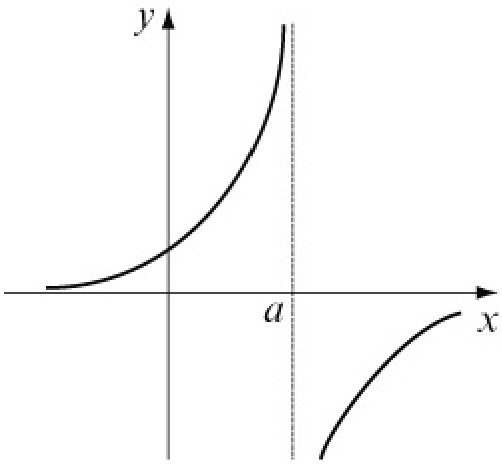

(c) many-to-one function




(d) many-to-one function

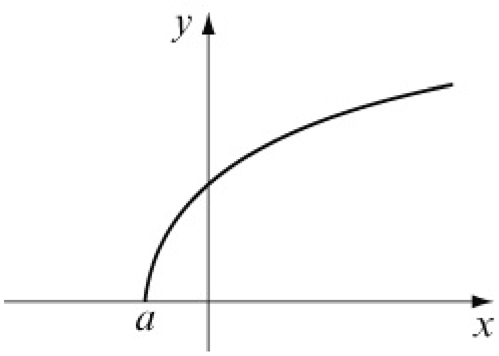



(e) not a function




*x* value *a* doesn't get mapped to any value of *y*.
It could be redefined as a function if the domain is said to exclude point *a*.

(f) not a function




*x* values less than *a* don't get mapped anywhere.
Again we could define the domain to be $x \geq a$ and then it would be a function.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 2**

### Question:

The following functions f($x$), g($x$) and h($x$) are defined by

$f(x) = 4(x-2) \quad \{x \in \mathbb{R}, x \geq 0\}$

$g(x) = x^3 + 1 \quad \{x \in \mathbb{R}\}$

$h(x) = 3^x \quad \{x \in \mathbb{R}\}$

(a) Find f(7), g(3) and $h(-2)$.

(b) Find the range of f($x$) and the range of g($x$).

(c) Find $g^{-1}(x)$.

(d) Find the composite function fg($x$).

(e) Solve $gh(a) = 244$.

### Solution:

(a) $f(7) = 4(7-2) = 4 \times 5 = 20$

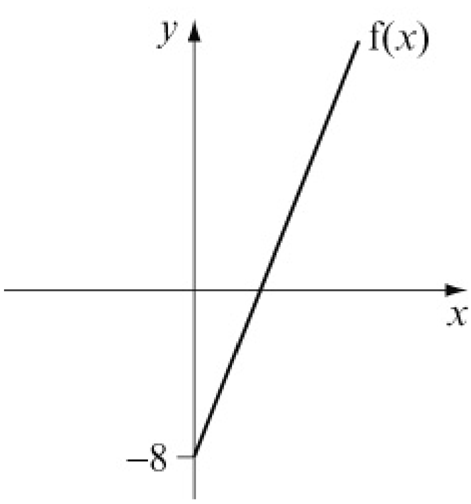
$g(3) = 3^3 + 1 = 27 + 1 = 28$

$h(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(b) $f(x) = 4(x-2) = 4x - 8$

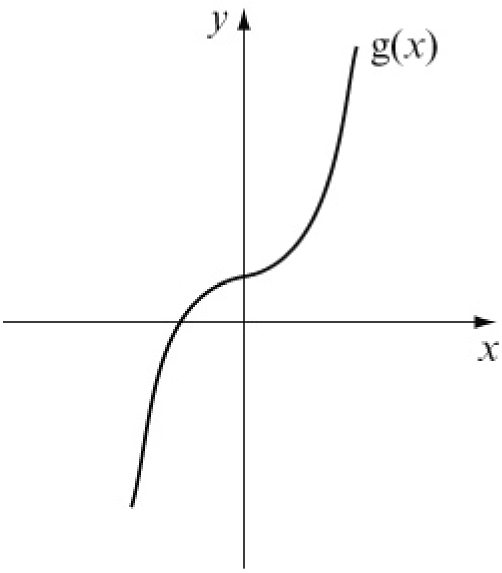
This is a straight line with gradient 4 and intercept $-8$.

The domain tells us that $x \geq 0$.



The range of f($x$) is $f(x) \in \mathbb{R}, f(x) \geq -8$

$g(x) = x^3 + 1$




The range of g($x$) is $g(x) \in \mathbb{R}$

(c) Let $y = x^3 + 1$ (change the subject of the formula)

$y - 1 = x^3$

$\sqrt[3]{y-1} = x$

Hence $g^{-1}(x) = \sqrt[3]{x-1}$ $\{ x \in \mathbb{R} \{$

(d) $fg(x) = f(x^3 + 1) = 4(x^3 + 1 - 2) = 4(x^3 - 1)$

(e) Find gh($x$) first.

$gh(x) = g(3^x) = (3^x)^3 + 1 = 3^{3x} + 1$

If $gh(a) = 244$

$3^{3a} + 1 = 244$

$3^{3a} = 243$

$3^{3a} = 3^5$

$3a = 5$

$a = \frac{5}{3}$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 3**

### Question:

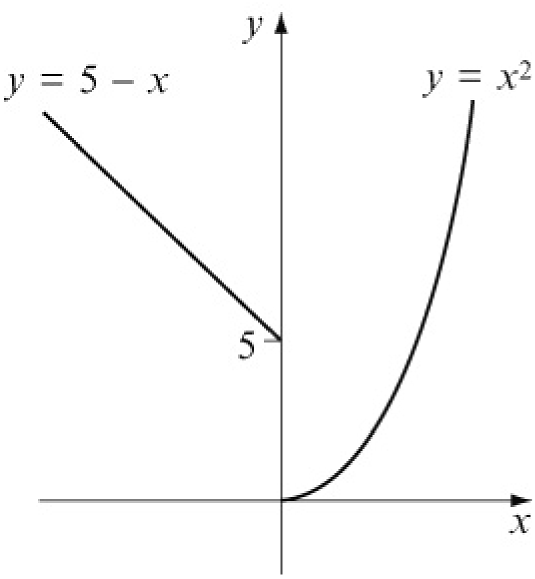
The function n($x$) is defined by

$$n(x) = \begin{cases} 5 - x & x \le 0 \\ x^2 & x > 0 \end{cases}$$

(a) Find $n(-3)$ and n(3).

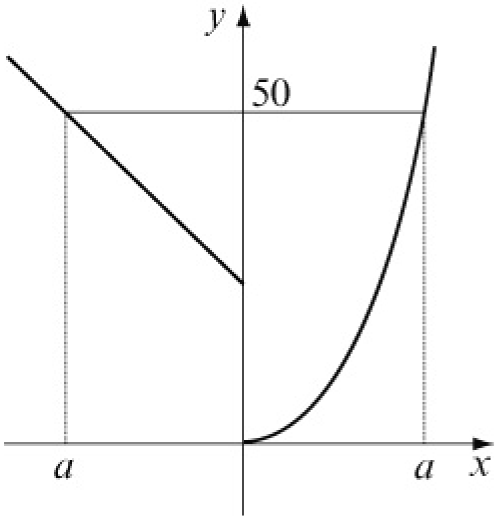
(b) Find the value(s) of $a$ such that $n(a) = 50$.

### Solution:




$y = 5 - x$ is a straight line with gradient $-1$ passing through 5 on the $y$ axis.
$y = x^2$ is a $\cup$ -shaped quadratic passing through (0, 0).
(a) $n(-3) = 5 - (-3) = 5 + 3 = 8$
$n(3) = 3^2 = 9$

(b) There are two values of $a$.




The negative value of *a* is where

$5 - a = 50$

$a = 5 - 50$

$a = -45$

The positive value of *a* is where

$a^2 = 50$

$a = \sqrt{50}$

$a = 5\sqrt{2}$

The values of *a* such that $n(a) = 50$ are $-45$ and $+5\sqrt{2}$.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 4**

### Question:

The function g(*x*) is defined as $g(x) = 2x + 7 \{ x \in \mathbb{R}, x \geq 0 \{$ .

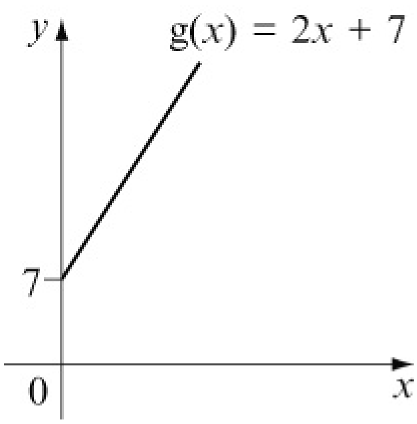
(a) Sketch g(*x*) and find the range.

(b) Determine $g^{-1}(x)$ , stating its domain.

(c) Sketch $g^{-1}(x)$ on the same axes as g(*x*), stating the relationship between the two graphs.

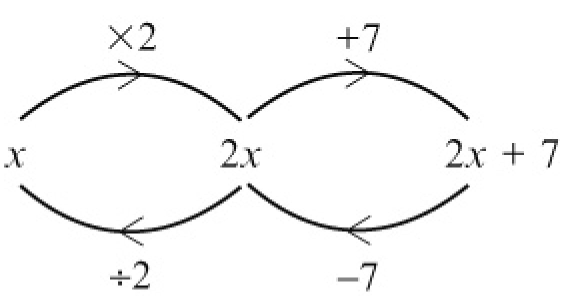
### Solution:

(a) $y = 2x + 7$ is a straight line of gradient 2 passing through 7 on the *y* axis.
The domain is given as $x \geq 0$.



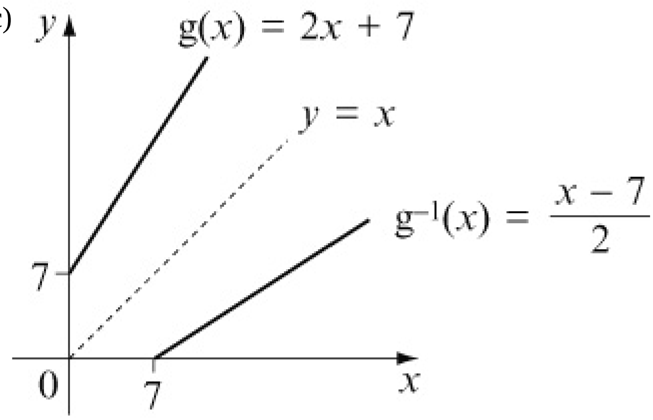

Hence the range is $g(x) \geq 7$

(b) The domain of the inverse function is $x \geq 7$.
To find the equation of the inverse function use a flow chart.



$g^{-1}(x) = \frac{x-7}{2}$ and has domain $x \geq 7$

(c)




$g^{-1}(x)$ is the reflection of g($x$) in the line $y = x$.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 5**

### Question:

The functions f and g are defined by

$f : x \rightarrow 4x - 1 \quad \{ x \in \mathbb{R} \{$

$g : x \rightarrow \frac{3}{2x - 1} \quad \left\{ x \in \mathbb{R}, \quad x \neq \frac{1}{2} \right\}$

Find in its simplest form:

(a) the inverse function $f^{-1}$

(b) the composite function gf, stating its domain

(c) the values of $x$ for which $2f(x) = g(x)$, giving your answers to 3 decimal places.

**[E]**

### Solution:

(a) $f : x \rightarrow 4x - 1$

Let $y = 4x - 1$ and change the subject of the formula.

$\Rightarrow \quad y + 1 = 4x$

$\Rightarrow \quad x = \frac{y + 1}{4}$

Hence $f^{-1} : x \rightarrow \frac{x + 1}{4}$

(b) $gf(x) = g(4x - 1) = \frac{3}{2(4x - 1) - 1} = \frac{3}{8x - 3}$

Hence $gf : x \rightarrow \frac{3}{8x - 3}$

The domain would include all the real numbers apart from $x = \frac{3}{8}$(i.e. where $8x - 3 = 0$).

(c) If $2f(x) = g(x)$

$2 \times (4x - 1) = \frac{3}{2x - 1}$

$8x - 2 = \frac{3}{2x - 1}$

$(8x - 2)(2x - 1) = 3$

$16x^2 - 12x + 2 = 3$

$16x^2 - 12x - 1 = 0$

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 16$, $b = -12$ and $c = -1$.

Then $x = \frac{12 \pm \sqrt{144 + 64}}{32} = \frac{12 \pm \sqrt{208}}{32} = 0.826, \quad -0.076$

Values of $x$ are $-0.076$ and $0.826$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 6**

### Question:

The function f($x$) is defined by

$$f(x) = \begin{cases} -x & x \le 1 \\ x-2 & x > 1 \end{cases}$$

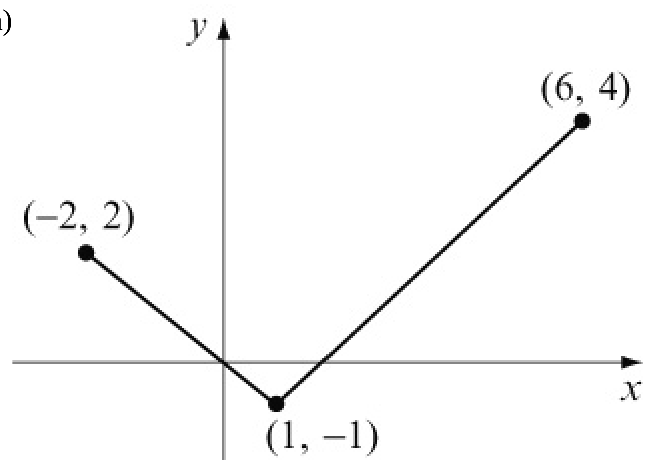
(a) Sketch the graph of f($x$) for $-2 \le x \le 6$.

(b) Find the values of $x$ for which $f(x) = -\frac{1}{2}$.

**[E]**

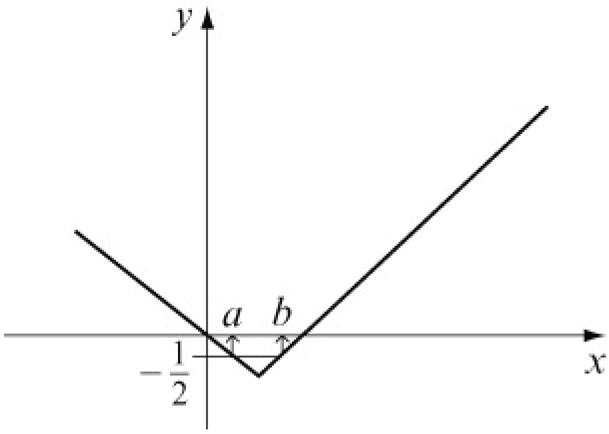
### Solution:

(a)




For $x \le 1$, $f(x) = -x$
This is a straight line of gradient $-1$.
At point $x = 1$, its $y$ coordinate is $-1$.
For $x > 1$, $f(x) = x - 2$
This is a straight line of gradient $+1$.
At point $x = 1$, its $y$ coordinate is also $-1$.
The graph is said to be **continuous**.

(b) There are two values at which $f(x) = -\frac{1}{2}$(see graph).




Point *a* is where

$-x = -\frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2}$

Point *b* is where

$x - 2 = -\frac{1}{2} \quad \Rightarrow \quad x = 1\frac{1}{2}$

The values of *x* for which $f(x) = -\frac{1}{2}$ are $\frac{1}{2}$ and $1\frac{1}{2}$.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 7**

### Question:

The function f is defined by

$$f : x \to \frac{2x+3}{x-1} \quad \left\{ x \in \mathbb{R},\ x > 1 \right\}$$

(a) Find $f^{-1}(x)$ .

(b) Find (i) the range of $f^{-1}(x)$
(ii) the domain of $f^{-1}(x)$ .

**[E]**

### Solution:

(a) To find $f^{-1}(x)$ change the subject of the formula.

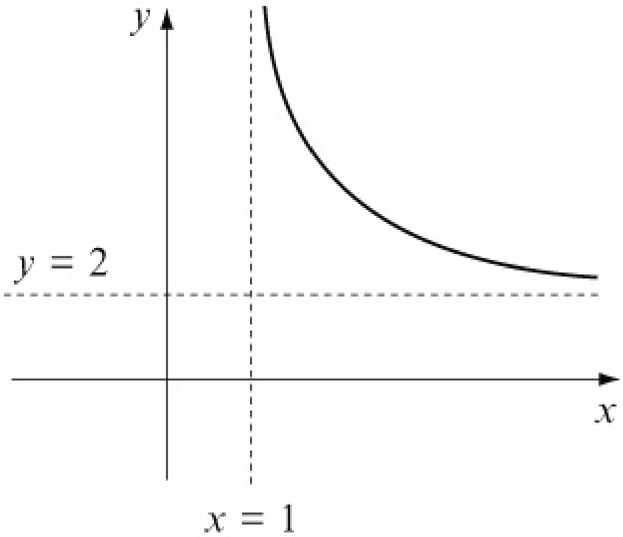
Let $y = \frac{2x+3}{x-1}$

$y(x-1) = 2x+3$
$yx - y = 2x + 3$
$yx - 2x = y + 3$
$x(y-2) = y + 3$
$x = \frac{y+3}{y-2}$

Therefore $f^{-1} : x \to \frac{x+3}{x-2}$

(b) f($x$) has domain $\{ x \in \mathbb{R},\ x > 1 \{$ and range $\{ f(x) \in \mathbb{R},\ f(x) > 2 \{$

As $x \to \infty$ , $y \to \frac{2x}{x} = 2$




So $f^{-1}(x)$ has domain $\{ x \in \mathbb{R},\ x > 2 \}$ and range $\left\{ f^{-1}(x) \in \mathbb{R},\ f^{-1}\left( x \right) > 1 \right\}$

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 8**

### Question:

The functions f and g are defined by

$f : x \rightarrow \frac{x}{x-2} \quad \left\{ x \in \mathbb{R}, \quad x \neq 2 \right\}$

$g : x \rightarrow \frac{3}{x} \quad \left\{ x \in \mathbb{R}, \quad x \neq 0 \right\}$

(a) Find an expression for $f^{-1}(x)$ .

(b) Write down the range of $f^{-1}(x)$ .

(c) Calculate gf(1.5).

(d) Use algebra to find the values of $x$ for which $g(x) = f(x) + 4$ .

**[E]**

### Solution:

(a) To find $f^{-1}(x)$ change the subject of the formula.

Let $y = \frac{x}{x-2}$

$y(x-2) = x$

$yx - 2y = x$ (rearrange)

$yx - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1}$

It must always be rewritten as a function in $x$:

$f^{-1}\left(x\right) = \frac{2x}{x-1}$

(b) The range of $f^{-1}(x)$ is the domain of $f(x)$ .

Hence range is $\{ f^{-1}(x) \in \mathbb{R}, \quad f^{-1}(x) \neq 2 \{$ .

(c) $gf(1.5) = g\left(\frac{1.5}{1.5-2}\right) = g\left(\frac{1.5}{-0.5}\right) = g(-3) = \frac{3}{-3} = -1$

(d) If $g(x) = f(x) + 4$

$\frac{3}{x} = \frac{x}{x-2} + 4 \qquad \left[\times x\left(x-2\right)\right]$

$3(x-2) = x \times x + 4x(x-2)$

$3x - 6 = x^2 + 4x^2 - 8x$

$0 = 5x^2 - 11x + 6$

$0 = (5x-6)(x-1)$

$\Rightarrow \quad x = \frac{6}{5},\ 1$

The values of $x$ for which $g(x) = f(x) + 4$ are $\frac{6}{5}$ and 1.

# Solutionbank
## Edexcel AS and A Level Modular Mathematics

**Exercise F, Question 9**

### Question:

The functions f and g are given by

$$f : x \rightarrow \frac{x}{x^2 - 1} - \frac{1}{x + 1} \left\{ x \in \mathbb{R}, \quad x > 1 \right\}$$

$$g : x \rightarrow \frac{2}{x} \left\{ x \in \mathbb{R}, \quad x > 0 \right\}$$

(a) Show that $f(x) = \frac{1}{(x-1)(x+1)}$.

(b) Find the range of $f(x)$ .

(c) Solve $gf(x) = 70$.

**[E]**

### Solution:

(a) $f(x) = \frac{x}{x^2 - 1} - \frac{1}{x + 1}$

$$= \frac{x}{(x+1)(x-1)} - \frac{1}{(x+1)}$$

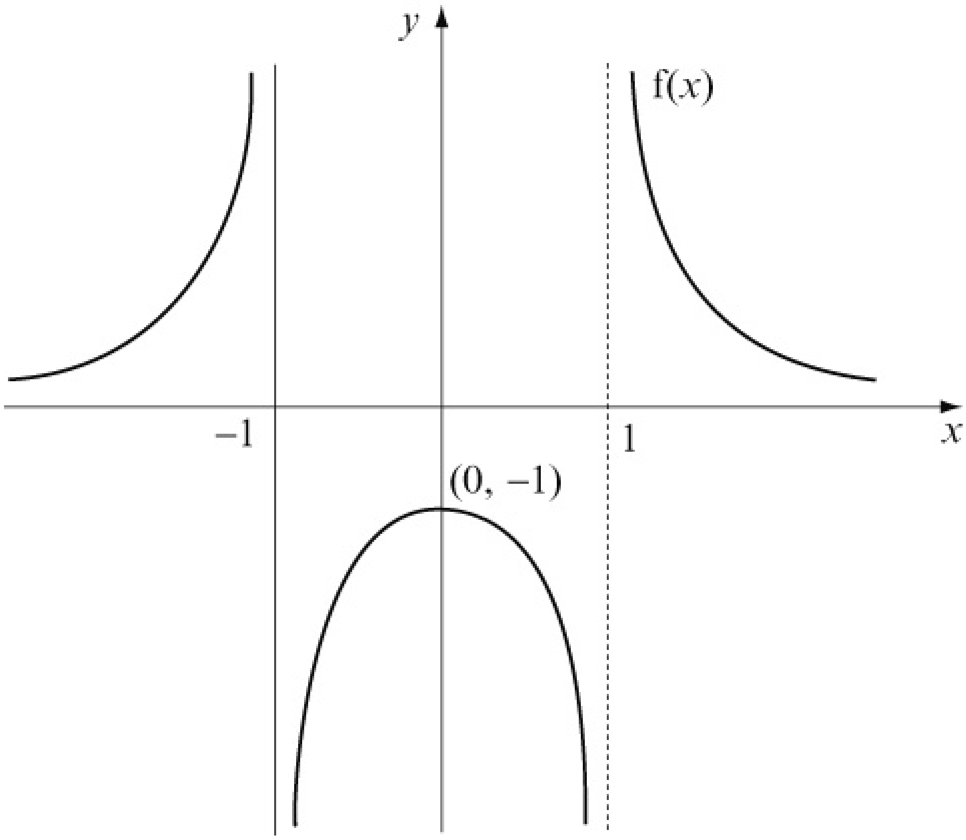
$$= \frac{x}{(x+1)(x-1)} - \frac{x-1}{(x+1)(x-1)}$$

$$= \frac{x - (x-1)}{(x+1)(x-1)}$$

$$= \frac{1}{(x+1)(x-1)}$$

(b) The range of $f(x)$ is the set of values that $y$ take.

By using a graphical calculator we can see that $y = f(x)$ $\left\{ x \in \mathbb{R}, \quad x \neq -1, \quad x \neq 1 \right\}$ is a symmetrical graph about the $y$ axis.




For $x > 1$, $f(x) > 0$

(c) $gf(x) = g\left[\frac{1}{(x-1)(x+1)}\right] = \frac{2}{\frac{1}{(x-1)(x+1)}} = 2 \times \frac{(x-1)(x+1)}{1} = 2\left(x-1\right)\left(x+1\right)$

If $gf(x) = 70$

$2(x-1)(x+1) = 70$

$(x-1)(x+1) = 35$

$x^2 - 1 = 35$

$x^2 = 36$

$x = \pm 6$