

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

(a) $y = |x - 1|$

(b) $y = |2x + 3|$

(c) $y = \left| \frac{1}{2}x - 5 \right|$

(d) $y = |7 - x|$

(e) $y = |x^2 - 7x - 8|$

(f) $y = |x^2 - 9|$

(g) $y = |x^3 + 1|$

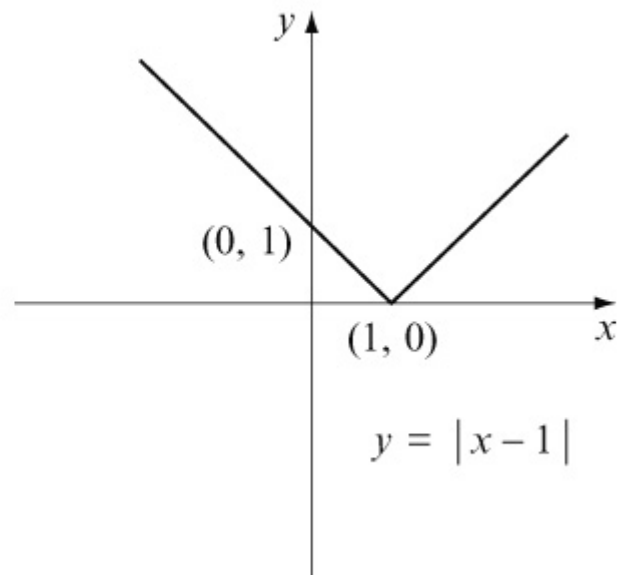
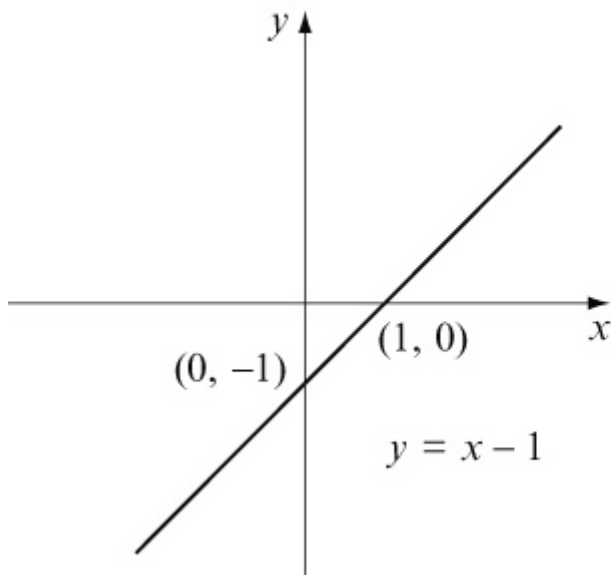
(h) $y = \left| \frac{12}{x} \right|$

(i) $y = -|x|$

(j) $y = -|3x - 1|$

Solution:

(a)

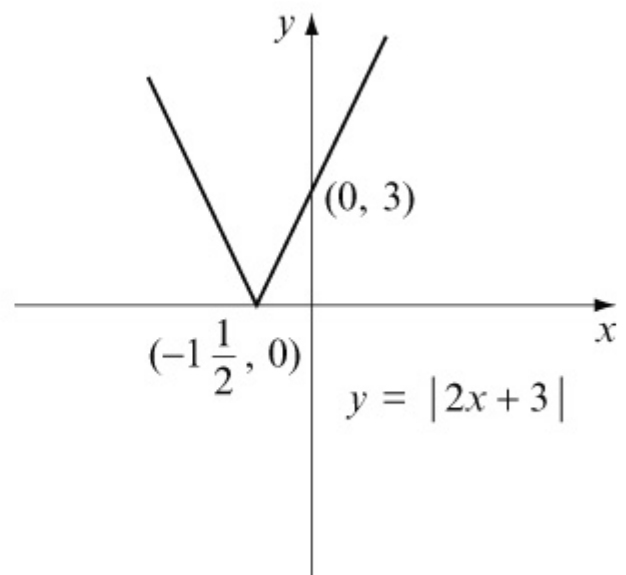
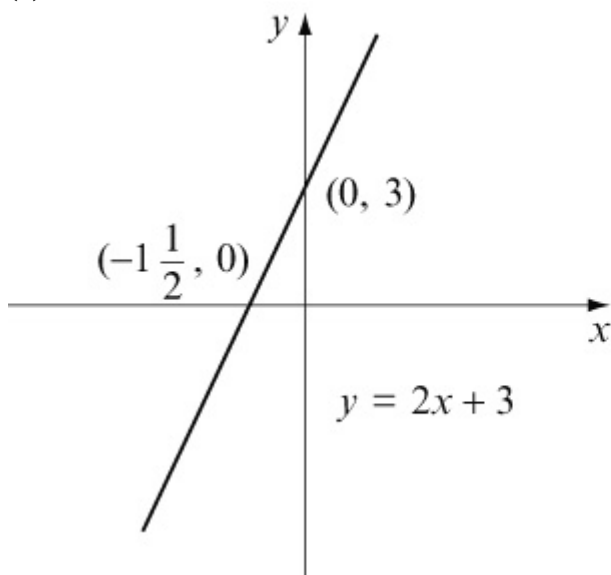


For $y = |x - 1|$:

When $x = 0$, $y = |-1| = 1$ (0, 1)

When $y = 0$, $x - 1 = 0 \Rightarrow x = 1$ (1, 0)

(b)

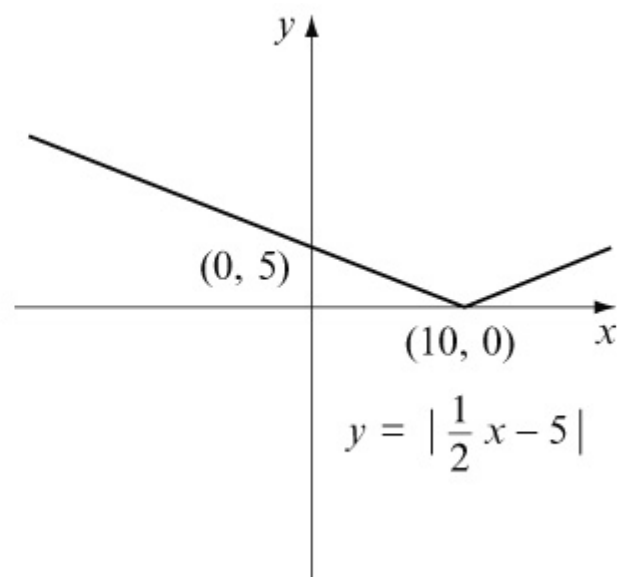
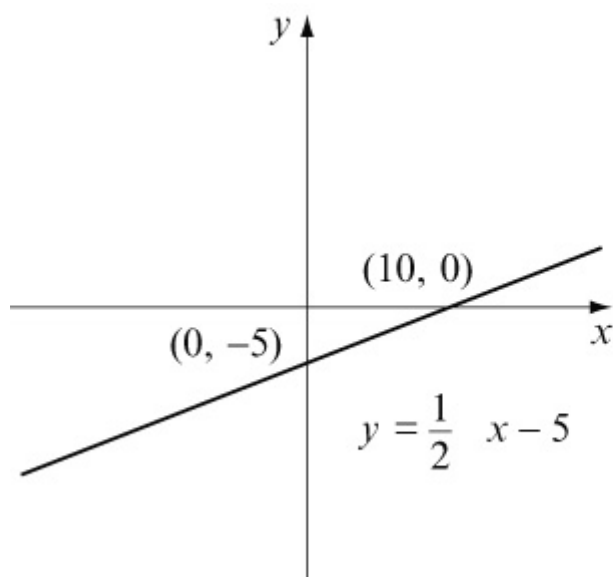


For $y = |2x + 3|$:

When $x = 0$, $y = |3| = 3$ (0, 3)

When $y = 0$, $2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$ $\left(-1\frac{1}{2}, 0\right)$

(c)

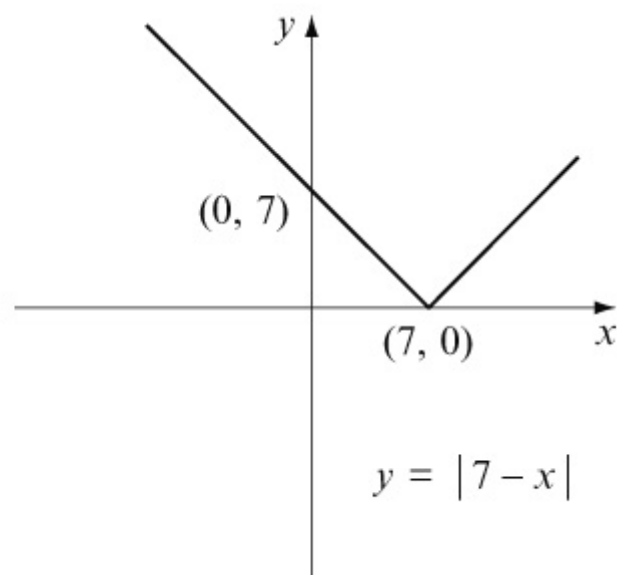
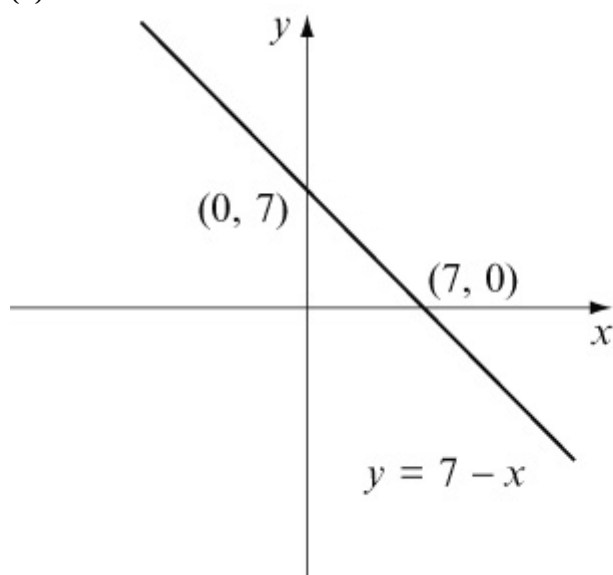


For $y = \left| \frac{1}{2}x - 5 \right|$:

When $x = 0$, $y = \left| -5 \right| = 5$ (0 , 5)

When $y = 0$, $\frac{1}{2}x - 5 = 0 \Rightarrow x = 10$ (10 , 0)

(d)



For $y = |7 - x|$:

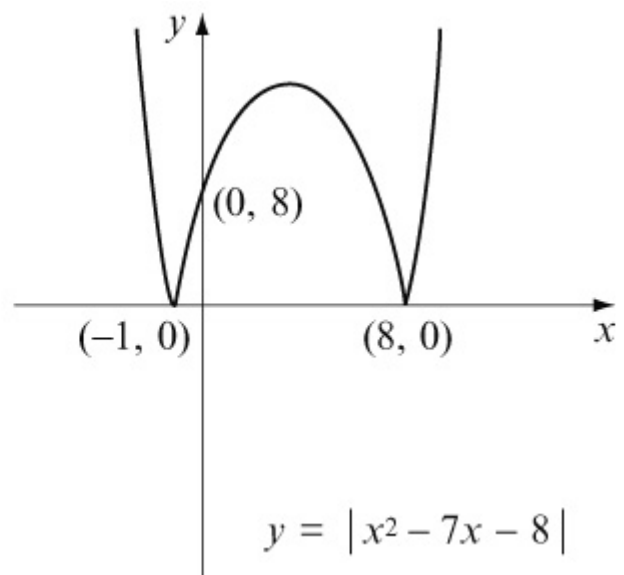
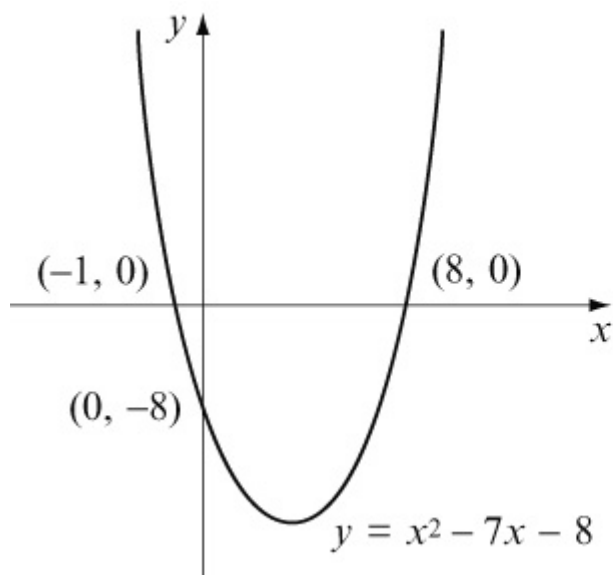
When $x = 0$, $y = |7| = 7$ (0 , 7)

When $y = 0$, $7 - x = 0 \Rightarrow x = 7$ (7 , 0)

(e) $x^2 - 7x - 8 = (x + 1)(x - 8)$

When $y = 0$, $(x + 1)(x - 8) = 0 \Rightarrow x = -1$ and $x = 8$

Curve crosses x-axis at (- 1 , 0) and (8, 0)



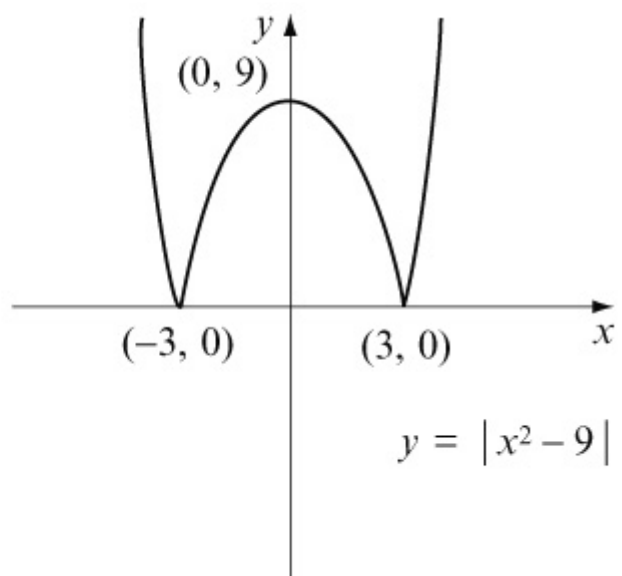
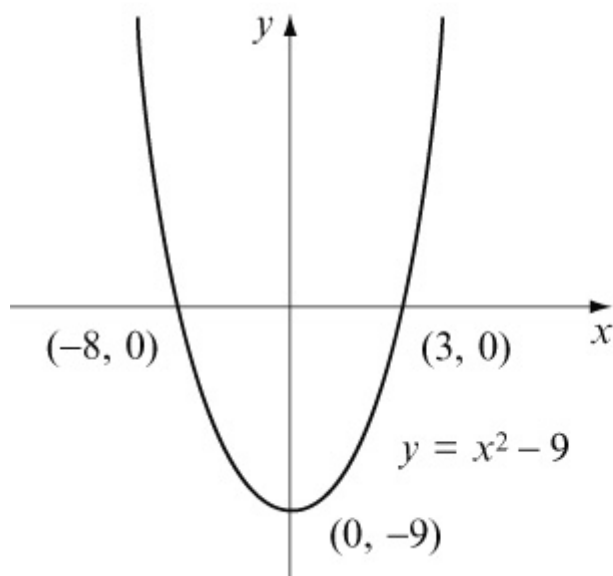
For $y = |x^2 - 7x - 8|$:

When $x = 0$, $y = |-8| = 8$ $(0, 8)$

$$(f) x^2 - 9 = (x + 3)(x - 3)$$

When $y = 0$, $(x + 3)(x - 3) = 0 \Rightarrow x = -3$ and $x = 3$

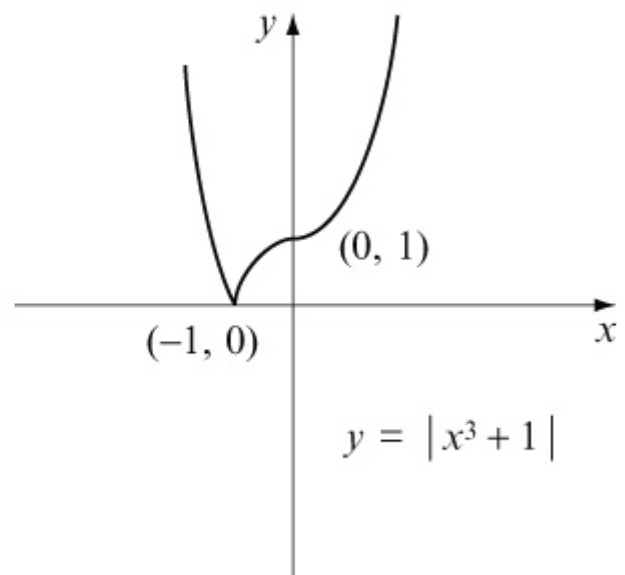
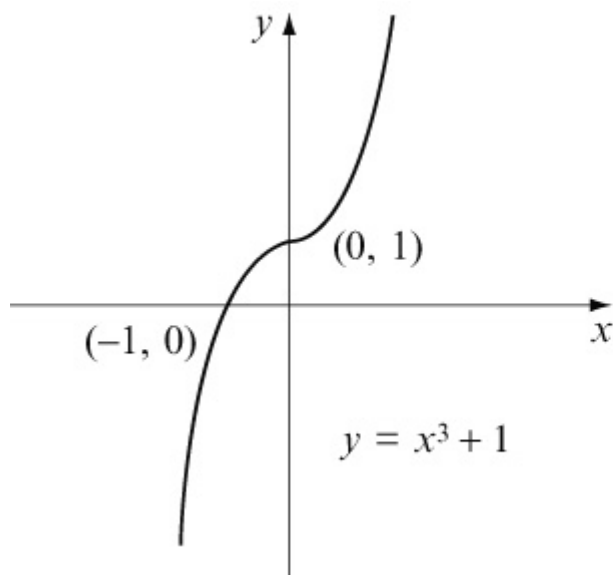
Curve crosses x-axis at $(-3, 0)$ and $(3, 0)$



For $y = |x^2 - 9|$:

When $x = 0$, $y = |-9| = 9$ $(0, 9)$

(g) The graph of $y = x^3 + 1$ is found by translating $y = x^3$ by $+1$ parallel to the y-axis.

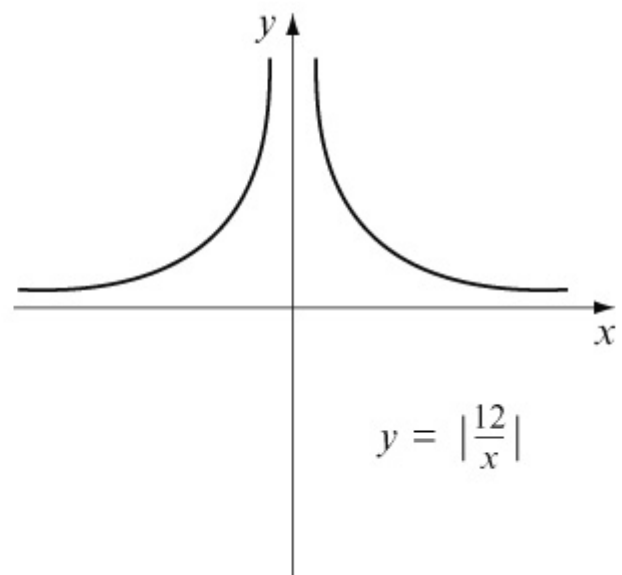
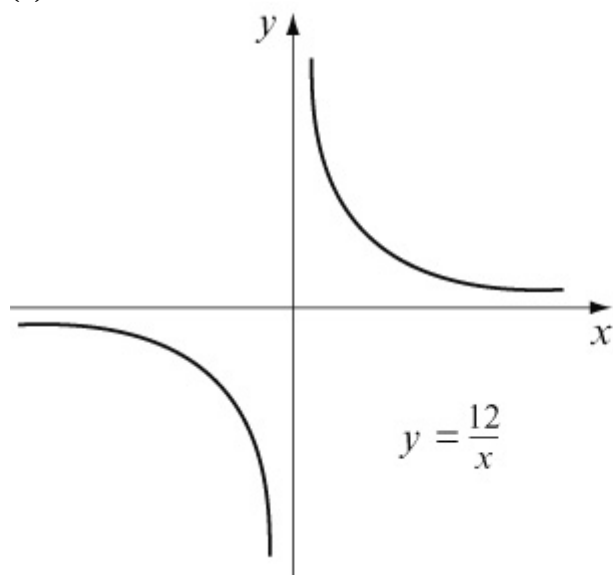


For $y = |x^3 + 1|$:

When $x = 0$, $y = |1| = 1$ $(0, 1)$

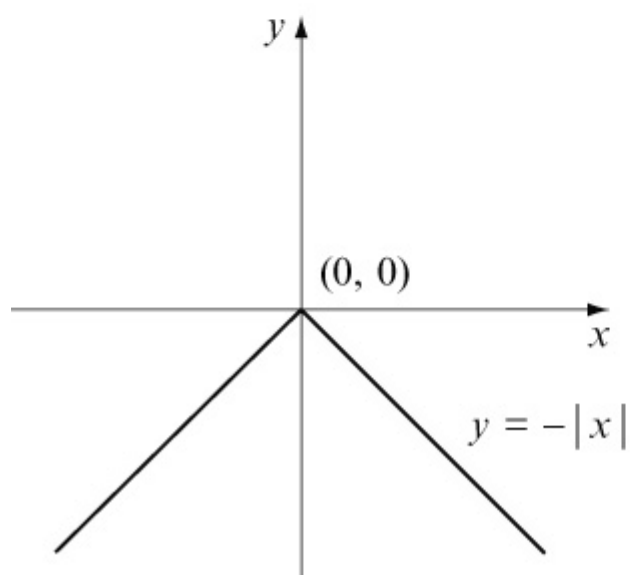
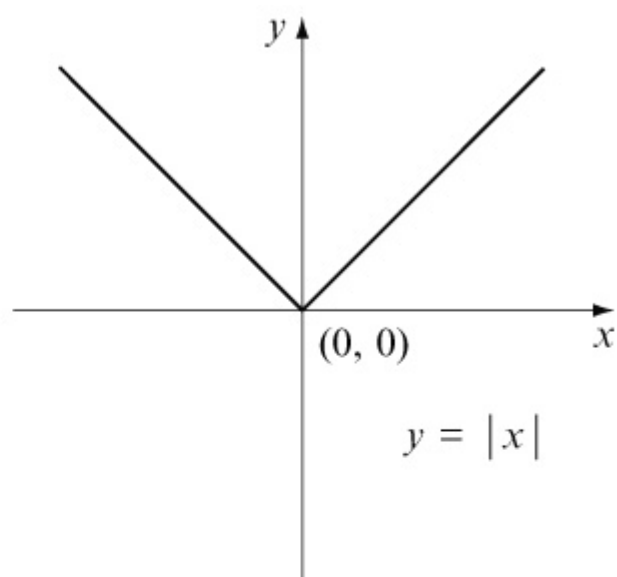
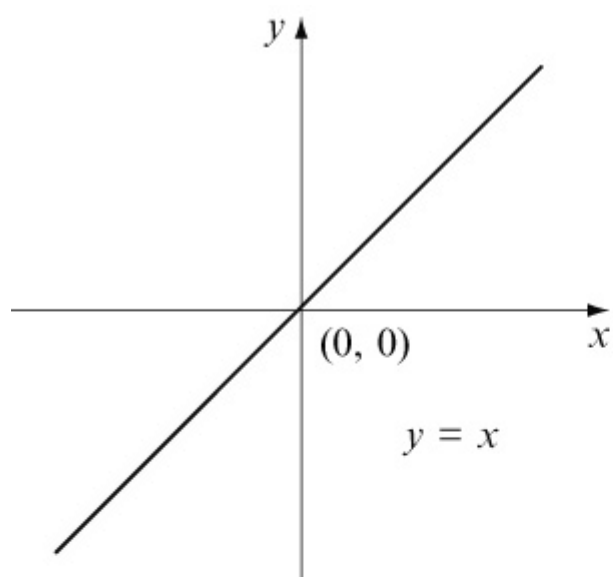
When $y = 0$, $x^3 + 1 = 0 \Rightarrow x^3 = -1 \Rightarrow x = -1$ $(-1, 0)$

(h)



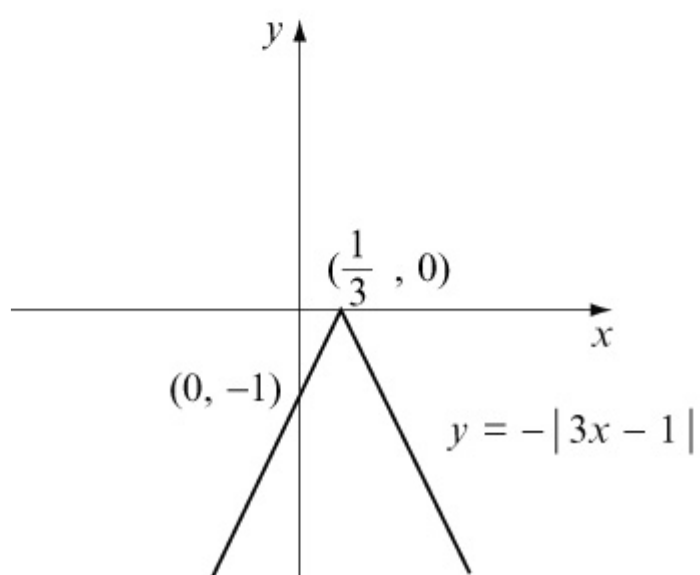
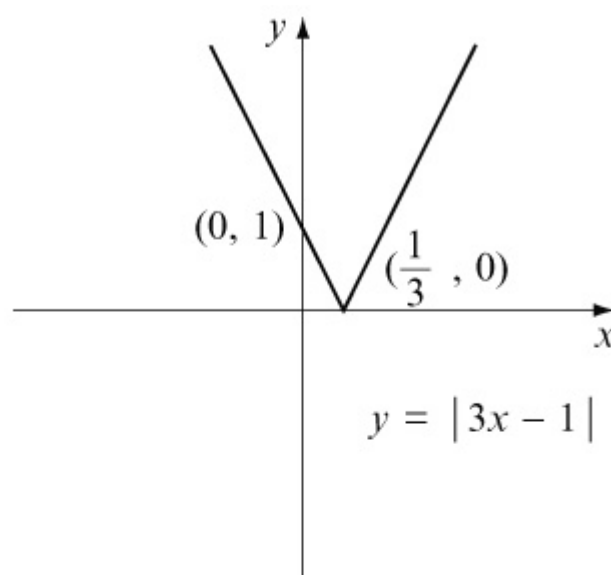
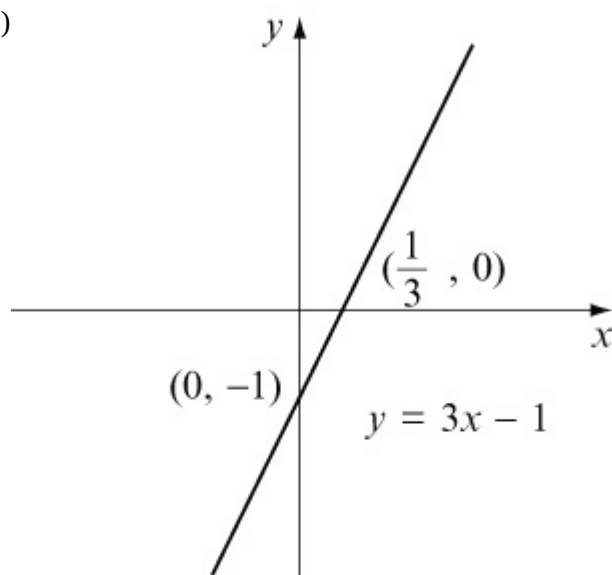
No intersections with the axes (the axes are asymptotes).

(i)



Passes through the origin $(0, 0)$

(i)



For $y = -|3x - 1|$:

When $x = 0$, $y = -|-1| = -1$ $(0, -1)$

When $y = 0$, $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$ $(\frac{1}{3}, 0)$

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Exercise A, Question 2

Question:

Sketch the graph of each of the following. In each case, write down the coordinates of any points at which the graph meets the coordinate axes.

(a) $y = |\cos x|$, $0 \leq x \leq 2\pi$

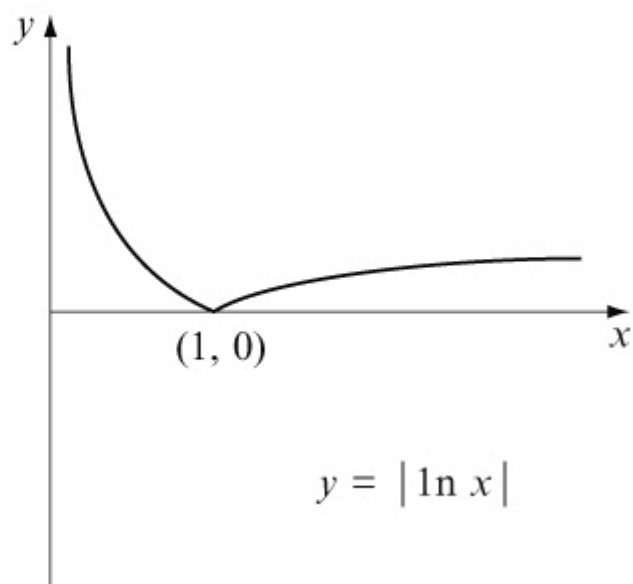
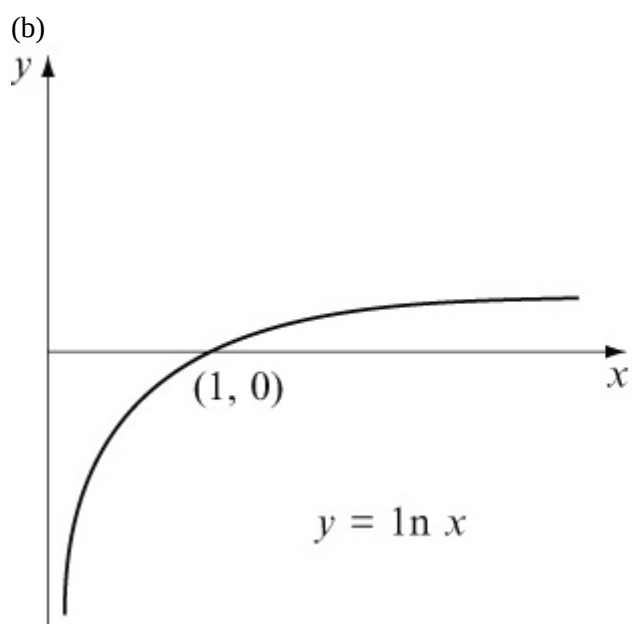
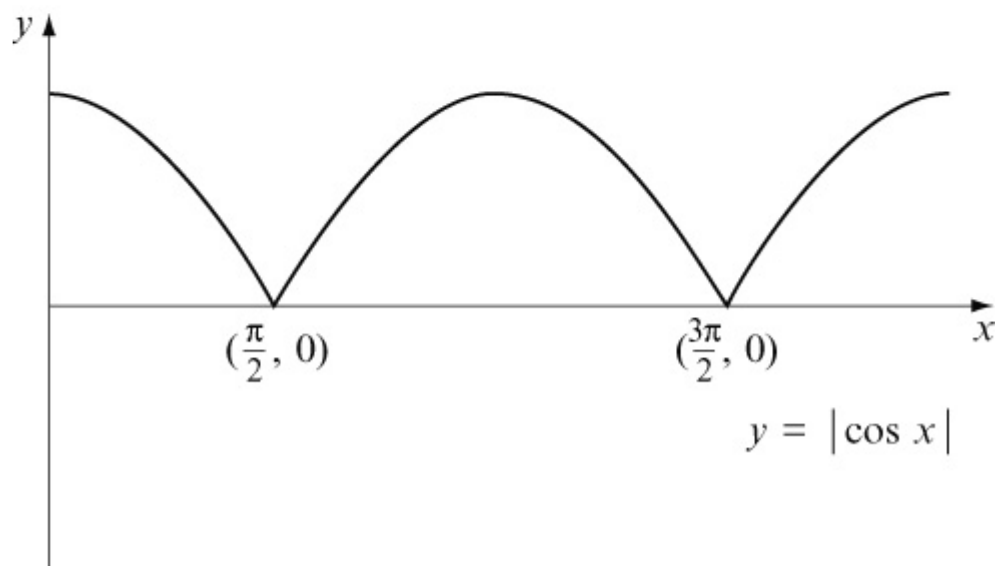
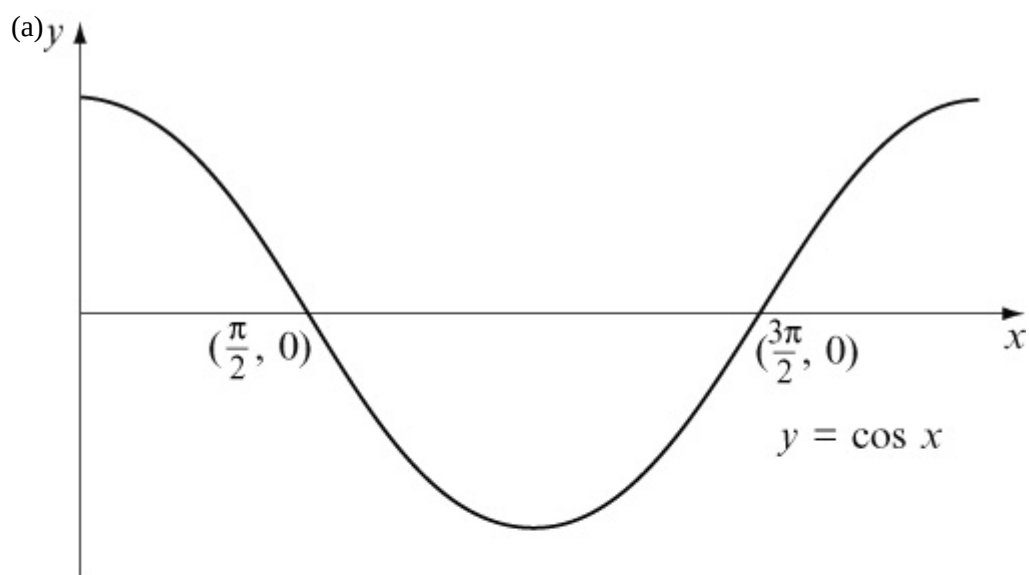
(b) $y = |\ln x|$, $x > 0$

(c) $y = |2^x - 2|$

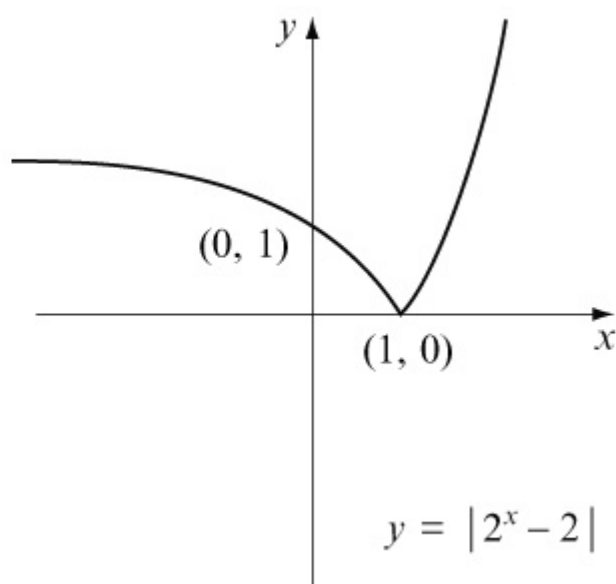
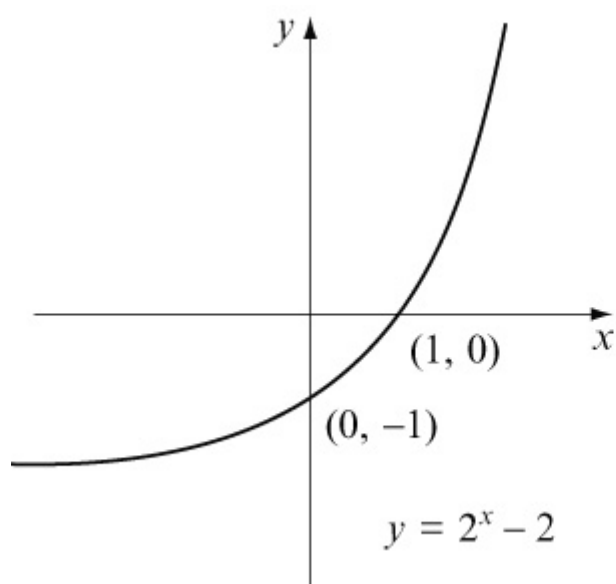
(d) $y = |100 - 10^x|$

(e) $y = |\tan 2x|$, $0 < x < 2\pi$

Solution:



(c)

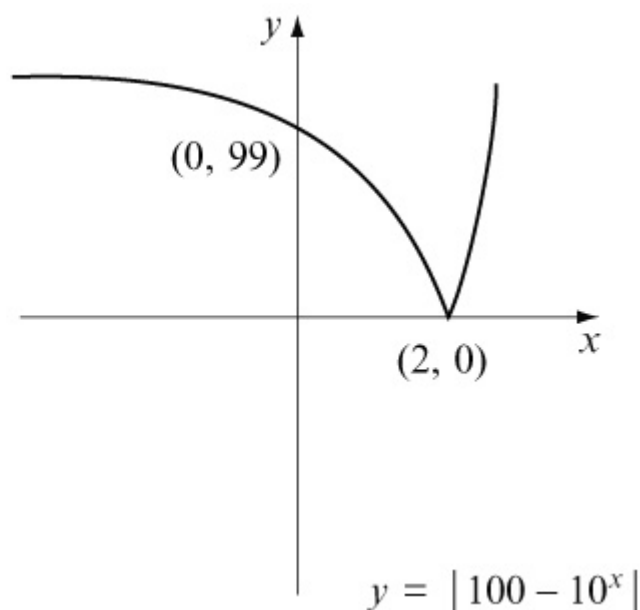
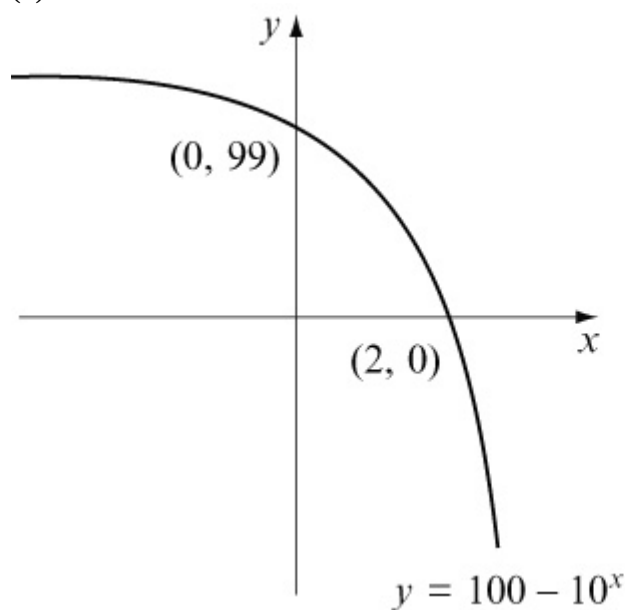


For $y = |2^x - 2|$:

When $x = 0$, $y = |2^0 - 2| = |-1| = 1$ $(0, 1)$

When $y = 0$, $2^x - 2 = 0 \Rightarrow 2^x = 2 \Rightarrow x = 1$ $(1, 0)$

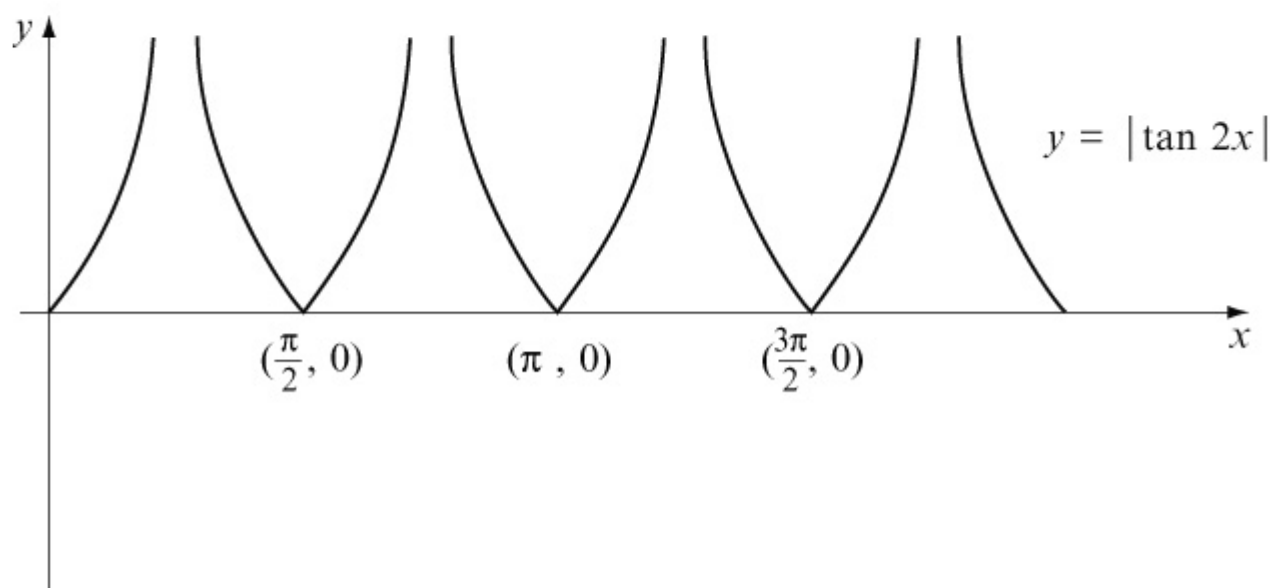
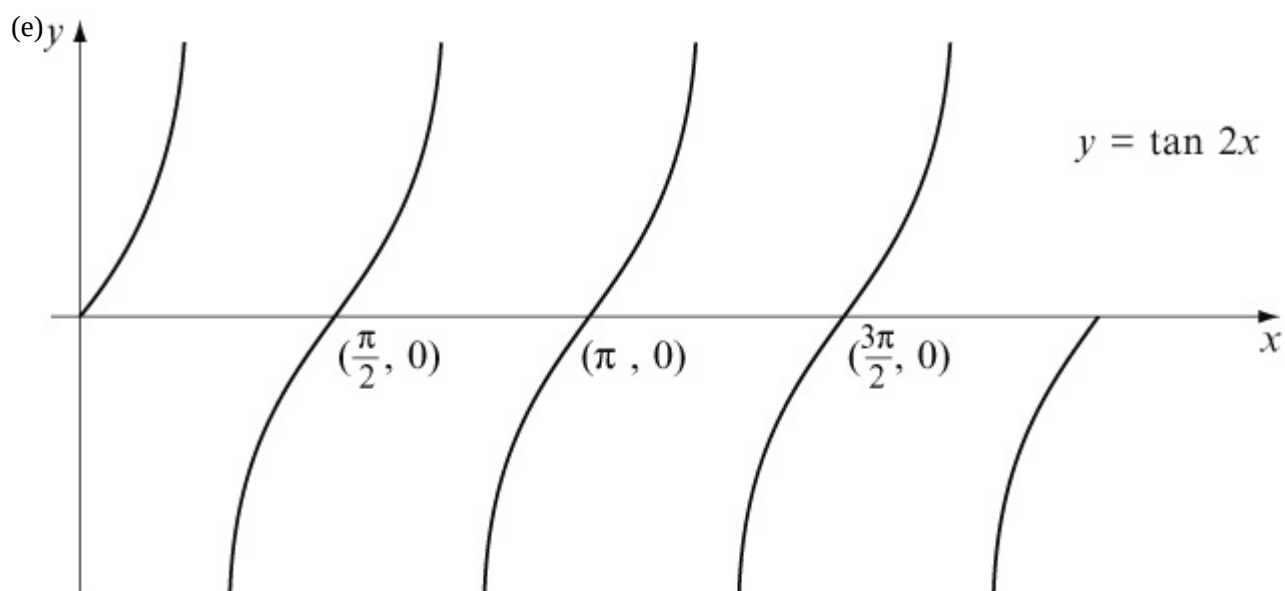
(d)



For $y = |100 - 10^x|$:

When $x = 0$, $y = |100 - 10^0| = |99| = 99$ $(0, 99)$

When $y = 0$, $100 - 10^x = 0 \Rightarrow 10^x = 100 \Rightarrow x = 2$ $(2, 0)$



For $y = |\tan 2x|$:

When $x = 0$, $y = |\tan 0| = 0$

When $y = 0$, $\tan 2x = 0$

$$\Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \dots \quad \left(\frac{\pi}{2}, 0 \right), (\pi, 0), \left(\frac{3\pi}{2}, 0 \right)$$

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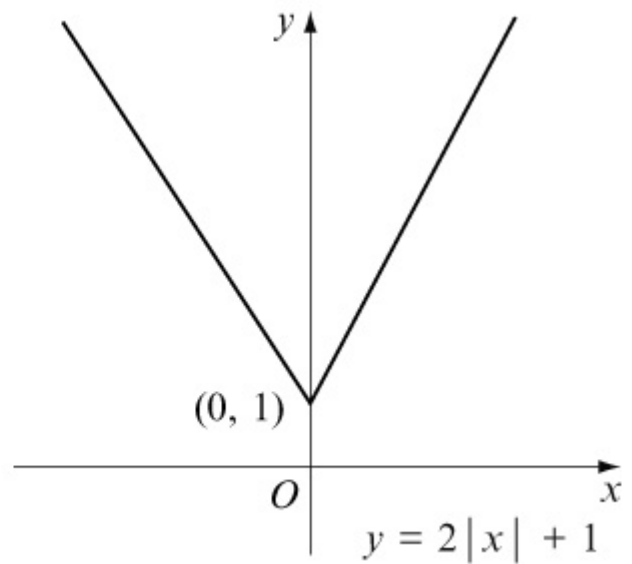
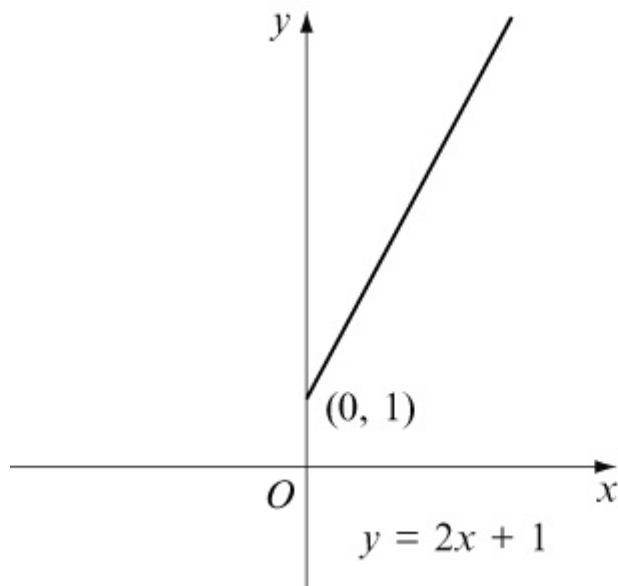
Exercise B, Question 1

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$$y = 2|x| + 1$$

Solution:



For $y = 2|x| + 1$:

When $x = 0$, $y = 1$ $(0, 1)$

When $y = 0$, $2|x| + 1 = 0$

$$\Rightarrow |x| = -\frac{1}{2}$$

No values ($|x|$ cannot be negative).

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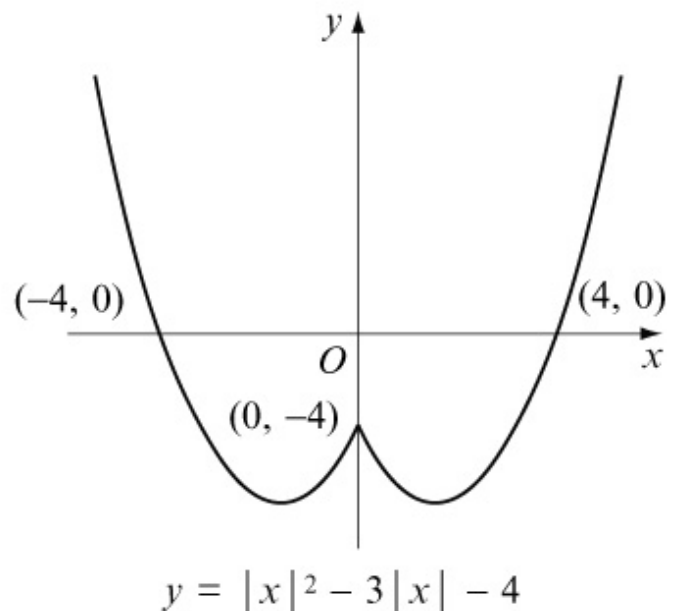
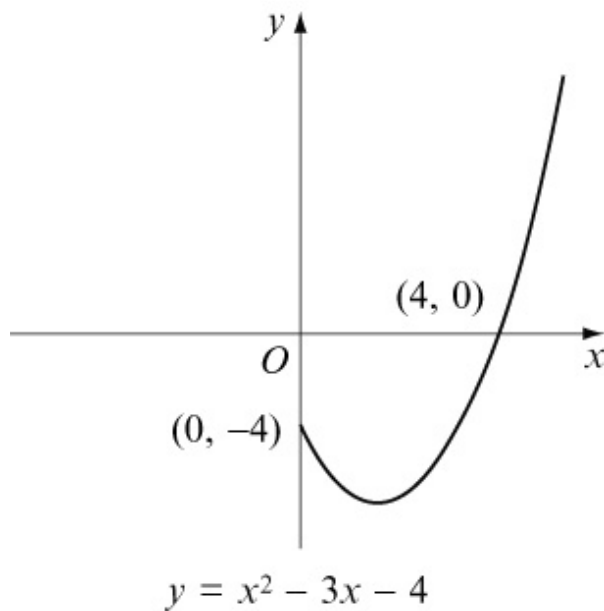
Exercise B, Question 2

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$$y = |x|^2 - 3|x| - 4$$

Solution:



For $y = |x|^2 - 3|x| - 4$:

When $x = 0$, $y = -4$ $(0, -4)$

When $y = 0$, $|x|^2 - 3|x| - 4 = 0$

$$\Rightarrow (|x| + 1)(|x| - 4) = 0$$

$$\Rightarrow |x| = 4$$

$$\Rightarrow x = 4 \text{ or } -4 \quad (-4, 0) \text{ and } (4, 0)$$

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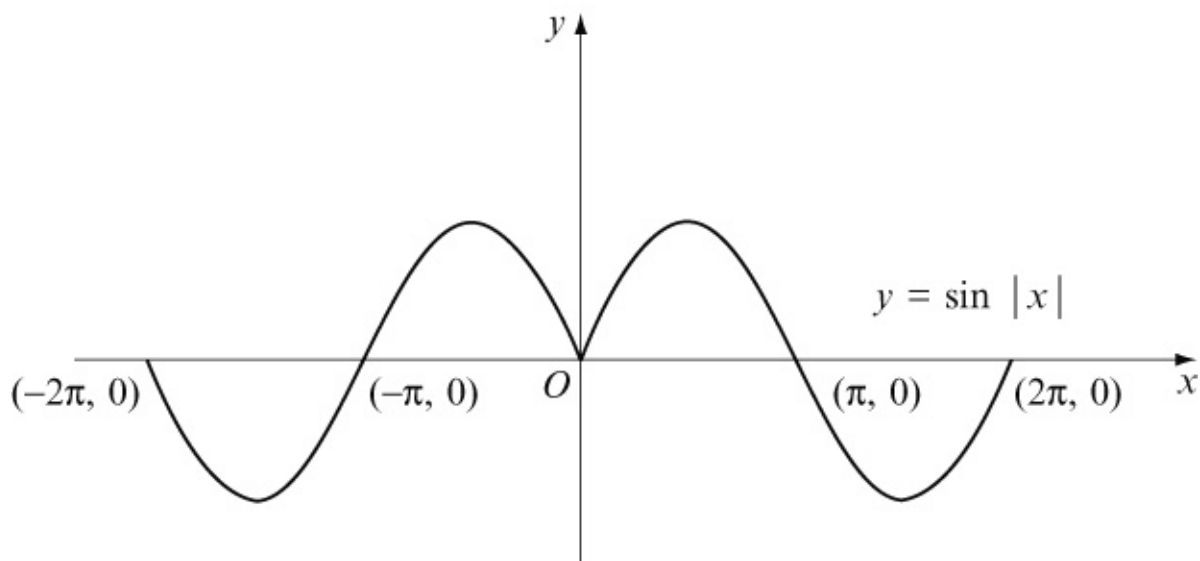
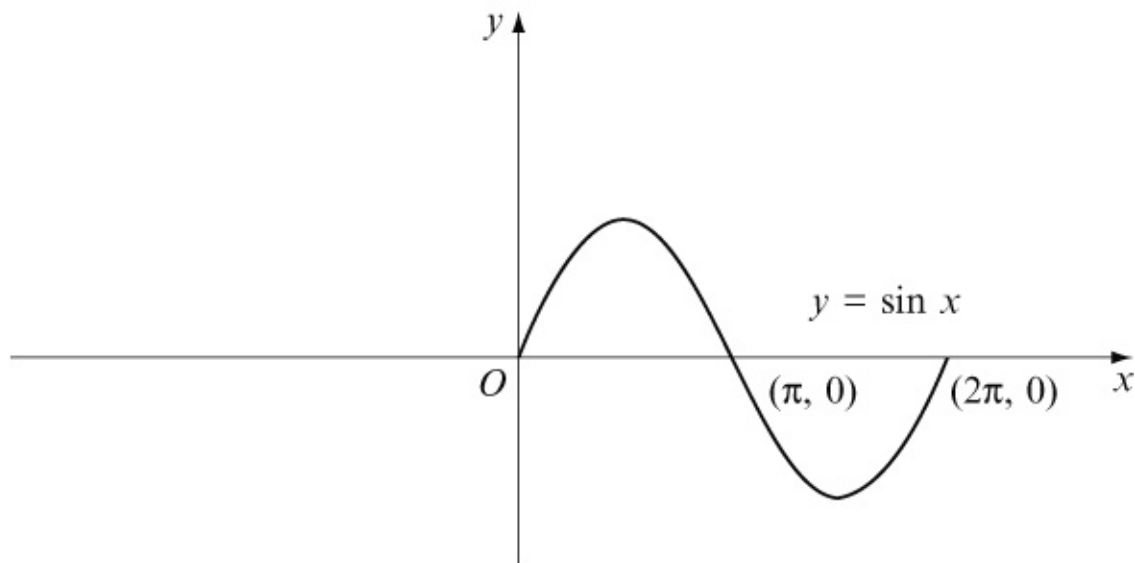
Exercise B, Question 3

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$$y = \sin |x|, \quad -2\pi \leq x \leq 2\pi$$

Solution:



...

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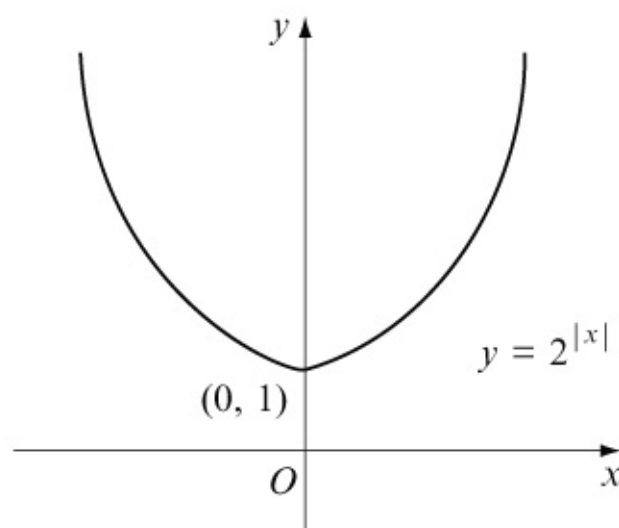
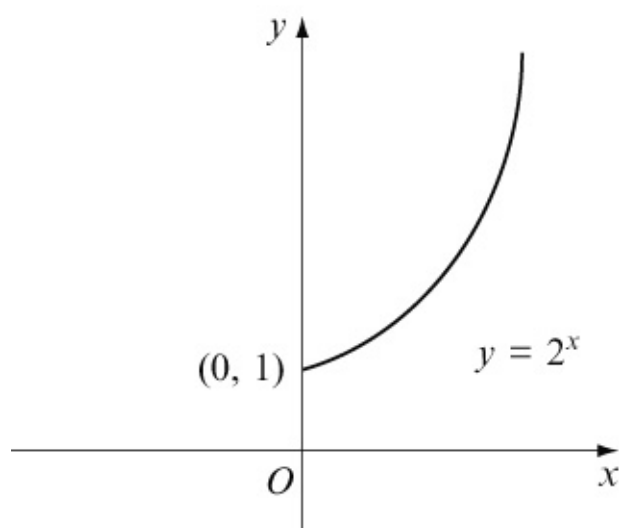
Exercise B, Question 4

Question:

Sketch the following graph and write down the coordinates of any points at which the graph meets the coordinate axes.

$$y = 2^{|x|}$$

Solution:



...

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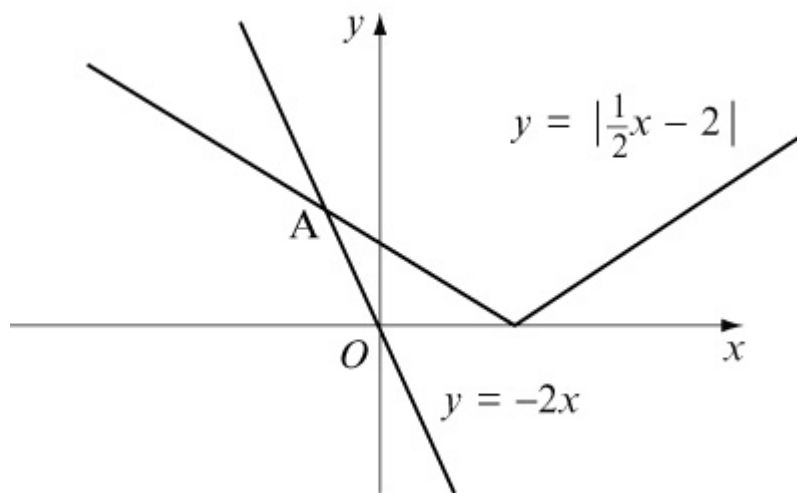
Exercise C, Question 1

Question:

On the same diagram, sketch the graphs of $y = -2x$ and $y = \left| \frac{1}{2}x - 2 \right|$.

Solve the equation $-2x = \left| \frac{1}{2}x - 2 \right|$.

Solution:



Intersection point A is on the reflected part of $y = \frac{1}{2}x - 2$.

$$-\left(\frac{1}{2}x - 2\right) = -2x$$

$$-\frac{1}{2}x + 2 = -2x$$

$$2x - \frac{1}{2}x = -2$$

$$\frac{3}{2}x = -2$$

$$x = -\frac{4}{3}$$

Solutionbank

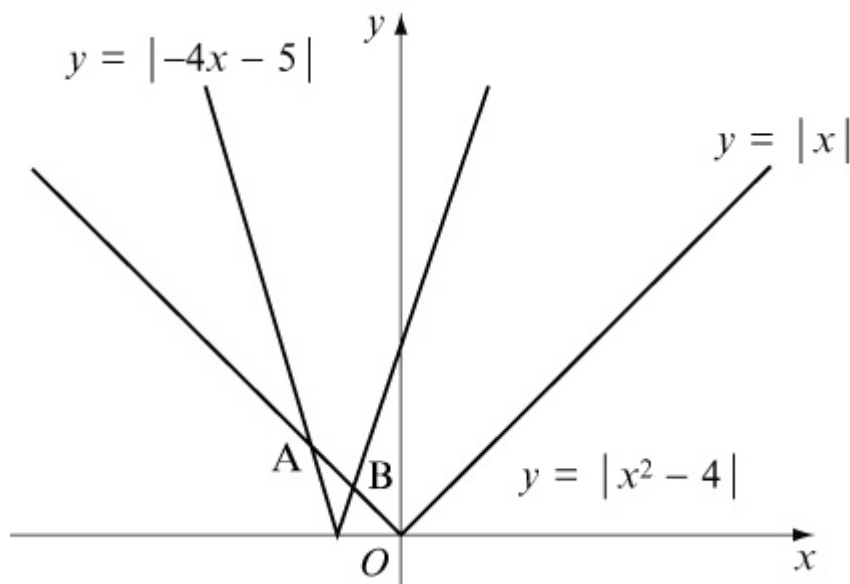
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Exercise C, Question 2

Question:

On the same diagram, sketch the graphs of $y = |x|$ and $y = |-4x - 5|$.
Solve the equation $|x| = |-4x - 5|$.

Solution:



Intersection point A is on the reflected part of $y = x$.

$$-x = -4x - 5$$

$$4x - x = -5$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

Intersection point B is on the reflected part of $y = x$ and also on the reflected part of $y = -4x - 5$.

$$-x = -(-4x - 5)$$

$$-x = 4x + 5$$

$$-x - 4x = 5$$

$$-5x = 5$$

$$x = -1$$

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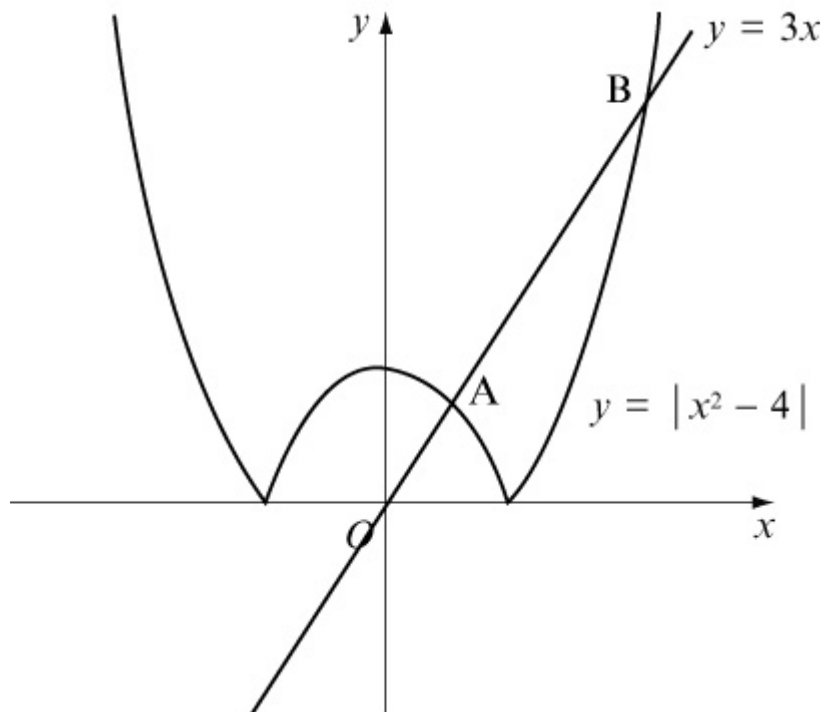
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Exercise C, Question 3

Question:

On the same diagram, sketch the graphs of $y = 3x$ and $y = |x^2 - 4|$. Solve the equation $3x = |x^2 - 4|$.

Solution:



Intersection point A is on the reflected part of $y = x^2 - 4$.

$$3x = -(x^2 - 4)$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0 \quad (x = -4 \text{ is not valid})$$

$$x = 1$$

Intersection point B:

$$3x = x^2 - 4$$

$$0 = x^2 - 3x - 4$$

$$0 = (x - 4)(x + 1) \quad (x = -1 \text{ is not valid})$$

$$x = 4$$

Solutionbank

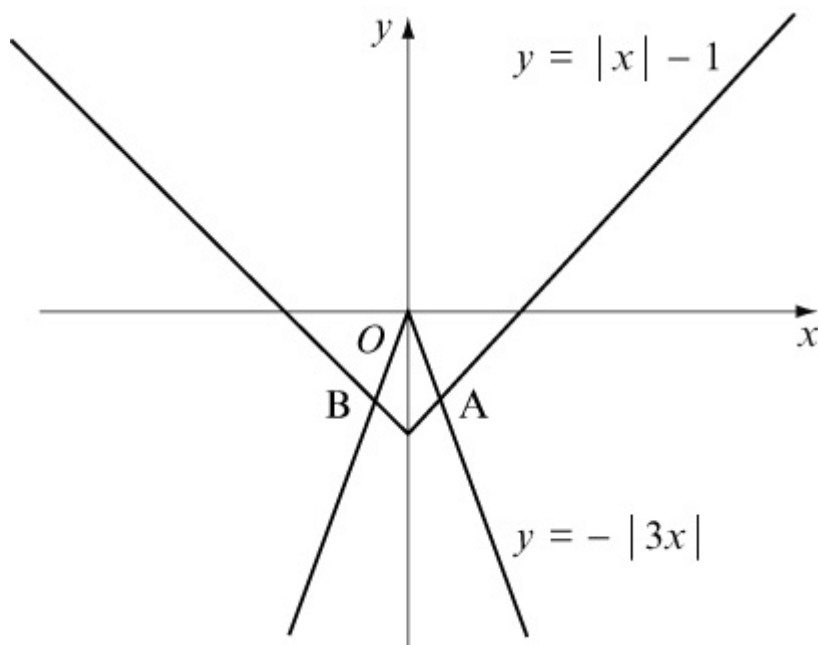
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Exercise C, Question 4

Question:

On the same diagram, sketch the graphs of $y = |x| - 1$ and $y = -|3x|$.
Solve the equation $|x| - 1 = -|3x|$.

Solution:



Intersection point A:

$$x - 1 = -3x$$

$$3x + x = 1$$

$$x = \frac{1}{4}$$

Intersection point B is on the reflected part of both graphs.

$$-(x) - 1 = -(-3x)$$

$$-x - 1 = 3x$$

$$-4x = 1$$

$$x = -\frac{1}{4}$$

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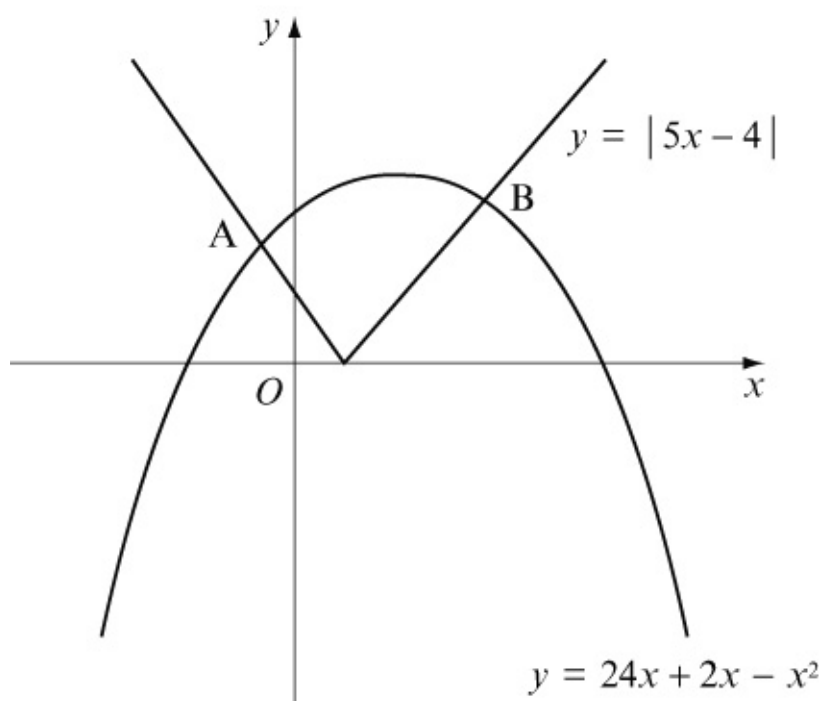
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Exercise C, Question 5

Question:

On the same diagram, sketch the graphs of $y = 24 + 2x - x^2$ and $y = |5x - 4|$.
Solve the equation $24 + 2x - x^2 = |5x - 4|$. (Answers to 2 d.p. where appropriate).

Solution:



Intersection point A is on the reflected part of $y = 5x - 4$.

$$-(5x - 4) = 24 + 2x - x^2$$

$$-5x + 4 = 24 + 2x - x^2$$

$$x^2 - 7x - 20 = 0$$

$$x = \frac{7 \pm \sqrt{49 + 80}}{2} \quad (\text{positive solution not valid})$$

$$x = -2.18 \text{ (2 d.p.)}$$

Intersection point B:

$$5x - 4 = 24 + 2x - x^2$$

$$x^2 + 3x - 28 = 0$$

$$(x + 7)(x - 4) = 0 \quad (x = -7 \text{ is not valid})$$

$$x = 4$$

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Exercise D, Question 1

Question:

Using combinations of transformations, sketch the graph of each of the following:

(a) $y = 2x^2 - 4$

(b) $y = 3(x + 1)^2$

(c) $y = \frac{3}{x} - 2$

(d) $y = \frac{3}{x - 2}$

(e) $y = 5 \sin(x + 30^\circ), 0 \leq x \leq 360^\circ$

(f) $y = \frac{1}{2}e^x + 4$

(g) $y = |4x| + 1$

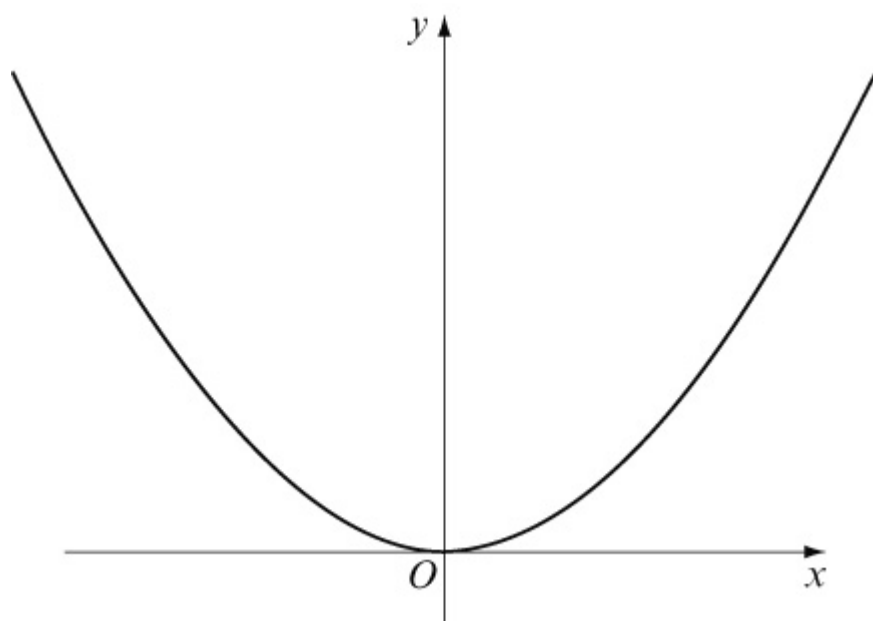
(h) $y = 2x^3 - 3$

(i) $y = 3 \ln(x - 2), x > 2$

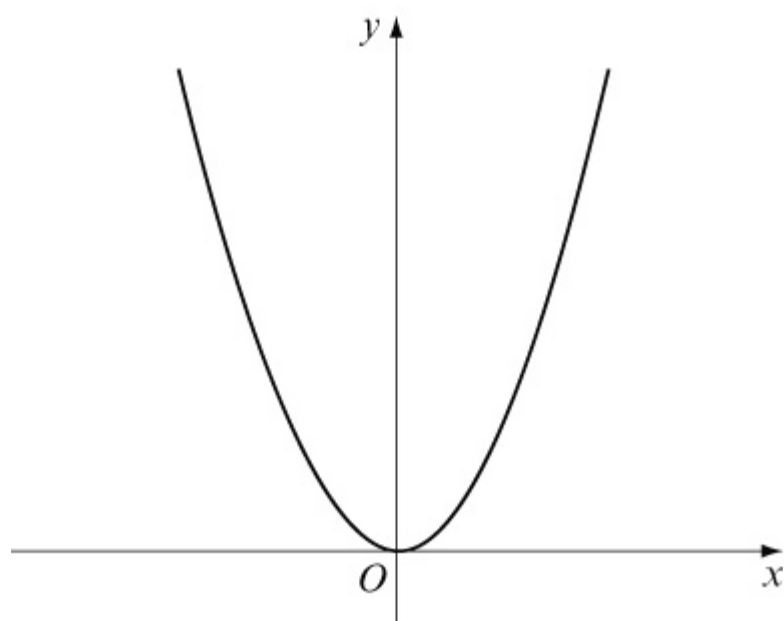
(j) $y = |2e^x - 3|$

Solution:

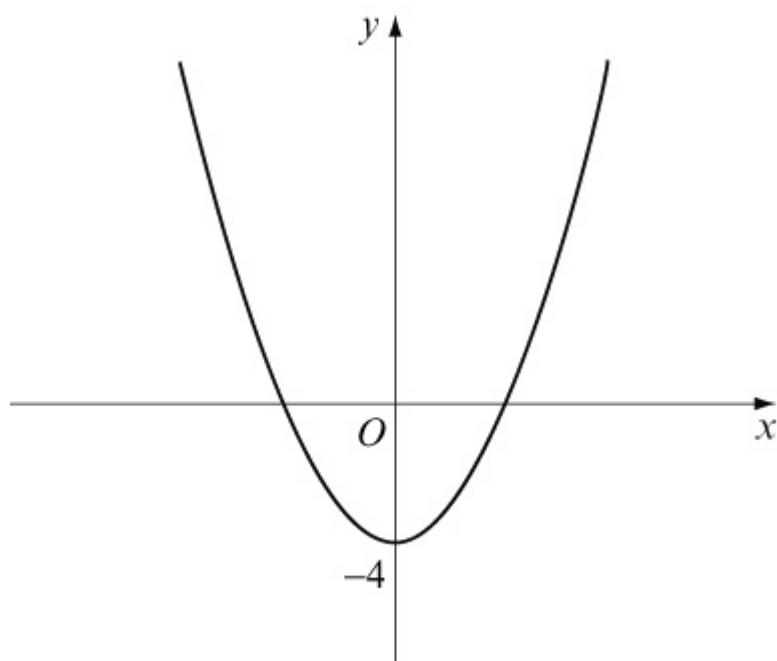
(a) $y = x^2$



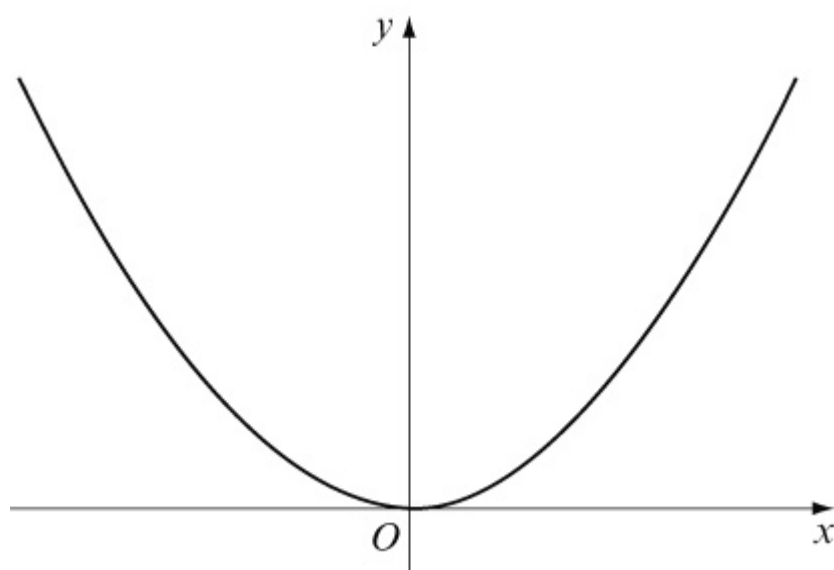
$y = 2x^2$. Vertical stretch, scale factor 2.



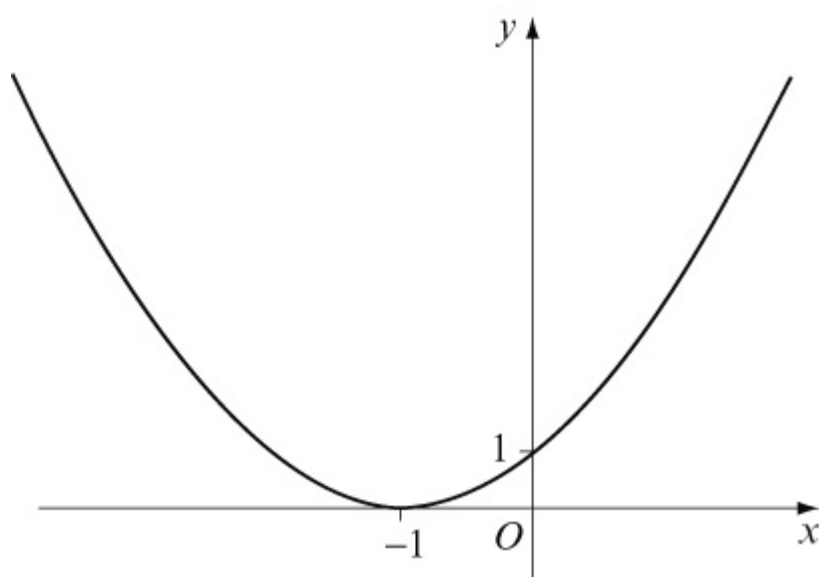
$y = 2x^2 - 4$. Vertical translation of -4 .



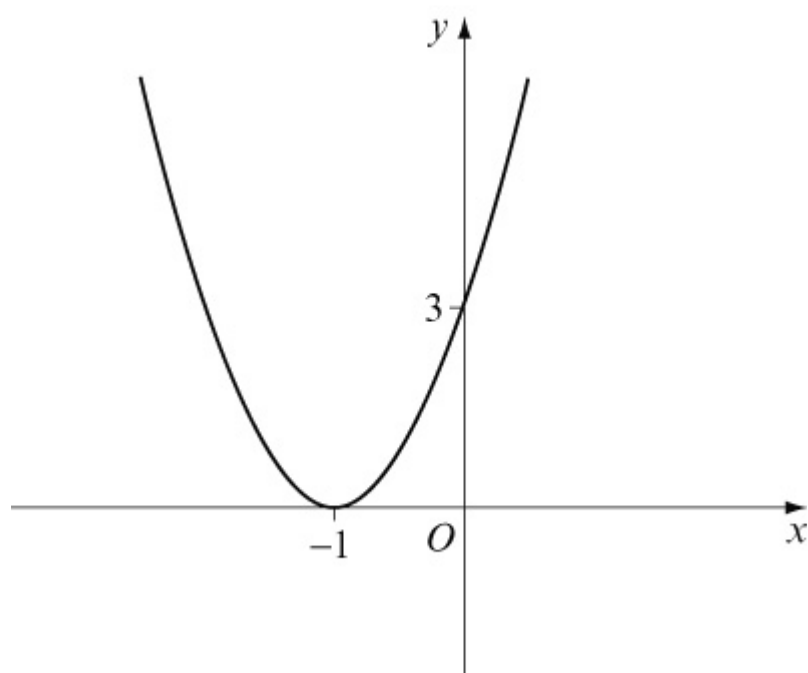
(b) $y = x^2$



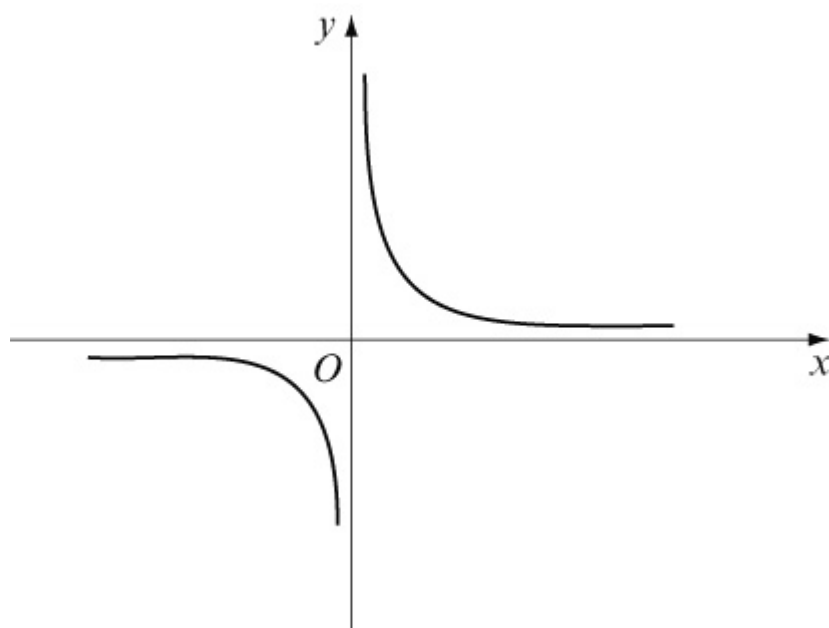
$y = (x + 1)^2$. Horizontal translation of -1 .



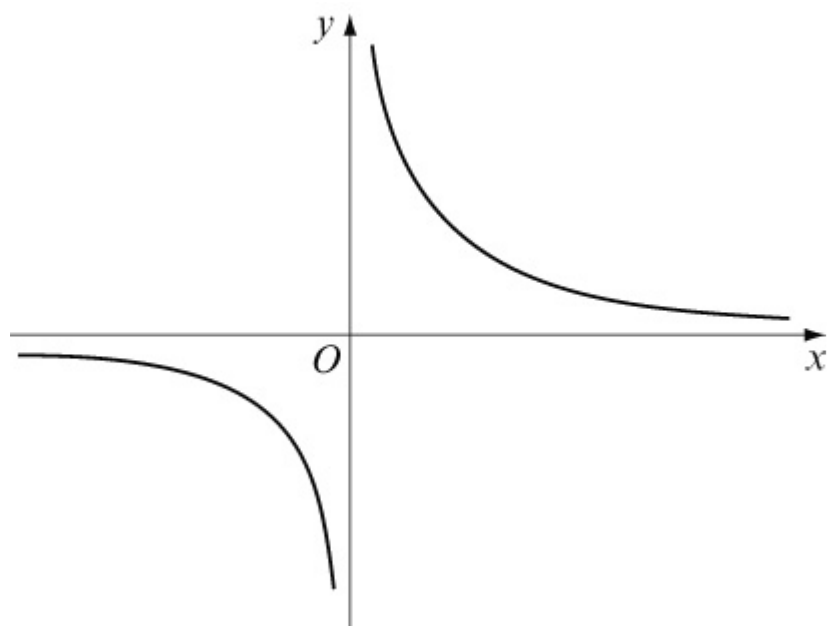
$y = 3(x + 1)^2$. Vertical stretch, scale factor 3.



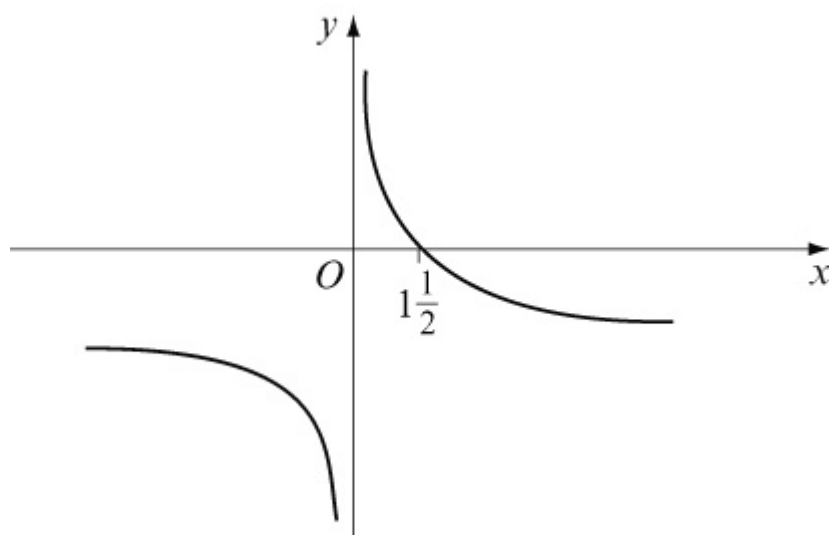
(c) $y = \frac{1}{x}$



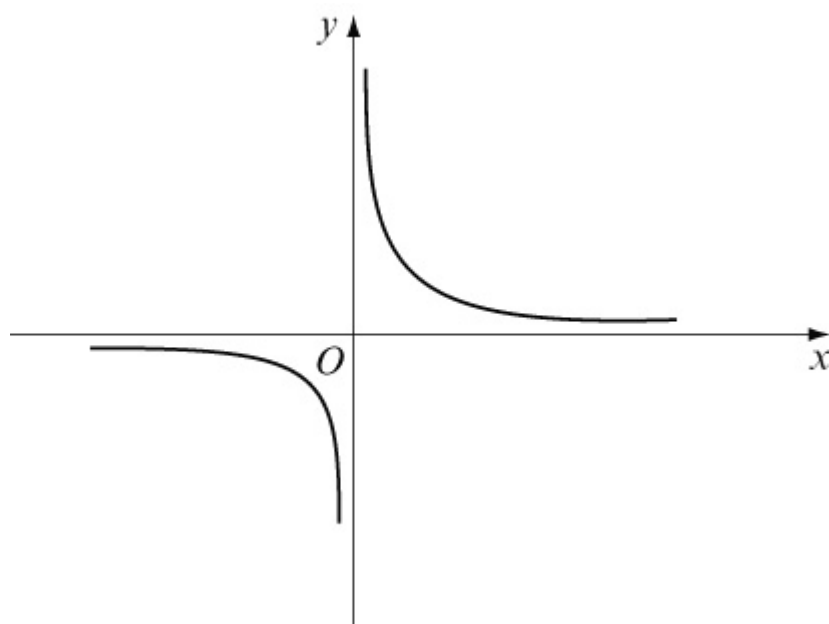
$y = \frac{3}{x}$. Vertical stretch, scale factor 3.



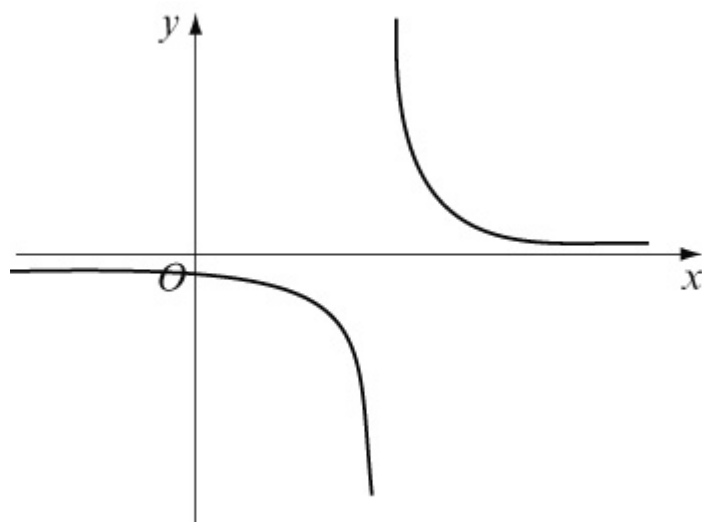
$y = \frac{3}{x} - 2$. Vertical translation of -2 .



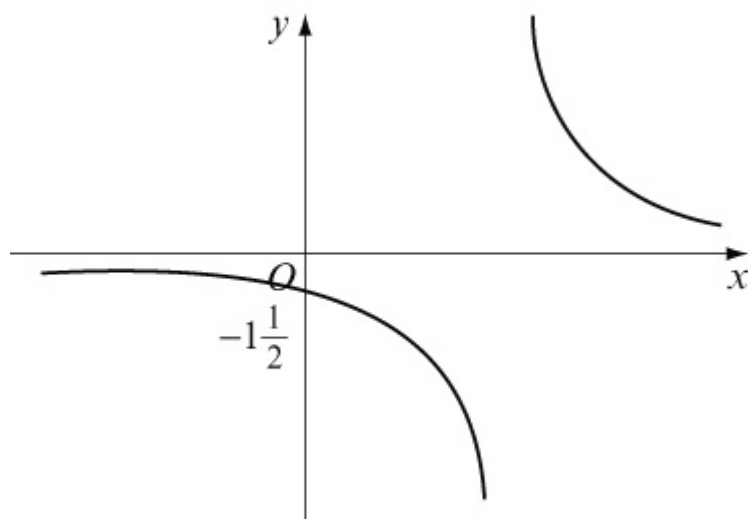
(d) $y = \frac{1}{x}$



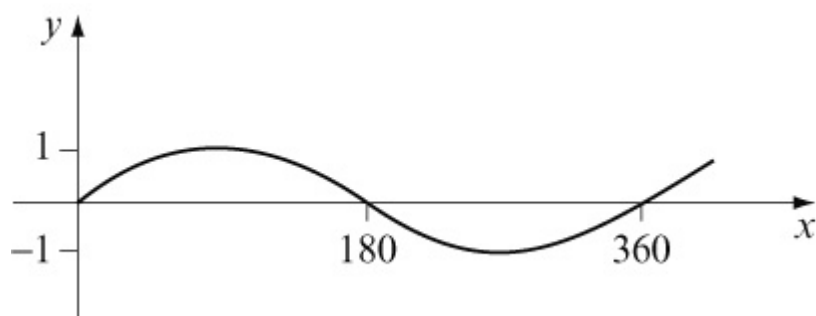
$y = \frac{1}{x-2}$. Horizontal translation of +2.



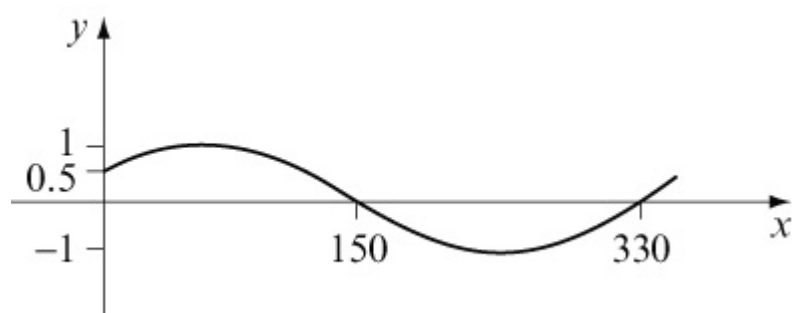
$y = \frac{3}{x-2}$. Vertical stretch, scale factor 3.



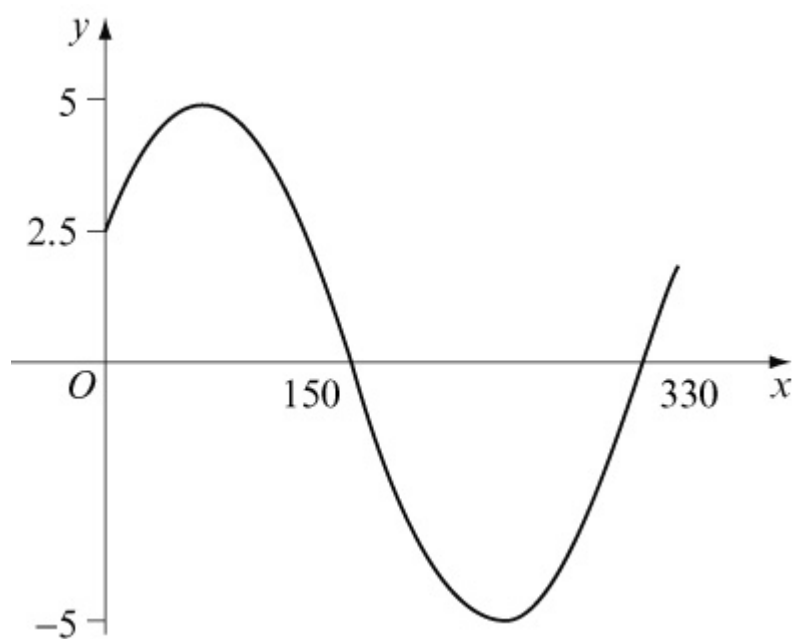
(e) $y = \sin x$



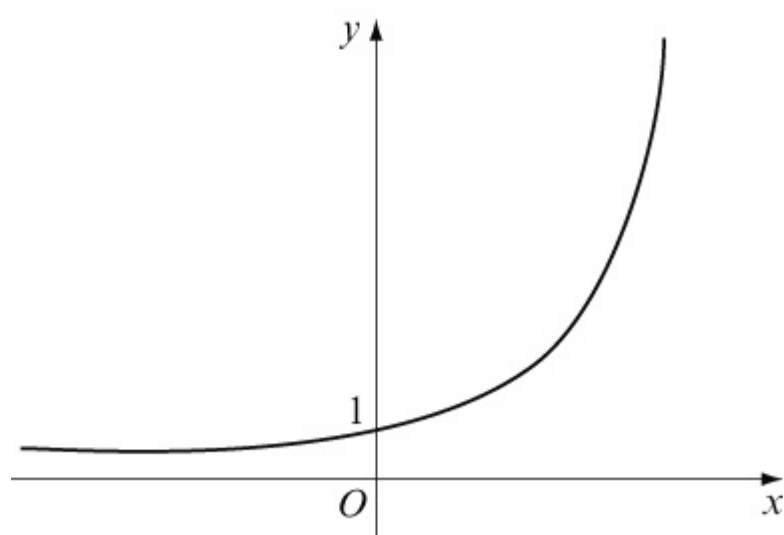
$y = \sin (x + 30^\circ)$. Horizontal translation of $- 30^\circ$



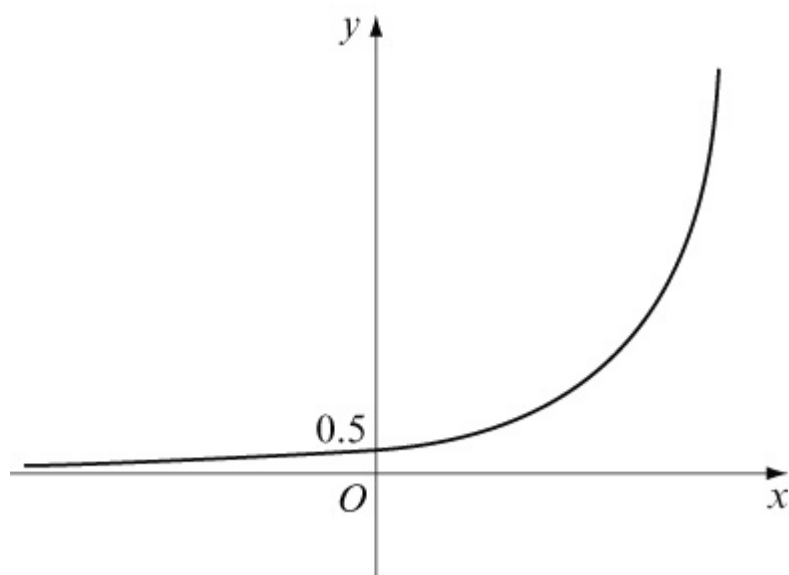
$y = 5 \sin (x + 30 ^\circ)$. Vertical stretch, scale factor 5.



(f) $y = e^x$

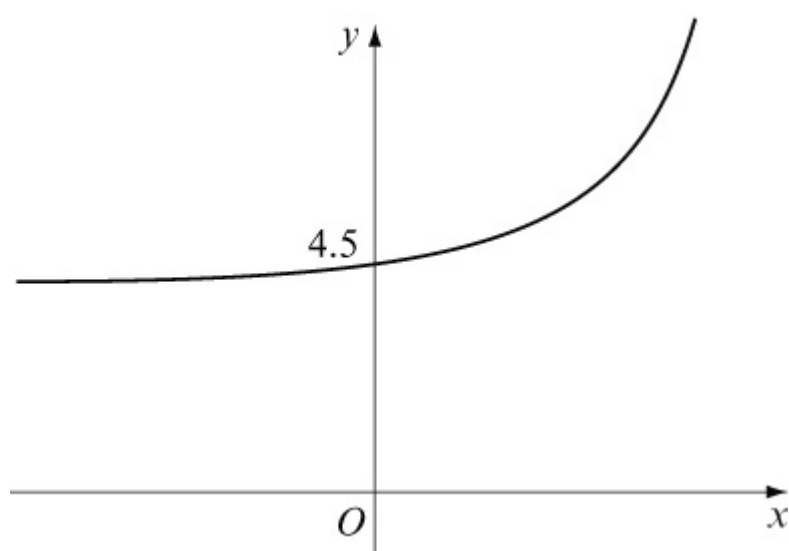


$y = \frac{1}{2}e^x$. Vertical stretch, scale factor $\frac{1}{2}$.

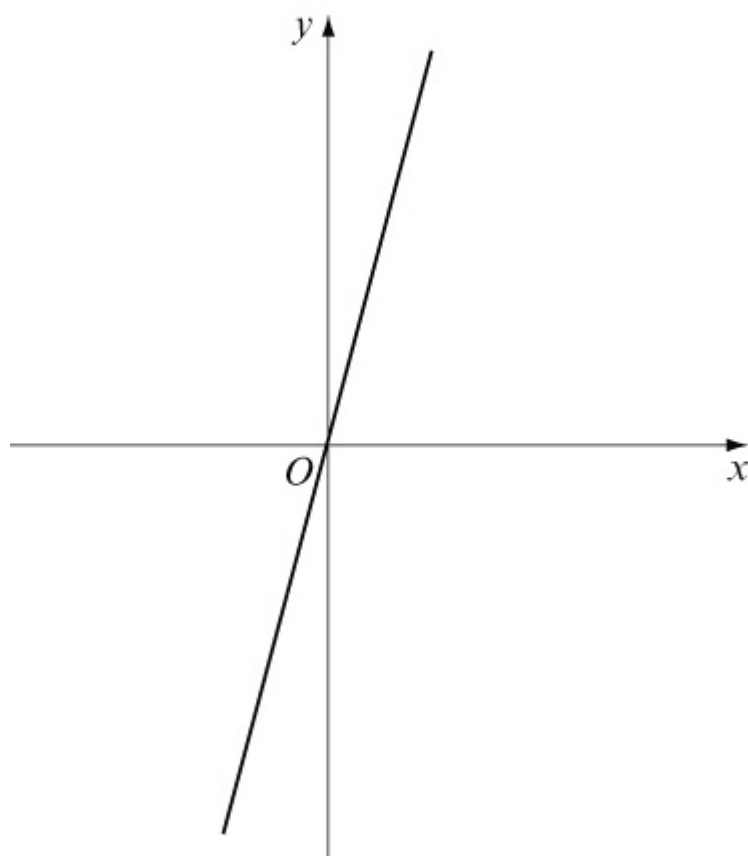


$y = \frac{1}{2}e^x + 4$. Vertical translation of $+4$.

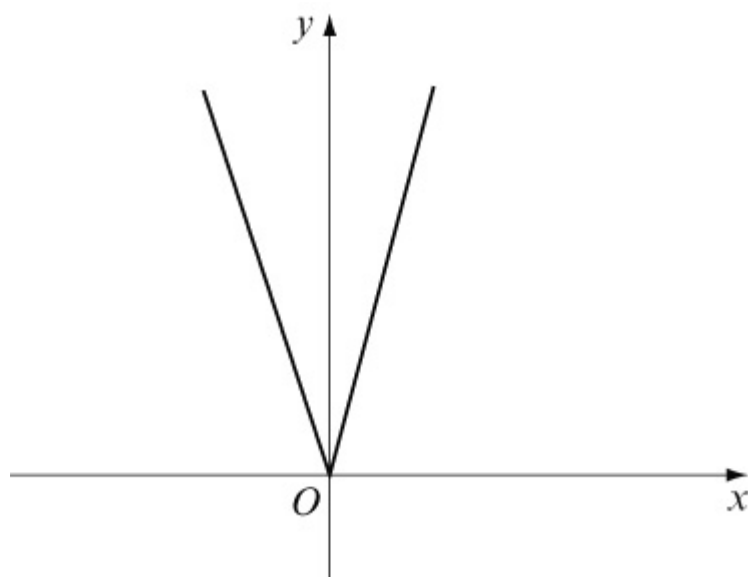
(When $x = 0$, $y = \frac{1}{2}e^0 + 4 = 4.5$).



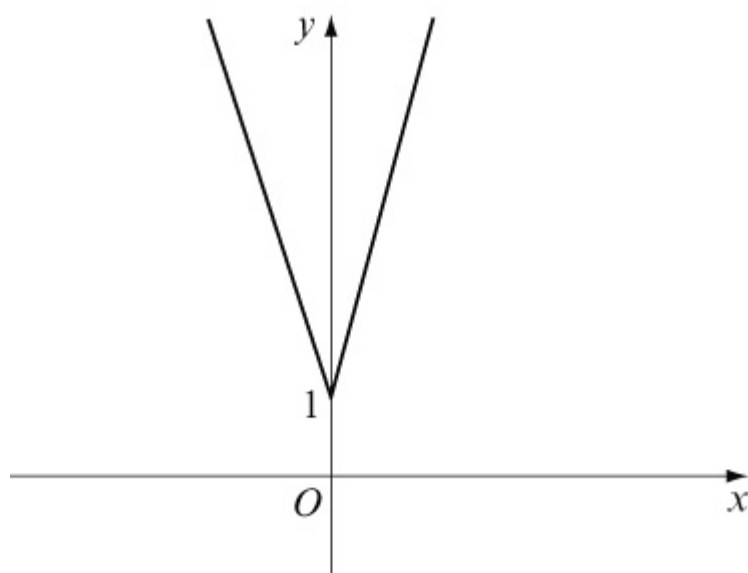
(g) $y = 4x$



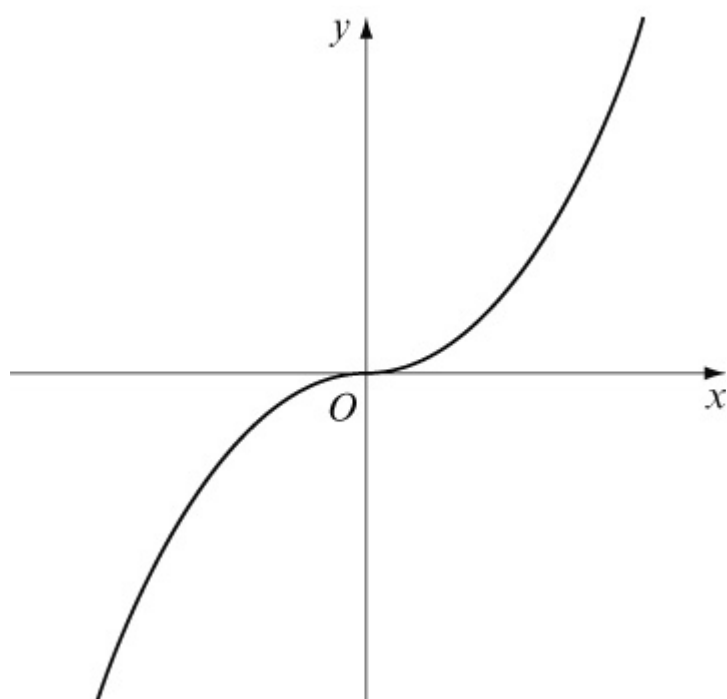
$y = |4x|$. For the part below the x -axis, reflect in the x -axis.



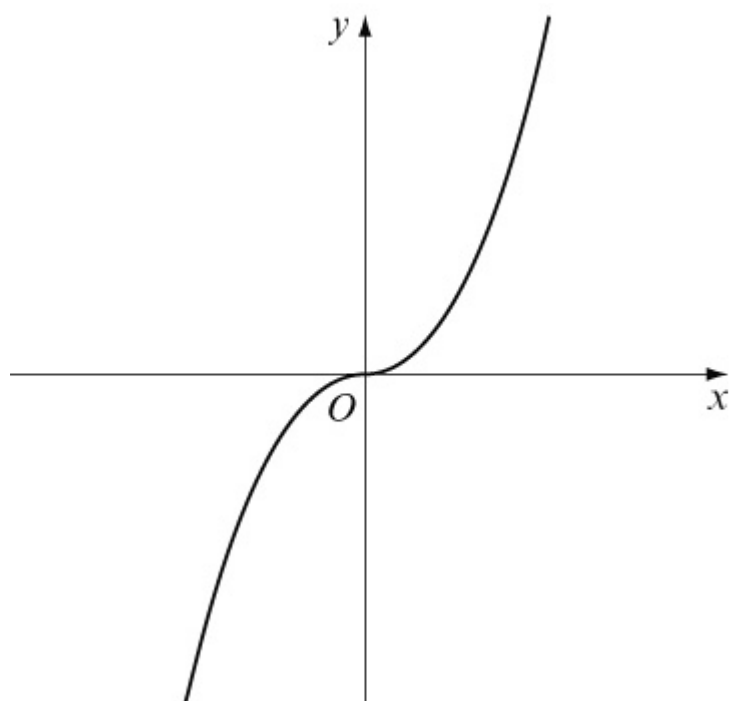
$y = |4x| + 1$. Vertical translation of $+1$.



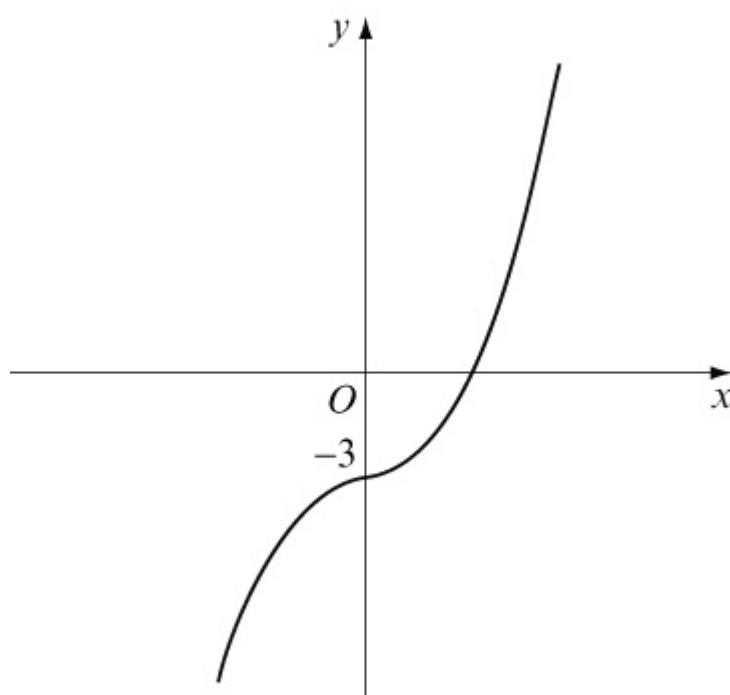
(h) $y = x^3$



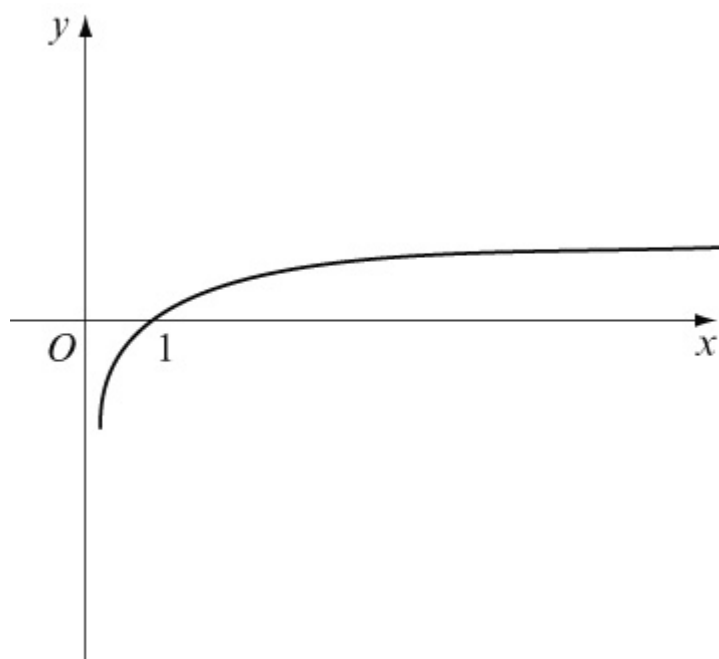
$y = 2x^3$. Vertical stretch, scale factor 2.



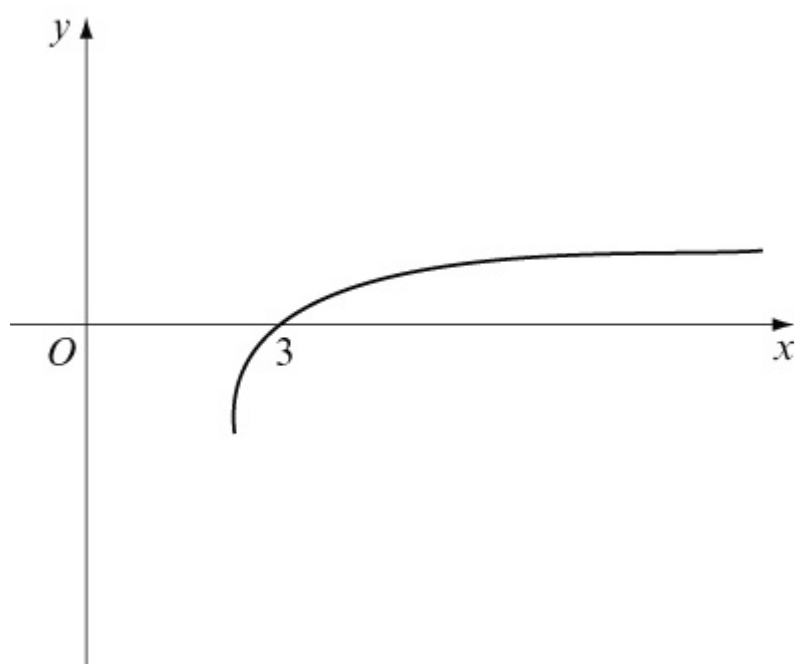
$y = 2x^3 - 3$. Vertical translation of -3 .



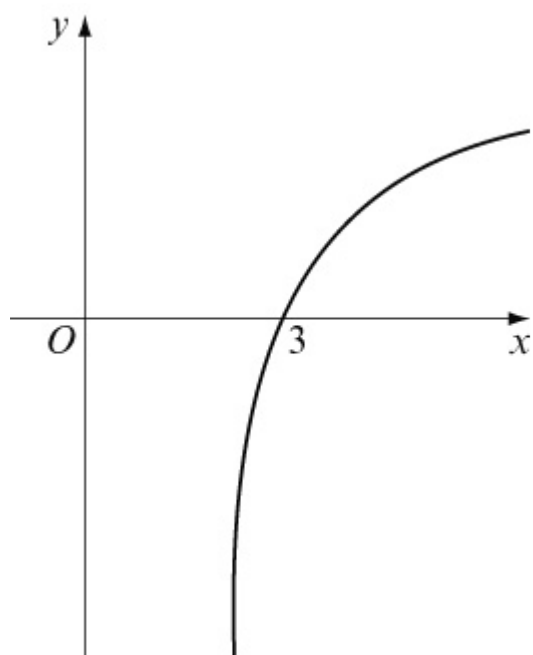
(i) $y = \ln x$



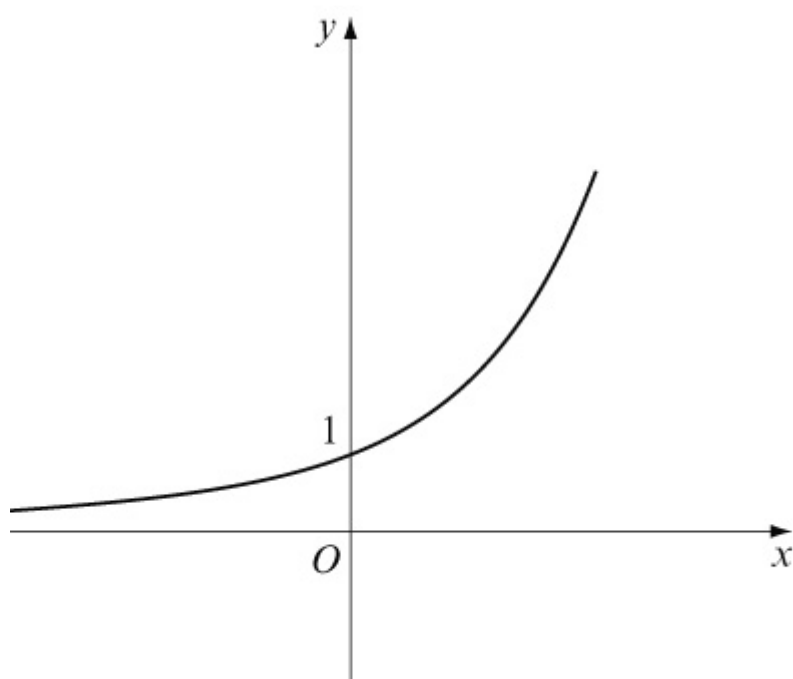
$y = \ln(x - 2)$. Horizontal translation of $+2$.



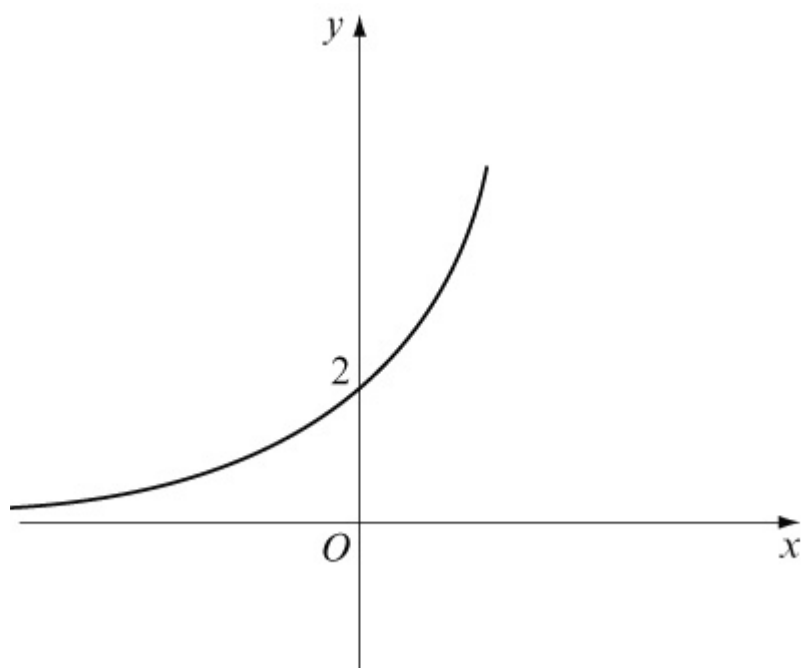
$y = 3\ln(x - 2)$. Vertical stretch, scale factor 3.



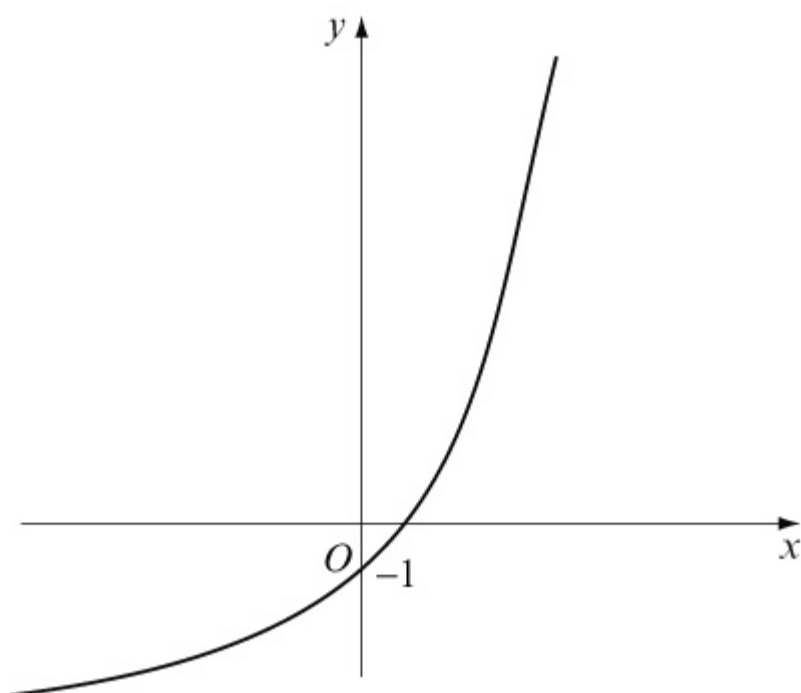
(i) $y = e^x$



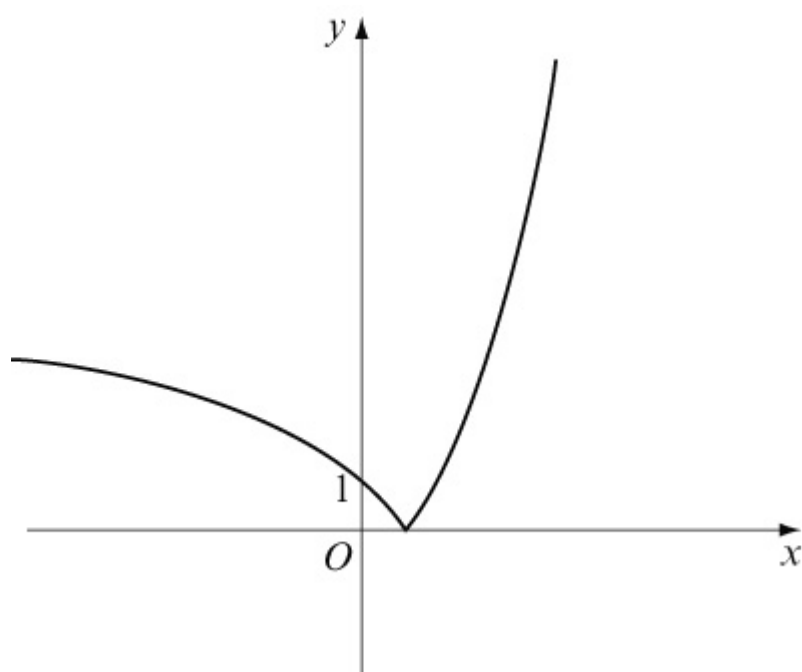
$y = 2e^x$. Vertical stretch, scale factor 2.



$y = 2e^x - 3$. Vertical translation of -3 .



$y = |2e^x - 3|$. For the part below the x -axis, reflect in the x -axis.



Solutionbank

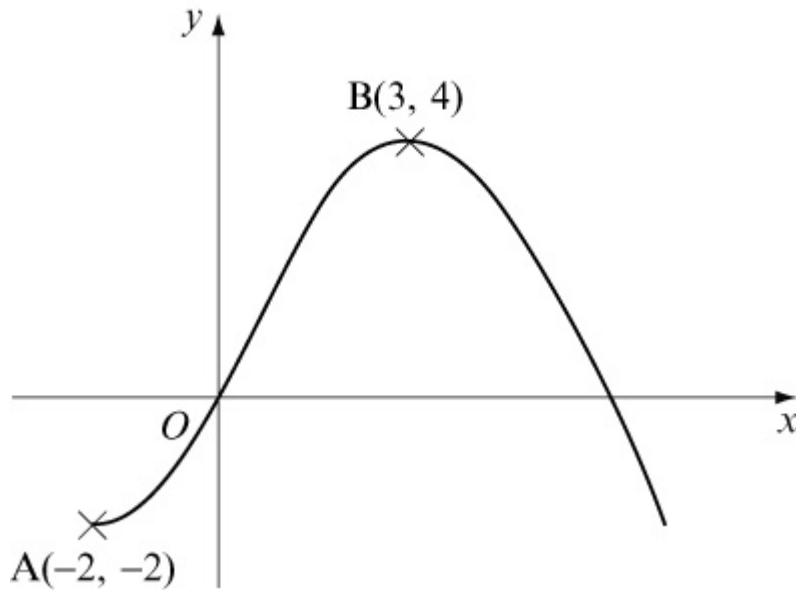
Edexcel AS and A Level Modular Mathematics

Exercise E, Question 1

Question:

The diagram shows a sketch of the graph of $y = f(x)$.

The curve passes through the origin O , the point $A(-2, -2)$ and the point $B(3, 4)$.



Sketch the graph of:

(a) $y = 3f(x) + 2$

(b) $y = f(x - 2) - 5$

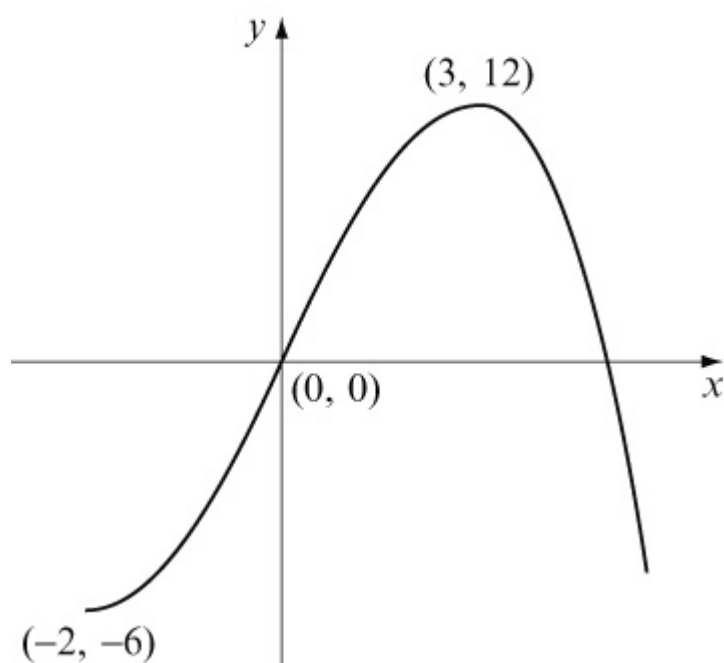
(c) $y = \frac{1}{2}f(x + 1)$

(d) $y = -f(2x)$

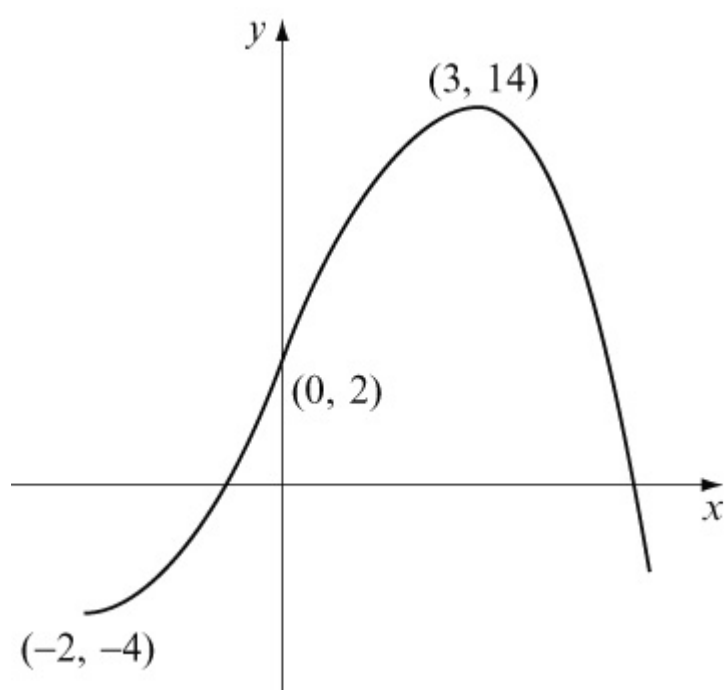
In each case, find the coordinates of the images of the points O , A and B .

Solution:

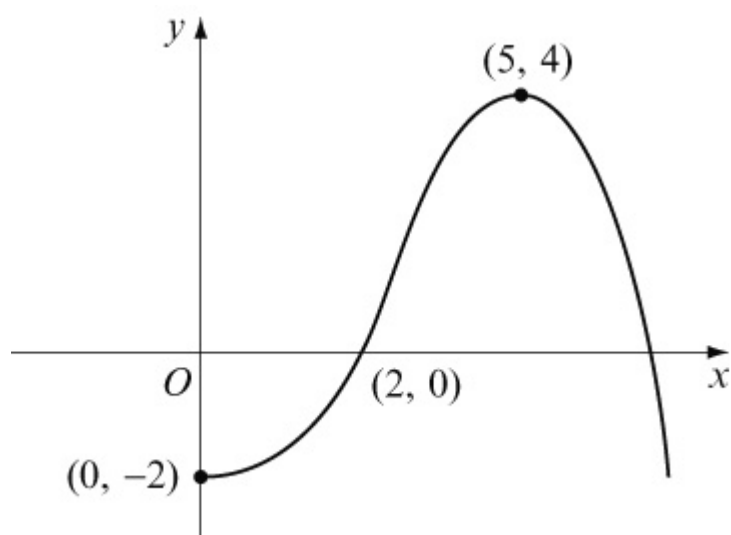
(a) $y = 3f(x)$. Vertical stretch, scale factor 3.



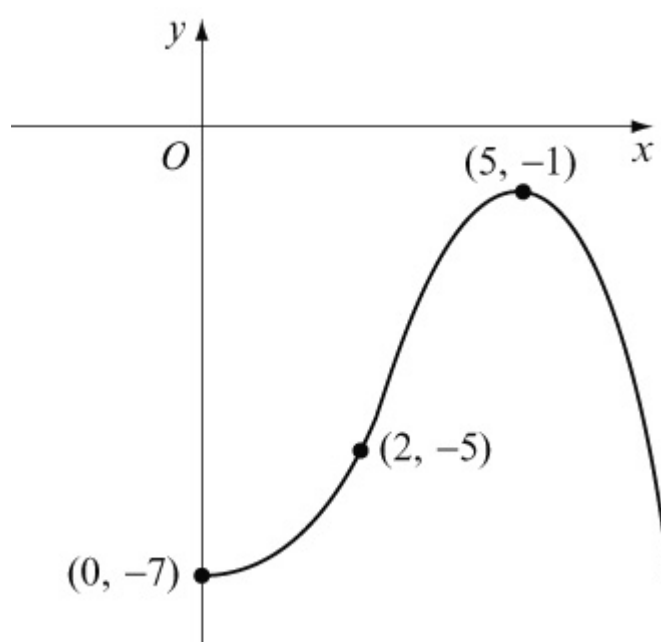
$y = 3f(x) + 2$. Vertical translation of +2.



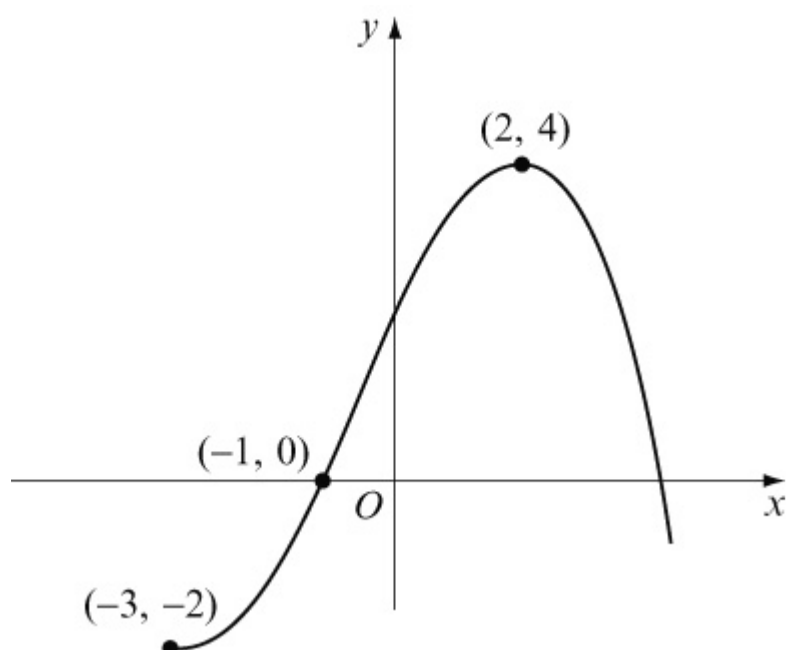
(b) $y = f(x - 2)$. Horizontal translation of +2.



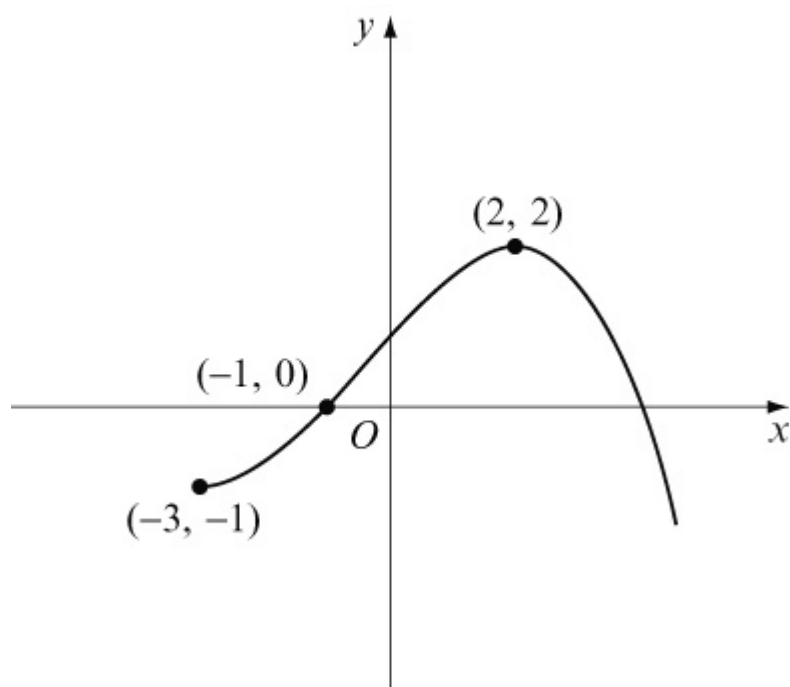
$y = f(x - 2) - 5$. Vertical translation of -5 .



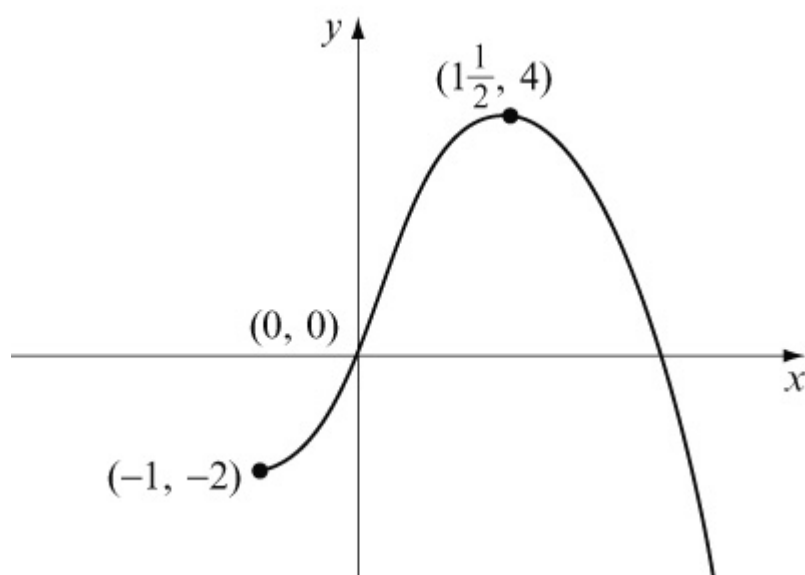
(c) $y = f(x + 1)$. Horizontal translation of -1 .



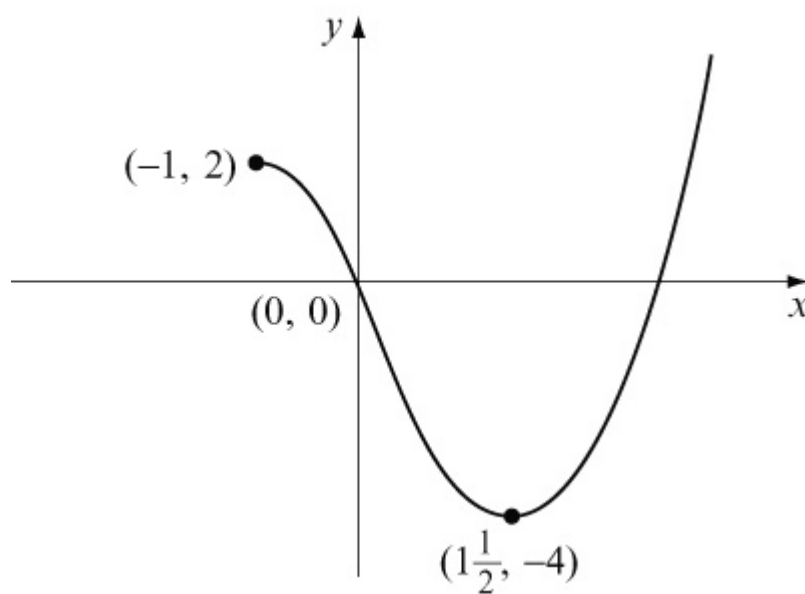
$y = \frac{1}{2}f(x + 1)$. Vertical stretch, scale factor $\frac{1}{2}$.



(d) $y = f(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



$y = -f(2x)$. Reflection in the x -axis.
(Vertical stretch, scale factor -1).



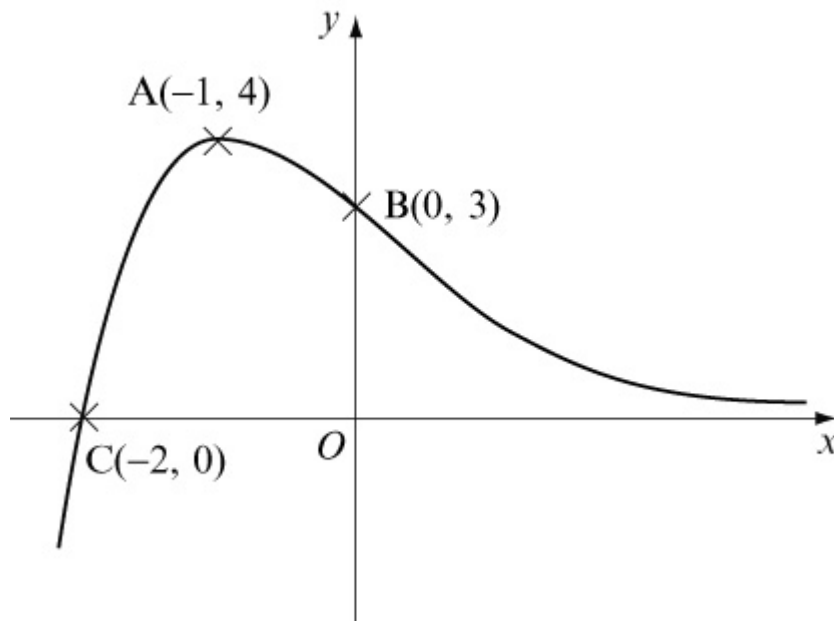
Solutionbank

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Exercise E, Question 2

Question:

The diagram shows a sketch of the graph of $y = f(x)$. The curve has a maximum at the point $A(-1, 4)$ and crosses the axes at the points $B(0, 3)$ and $C(-2, 0)$.



Sketch the graph of:

(a) $y = 3f(x - 2)$

(b) $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$

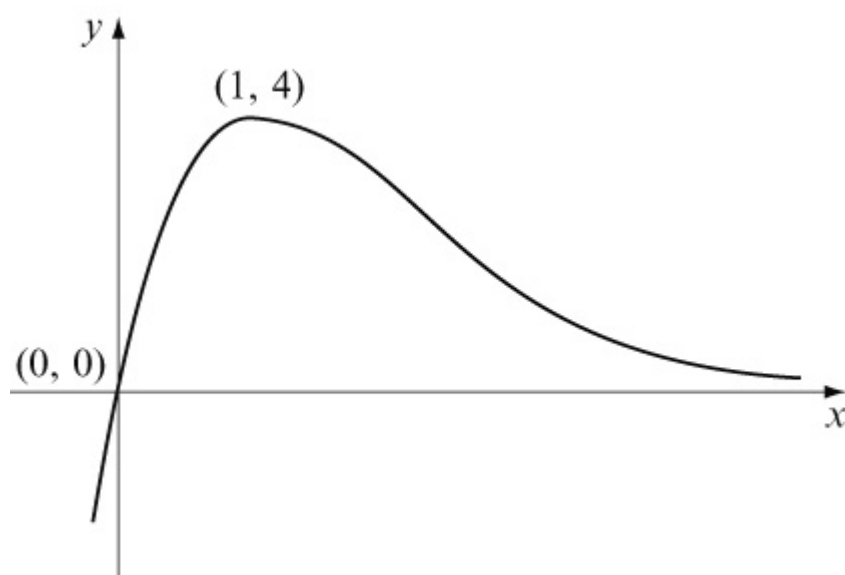
(c) $y = -f(x) + 4$

(d) $y = -2f(x + 1)$

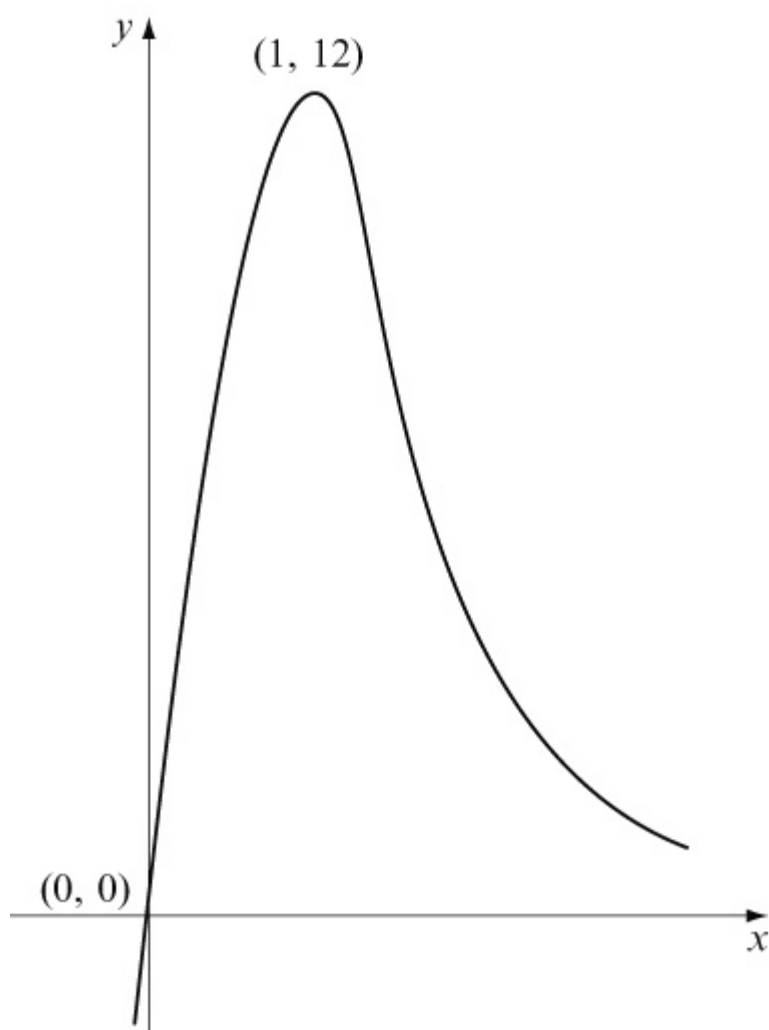
For each graph, find, where possible, the coordinates of the maximum or minimum and the coordinates of the intersection points with the axes.

Solution:

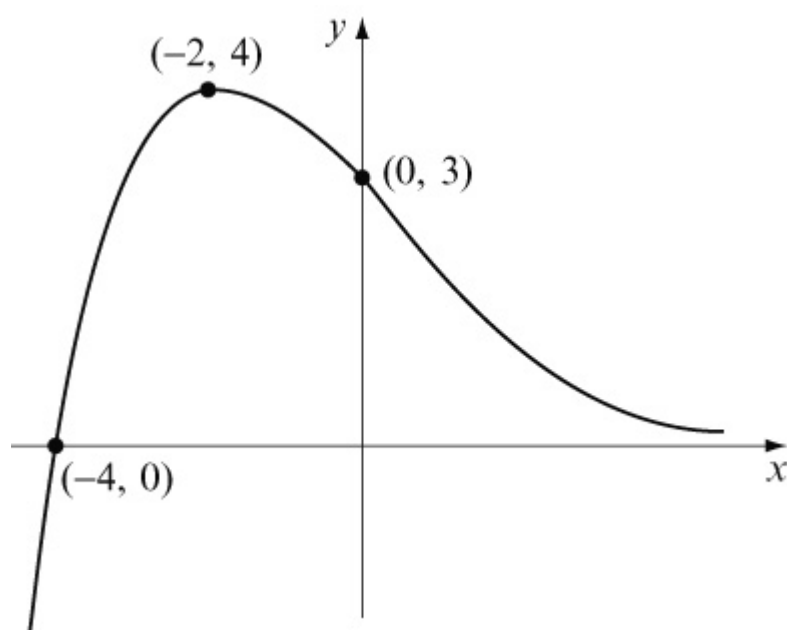
(a) $y = f(x - 2)$. Horizontal translation of $+2$.



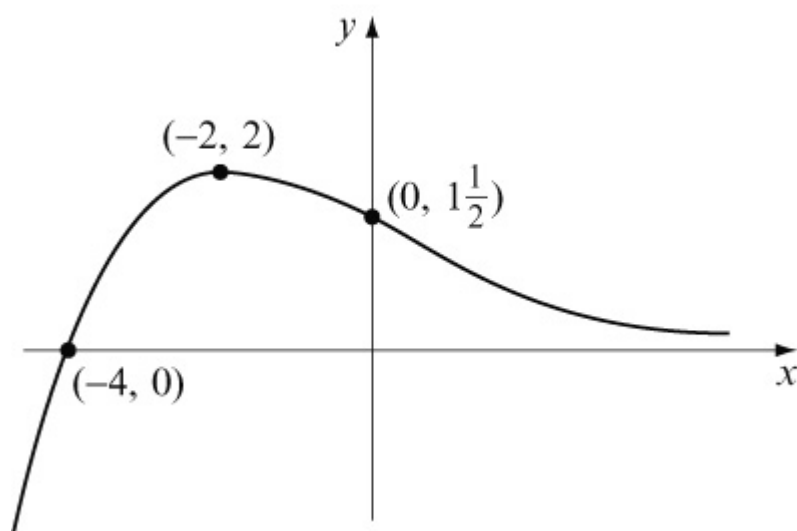
$y = 3f(x - 2)$. Vertical stretch, scale factor 3.



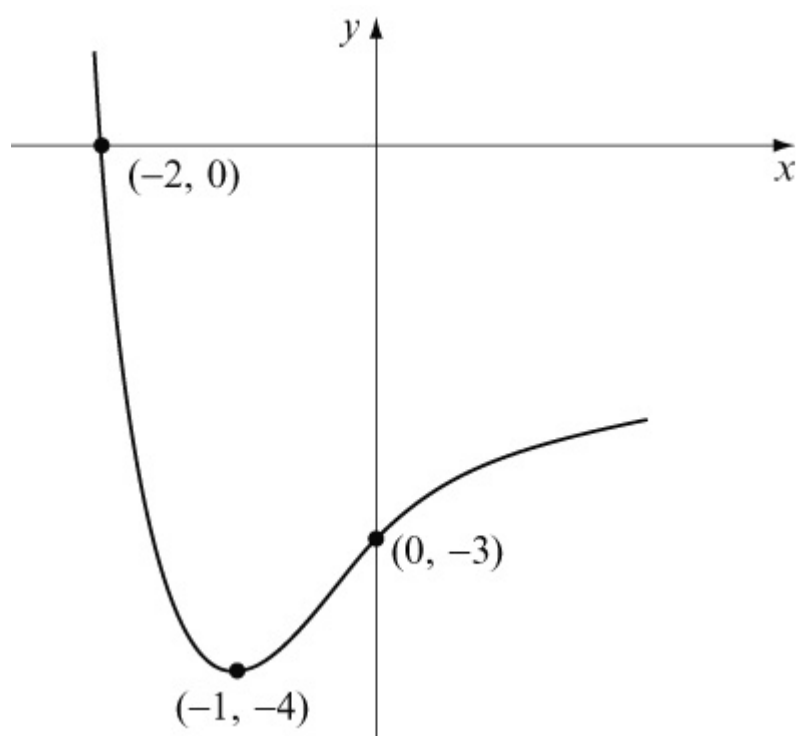
(b) $y = f\left(\frac{1}{2}x\right)$. Horizontal stretch, scale factor 2.



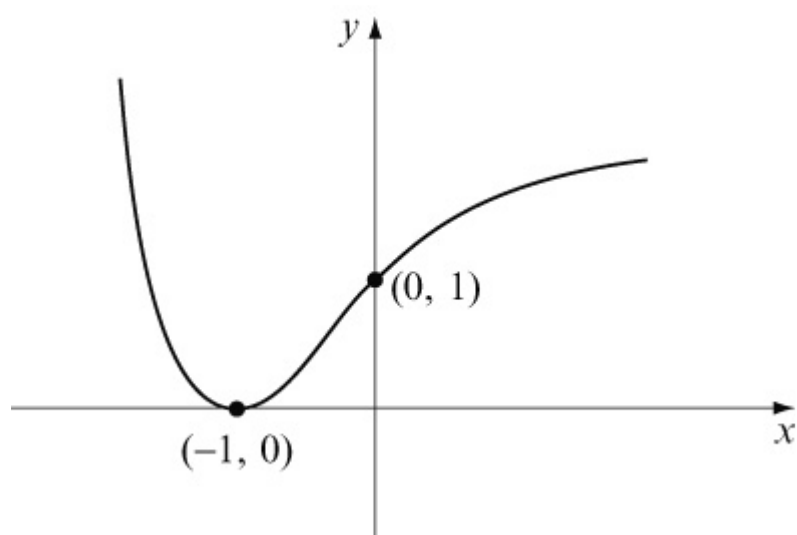
$y = \frac{1}{2}f\left(\frac{1}{2}x\right)$. Vertical stretch, scale factor $\frac{1}{2}$.



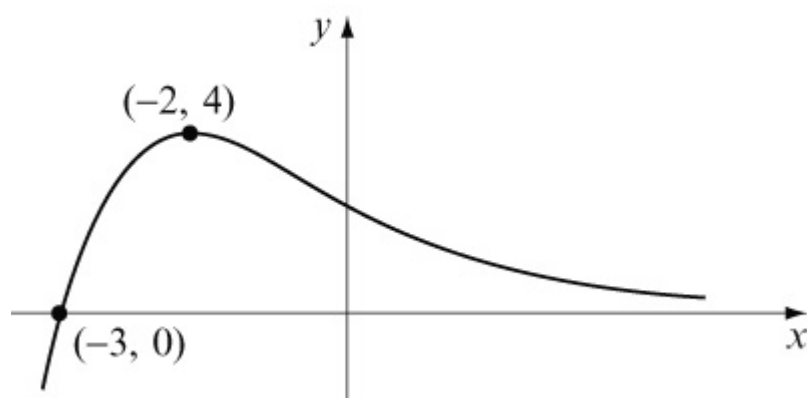
(c) $y = -f(x)$. Reflection in the x -axis. (Vertical stretch, scale factor -1).



$y = -f(x) + 4$. Vertical translation of $+4$.

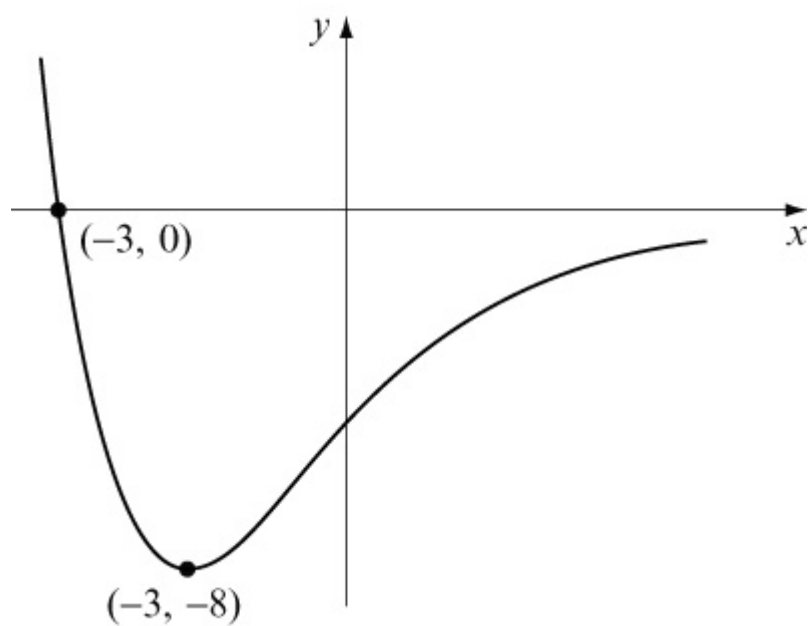


(d) $y = f(x + 1)$. Horizontal translation of -1 .



$$y = -2f(x + 1)$$

Reflection in the x -axis, and vertical stretch, scale factor 2.



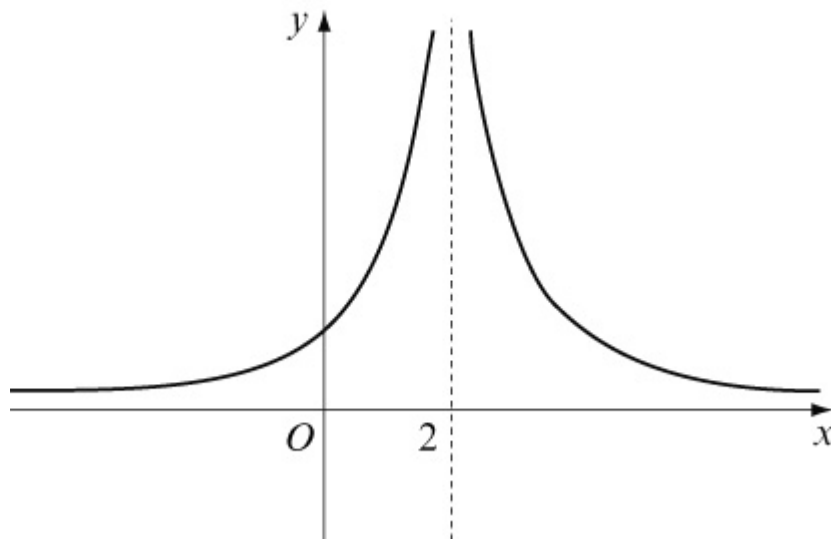
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Exercise E, Question 3

Question:

The diagram shows a sketch of the graph of $y = f(x)$. The lines $x = 2$ and $y = 0$ (the x -axis) are asymptotes to the curve.



Sketch the graph of:

(a) $y = 3f(x) - 1$

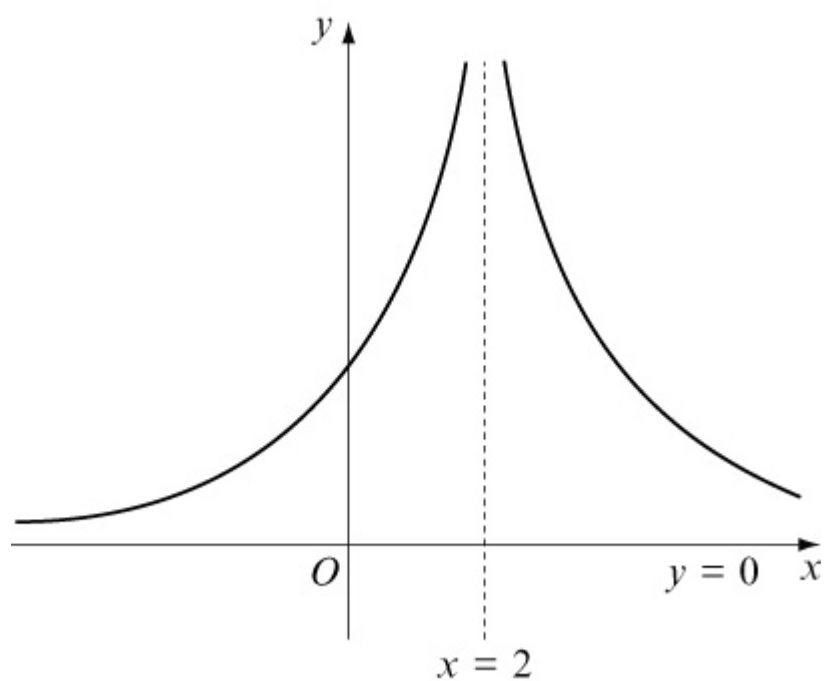
(b) $y = f(x + 2) + 4$

(c) $y = -f(2x)$

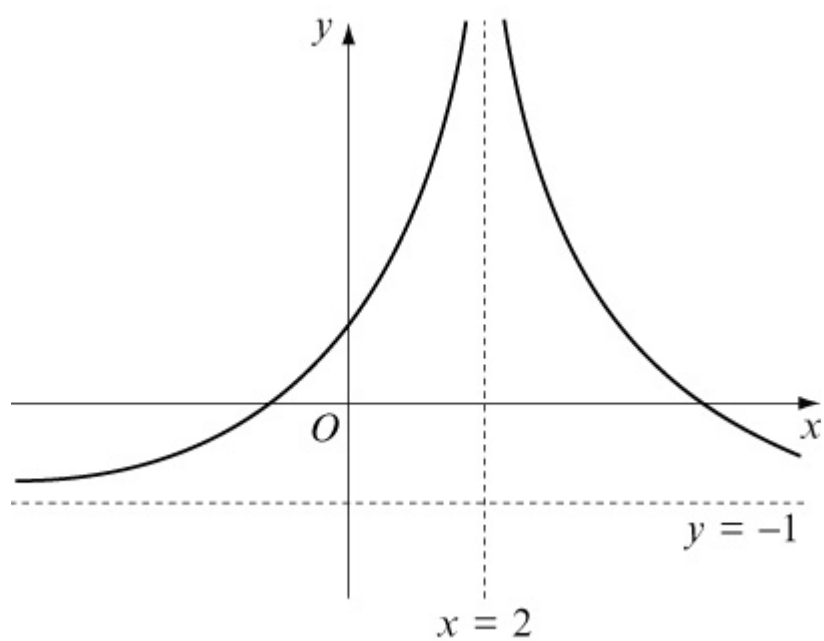
For each part, state the equations of the asymptotes.

Solution:

(a) $y = 3f(x)$. Vertical stretch, scale factor 3.

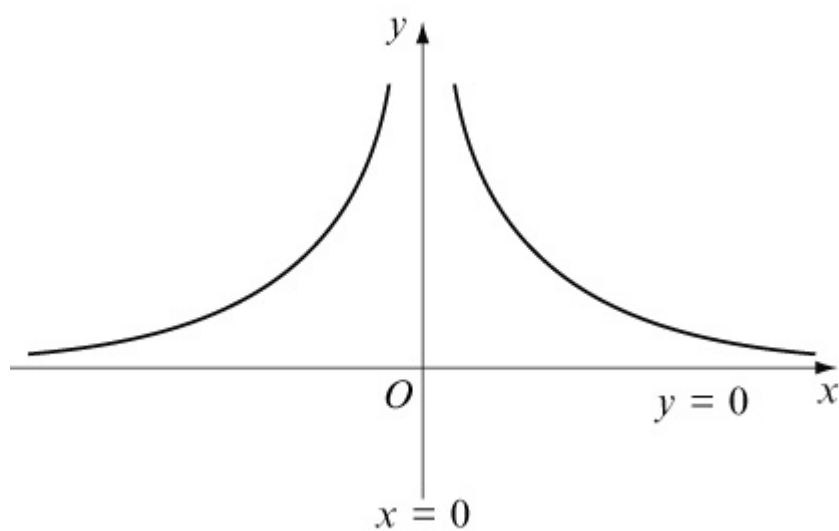


$y = 3f(x) - 1$. Vertical translation of -1 .

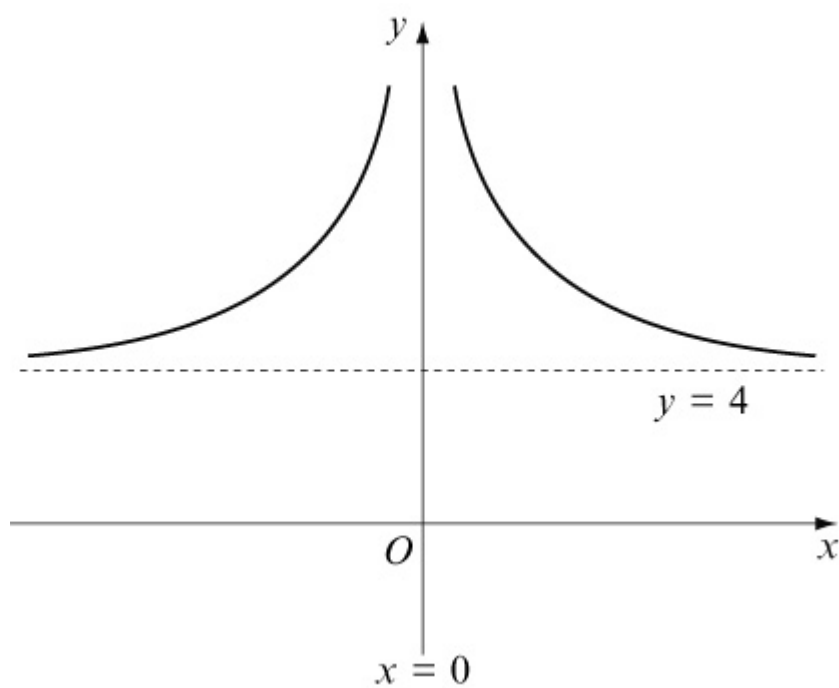


Asymptotes: $x = 2$, $y = -1$

(b) $y = f(x + 2)$. Horizontal translation of -2 .

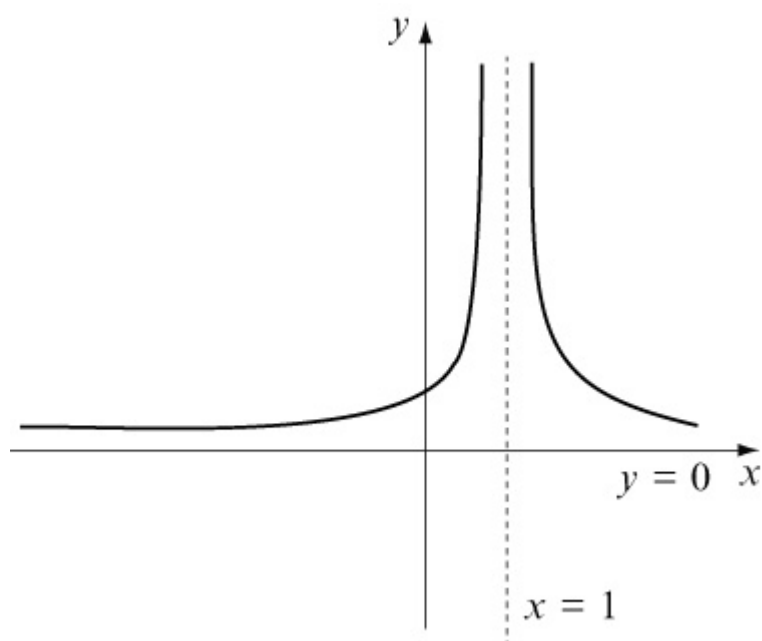


$y = f(x + 2) + 4$. Vertical translation of +4.

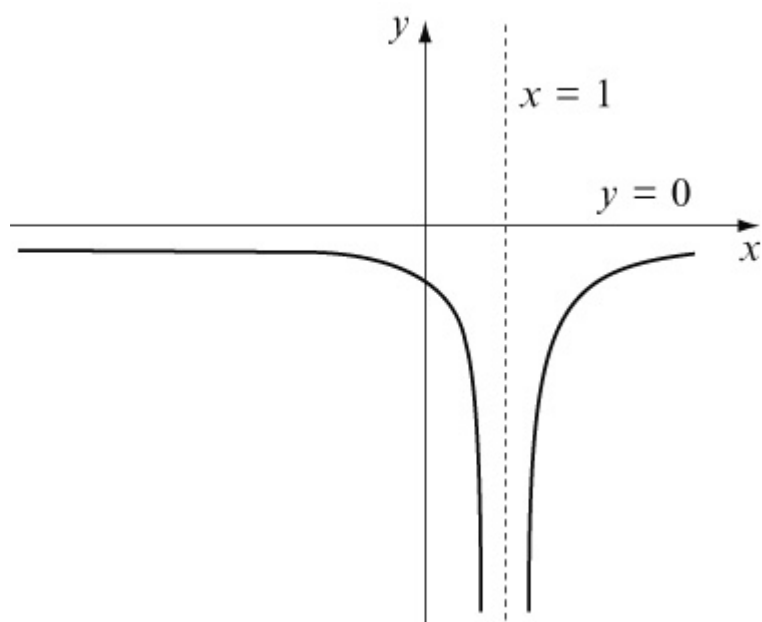


Asymptotes: $x = 0$, $y = 4$

(c) $y = f(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



$y = -f(2x)$. Reflection in the x -axis.



Asymptotes: $x = 1$, $y = 0$

Solutionbank

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Exercise F, Question 1

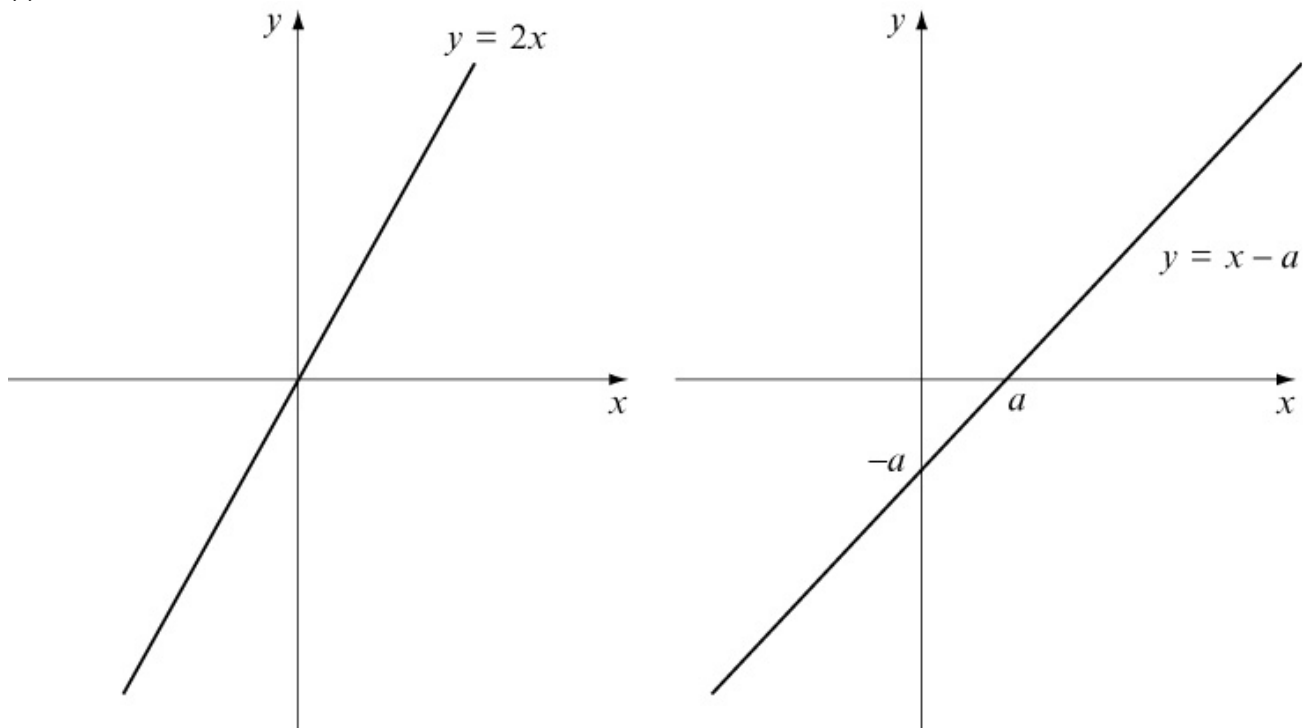
Question:

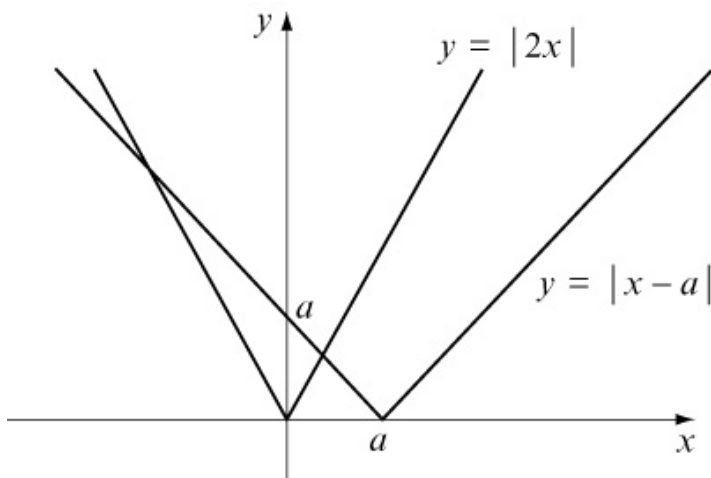
- (a) Using the same scales and the same axes, sketch the graphs of $y = |2x|$ and $y = |x - a|$, where $a > 0$.
- (b) Write down the coordinates of the points where the graph of $y = |x - a|$ meets the axes.
- (c) Show that the point with coordinates $(-a, 2a)$ lies on both graphs.
- (d) Find the coordinates, in terms of a , of a second point which lies on both graphs.

[E]

Solution:

(a)





(b) For $y = |x - a|$:

When $x = 0$, $y = |-a| = a$ $(0, a)$

When $y = 0$, $x - a = 0 \Rightarrow x = a$ $(a, 0)$

(c) For $y = |2x|$:

When $x = -a$, $y = |-2a| = 2a$

So $(-a, 2a)$ lies on $y = |2x|$.

For $y = |x - a|$:

When $x = -a$, $y = |-a - a| = |-2a| = 2a$

So $(-a, 2a)$ lies on $y = |x - a|$.

(d) The other intersection point is on the reflected part of $y = x - a$.

$$2x = -(x - a)$$

$$2x = -x + a$$

$$3x = a$$

$$x = \frac{a}{3}$$

$$\text{When } x = \frac{a}{3}, y = \left| \frac{2a}{3} \right| = \frac{2a}{3}$$

$\left(\frac{a}{3}, \frac{2a}{3} \right)$ lies on both graphs.

Solutionbank

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Exercise F, Question 2

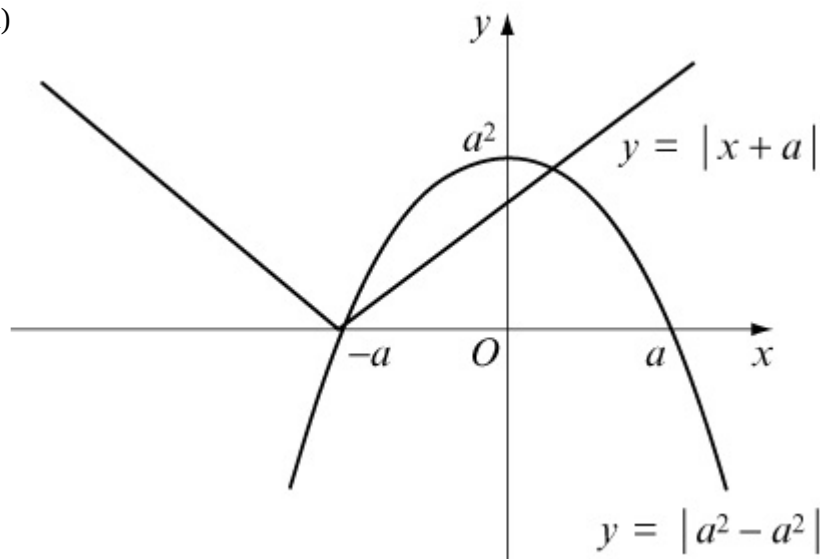
Question:

- (a) Sketch, on a single diagram, the graphs of $y = a^2 - x^2$ and $y = |x + a|$, where a is a constant and $a > 1$.
- (b) Write down the coordinates of the points where the graph of $y = a^2 - x^2$ cuts the coordinate axes.
- (c) Given that the two graphs intersect at $x = 4$, calculate the value of a .

[E]

Solution:

(a)



(b) For $y = a^2 - x^2$:

When $x = 0$, $y = a^2$ $(0, a^2)$

When $y = 0$, $a^2 - x^2 = 0$

$$\Rightarrow x^2 = a^2$$

$$\Rightarrow x = \pm a \quad (-a, 0) \text{ and } (a, 0)$$

(c) The graphs intersect on the non-reflected part of $y = x + a$.

$$a^2 - x^2 = x + a$$

Given that $x = 4$:

$$a^2 - 4^2 = 4 + a$$

$$a^2 - a - 20 = 0$$

$$(a - 5)(a + 4) = 0$$

Since $a > 1$, $a = 5$

Solutionbank

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Exercise F, Question 3

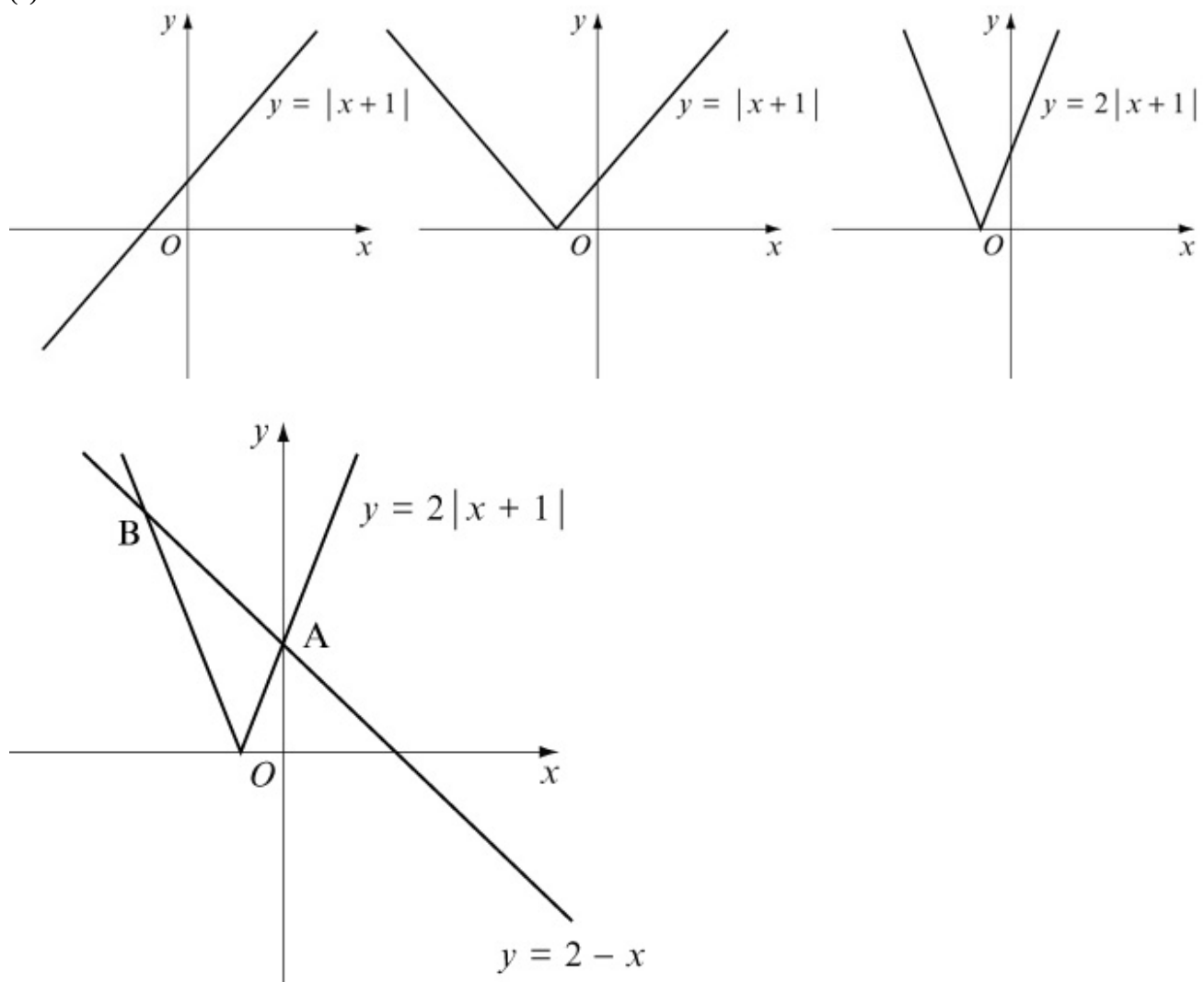
Question:

- (a) On the same axes, sketch the graphs of $y = 2 - x$ and $y = 2|x + 1|$.
- (b) Hence, or otherwise, find the values of x for which $2 - x = 2|x + 1|$.

[E]

Solution:

(a)



(b) Intersection point A:

$$2(x + 1) = 2 - x$$

$$2x + 2 = 2 - x$$

$$3x = 0$$

$$x = 0$$

Intersection point B is on the reflected part of the modulus graph.

$$\begin{aligned} -2(x + 1) &= 2 - x \\ -2x - 2 &= 2 - x \\ -x &= 4 \\ x &= -4 \end{aligned}$$

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Exercise F, Question 4

Question:

Functions f and g are defined by

$$f : x \rightarrow 4 - x \quad \{ x \in \mathbb{R} \}$$

$$g : x \rightarrow 3x^2 \quad \{ x \in \mathbb{R} \}$$

- (a) Find the range of g .
- (b) Solve $gf(x) = 48$.
- (c) Sketch the graph of $y = |f(x)|$ and hence find the values of x for which $|f(x)| = 2$.

[E]

Solution:

(a) $g(x) = 3x^2$

Since $x^2 \geq 0$ for all $x \in \mathbb{R}$, the range of $g(x)$ is $g(x) \geq 0$

(b) $gf(x) = 48$

$$gf(x) = g(4 - x) = 3(4 - x)^2$$

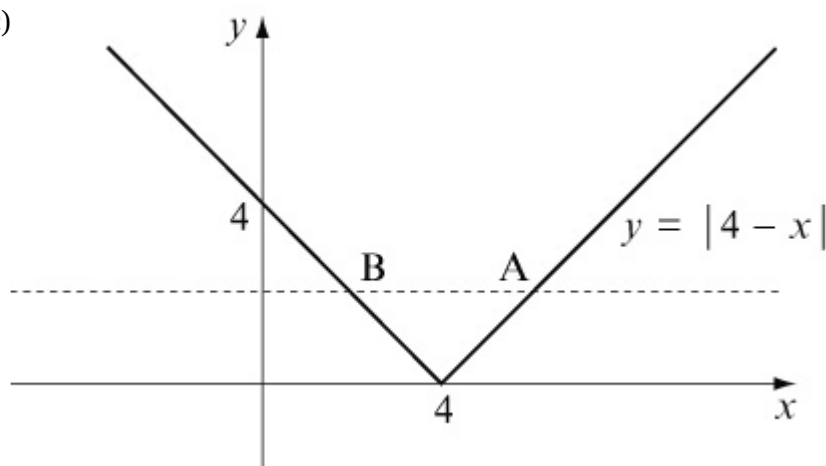
$$3(4 - x)^2 = 48$$

$$(4 - x)^2 = 16$$

Either $4 - x = 4$ or $4 - x = -4$

So $x = 0$ or $x = 8$

(c)



When $|f(x)| = 2$, $x = 2$ or $x = 6$ (from symmetry of graph).

[Or:

$$\text{At A: } -(4 - x) = 2 \Rightarrow -4 + x = 2 \Rightarrow x = 6$$

$$\text{At B: } 4 - x = 2 \Rightarrow x = 2]$$

Solutionbank

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Exercise F, Question 5

Question:

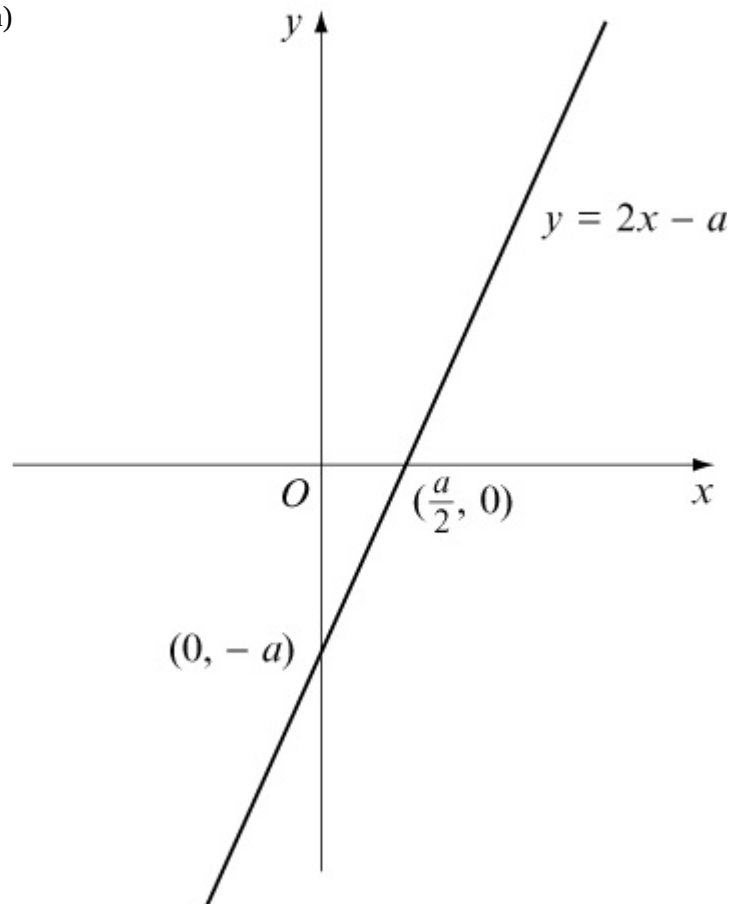
The function f is defined by $f : x \rightarrow |2x - a| \quad \{x \in \mathbb{R}\}$, where a is a positive constant.

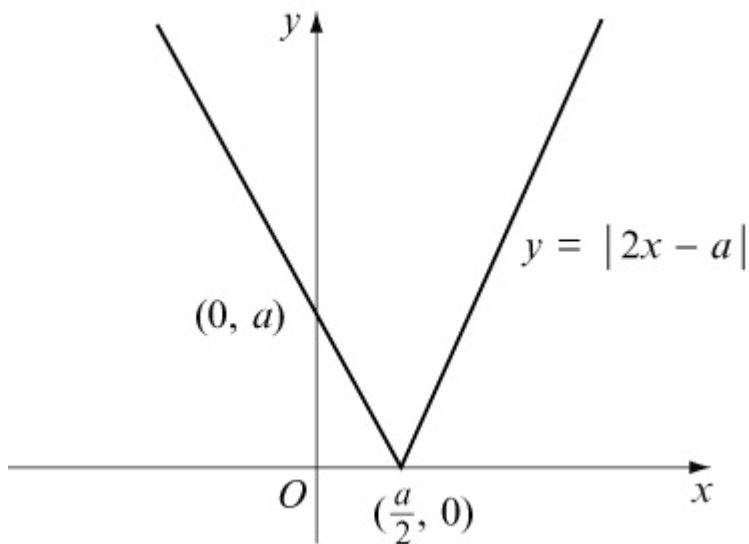
- (a) Sketch the graph of $y = f(x)$, showing the coordinates of the points where the graph cuts the axes.
- (b) On a separate diagram, sketch the graph of $y = f(2x)$, showing the coordinates of the points where the graph cuts the axes.
- (c) Given that a solution of the equation $f(x) = \frac{1}{2}x$ is $x = 4$, find the two possible values of a .

[E]

Solution:

(a)



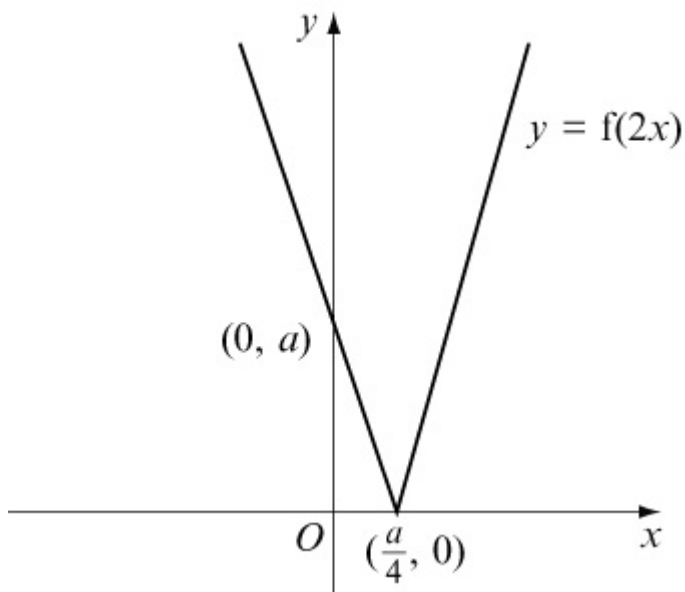


For $y = |2x - a|$:

When $x = 0$, $y = |-a| = a \quad (0, a)$

When $y = 0$, $2x - a = 0 \Rightarrow x = \frac{a}{2} \quad \left(\frac{a}{2}, 0 \right)$

(b) $y = f(2x)$. Horizontal stretch, scale factor $\frac{1}{2}$.



(c) $f(x) = \frac{1}{2}x$: $|2x - a| = \frac{1}{2}x$

Either $(2x - a) = \frac{1}{2}x$ or $-(2x - a) = \frac{1}{2}x$

$2x - a = \frac{1}{2}x \Rightarrow a = \frac{3}{2}x$

Given that $x = 4$, $a = 6$

$$- (2x - a) = \frac{1}{2}x \Rightarrow -2x + a = \frac{1}{2}x \Rightarrow a = \frac{5}{2}x$$

Given that $x = 4$, $a = 10$

Either $a = 6$ or $a = 10$

Solutionbank

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Exercise F, Question 6

Question:

(a) Sketch the graph of $y = |x - 2a|$, where a is a positive constant. Show the coordinates of the points where the graph meets the axes.

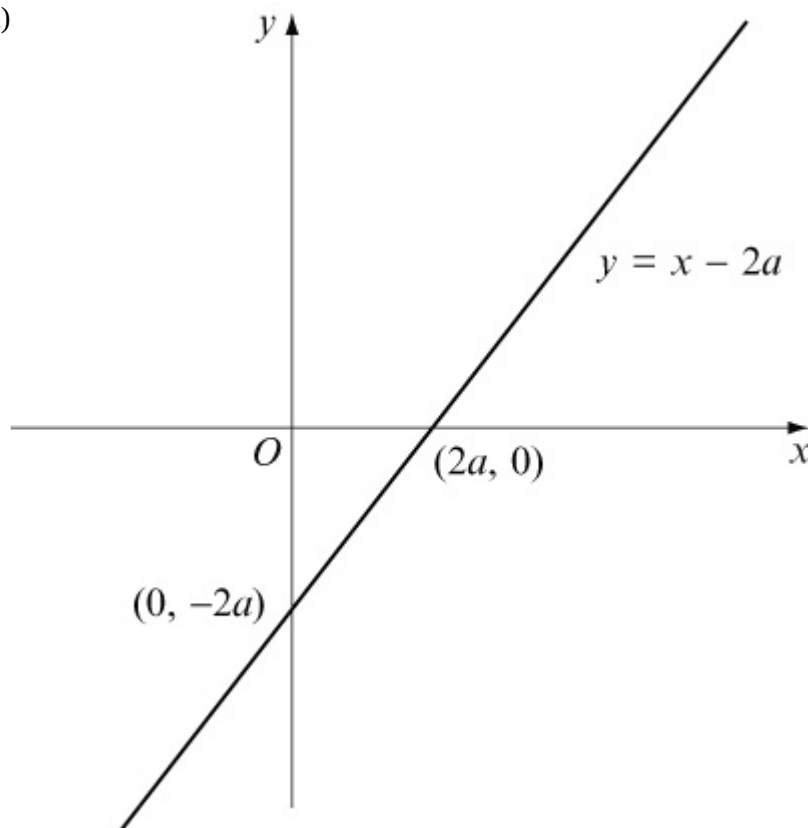
(b) Using algebra solve, for x in terms of a , $|x - 2a| = \frac{1}{3}x$.

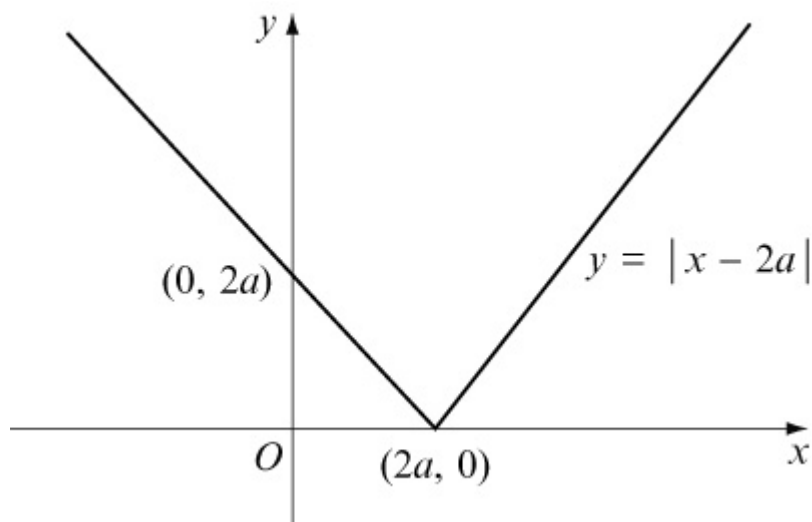
(c) On a separate diagram, sketch the graph of $y = a - |x - 2a|$, where a is a positive constant. Show the coordinates of the points where the graph cuts the axes.

[E]

Solution:

(a)





For $y = |x - 2a|$:

When $x = 0$, $y = |-2a| = 2a$ $(0, 2a)$

When $y = 0$, $x - 2a = 0 \Rightarrow x = 2a$ $(2a, 0)$

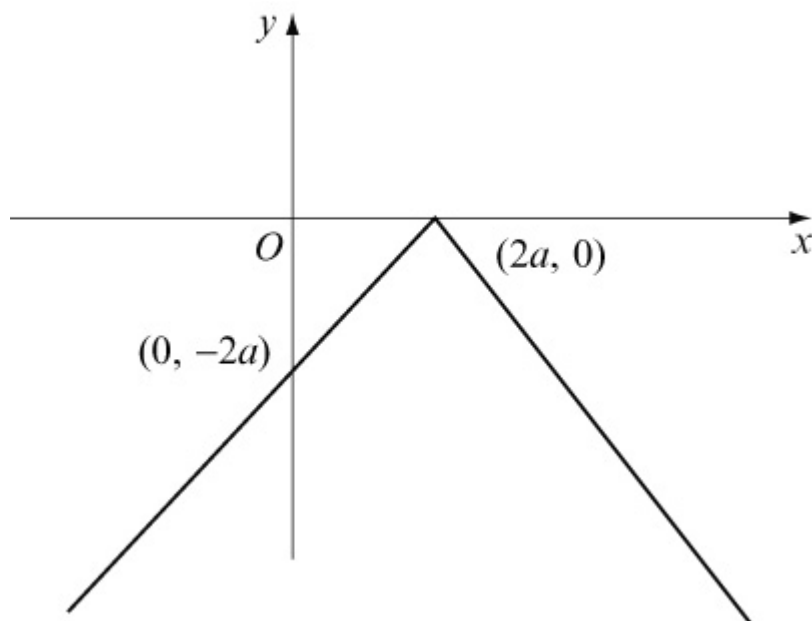
$$(b) \quad |x - 2a| = \frac{1}{3}x$$

Either $(x - 2a) = \frac{1}{3}x$ or $-(x - 2a) = \frac{1}{3}x$

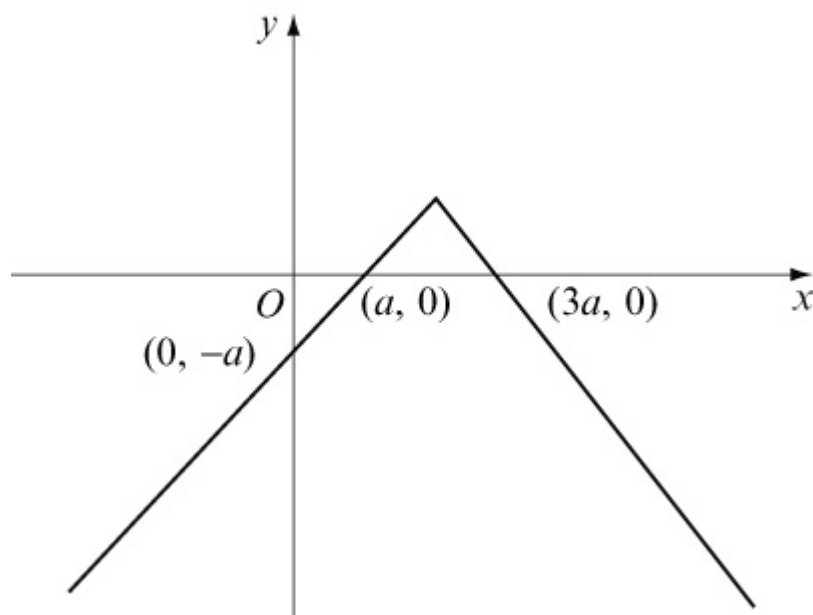
$$x - 2a = \frac{1}{3}x \Rightarrow x - \frac{1}{3}x = 2a \Rightarrow \frac{2}{3}x = 2a \Rightarrow x = 3a$$

$$-x + 2a = \frac{1}{3}x \Rightarrow \frac{4}{3}x = 2a \Rightarrow x = \frac{3}{2}a$$

(c) $y = -|x - 2a|$. Reflection in x-axis of $y = |x - 2a|$.



$y = a - |x - 2a|$. Vertical translation of $+a$.



For $y = a - |x - 2a|$:

When $x = 0$, $y = a - |-2a| = a - 2a = -a$ $(0, -a)$

When $y = 0$, $a - |x - 2a| = 0$

$$|x - 2a| = a$$

Either $x - 2a = a \Rightarrow x = 3a$ $(3a, 0)$

or $-(x - 2a) = a \Rightarrow -x + 2a = a \Rightarrow x = a$ $(a, 0)$

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise F, Question 7

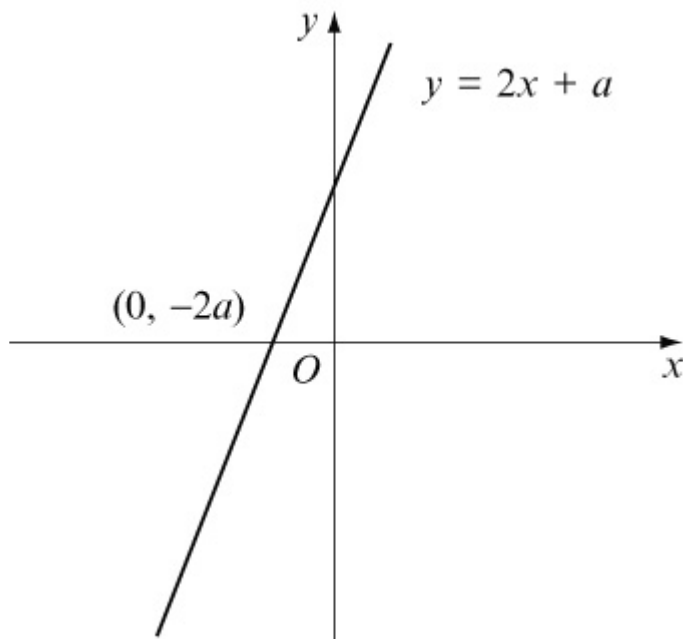
Question:

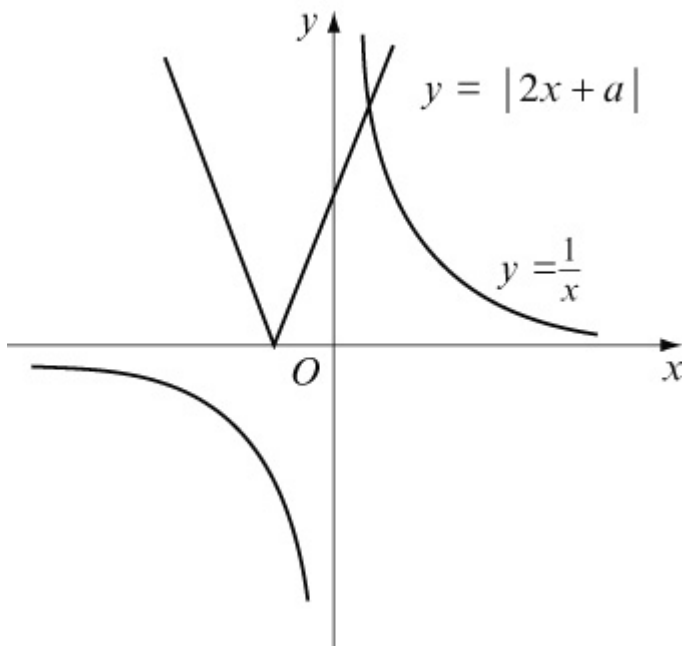
- (a) Sketch the graph of $y = |2x + a|$, $a > 0$, showing the coordinates of the points where the graph meets the coordinate axes.
- (b) On the same axes, sketch the graph of $y = \frac{1}{x}$.
- (c) Explain how your graphs show that there is only one solution of the equation $x |2x + a| - 1 = 0$.
- (d) Find, using algebra, the value of x for which $x |2x + a| - 1 = 0$.

[E]

Solution:

(a)(b)





For $y = |2x + a|$:

When $x = 0$, $y = |a| = a \quad (0, a)$

When $y = 0$, $2x + a = 0 \Rightarrow x = -\frac{a}{2} \quad \left(-\frac{a}{2}, 0\right)$

(c) Intersection of graphs is given by

$$|2x + a| = \frac{1}{x}$$

$$x|2x + a| = 1$$

$$x|2x + a| - 1 = 0$$

There is only one intersection point, so only one solution.

(d) The intersection point is on the non-reflected part of the modulus graph, so use $(2x + a)$ rather than $-(2x + a)$.

$$x(2x + a) - 1 = 0$$

$$2x^2 + ax - 1 = 0$$

$$x = \frac{-a \pm \sqrt{a^2 + 8}}{4}$$

As shown on the graph, x is positive, so

$$x = \frac{-a + \sqrt{a^2 + 8}}{4}$$

Solutionbank

Edexcel AS and A Level Modular Mathematics

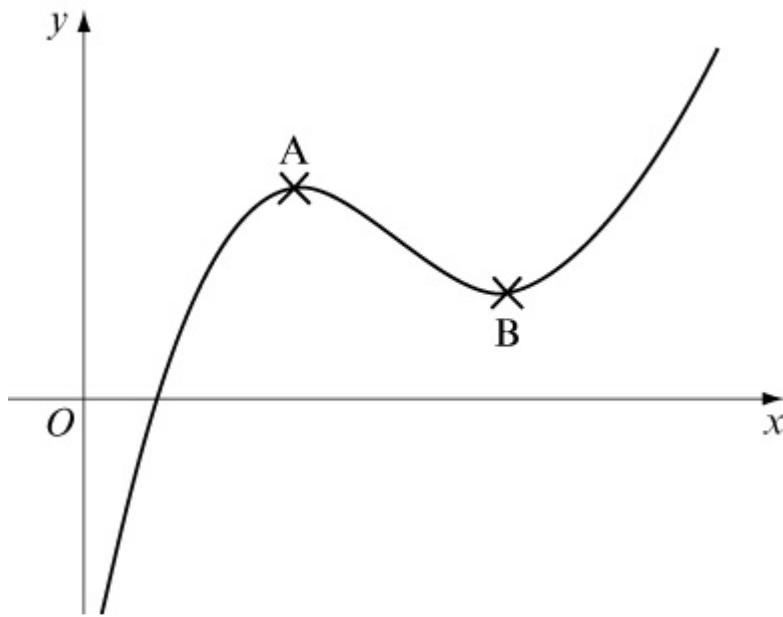
Exercise F, Question 8

Question:

The diagram shows part of the curve with equation $y = f(x)$, where

$$f(x) = x^2 - 7x + 5\ln x + 8 \quad x > 0$$

The points A and B are the stationary points of the curve.



(a) Using calculus and showing your working, find the coordinates of the points A and B.

(b) Sketch the curve with equation $y = -3f(x - 2)$.

(c) Find the coordinates of the stationary points of the curve with equation $y = -3f(x - 2)$. State, without proof, which point is a maximum and which point is a minimum.

[E]

Solution:

$$(a) f(x) = x^2 - 7x + 5\ln x + 8$$

$$f'(x) = 2x - 7 + \frac{5}{x}$$

At stationary points, $f'(x) = 0$:

$$2x - 7 + \frac{5}{x} = 0$$

$$2x^2 - 7x + 5 = 0$$

$$(2x - 5)(x - 1) = 0$$

$$x = \frac{5}{2}, x = 1$$

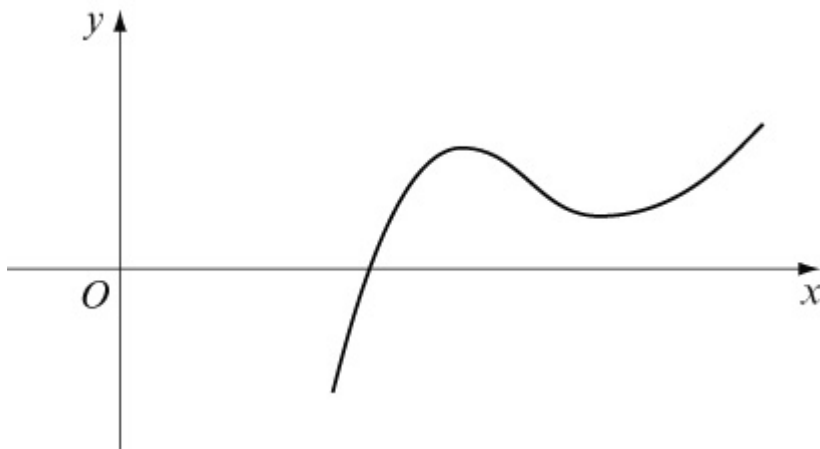
Point A: $x = 1, f(x) = 1 - 7 + 5 \ln 1 + 8 = 2$

A is (1, 2)

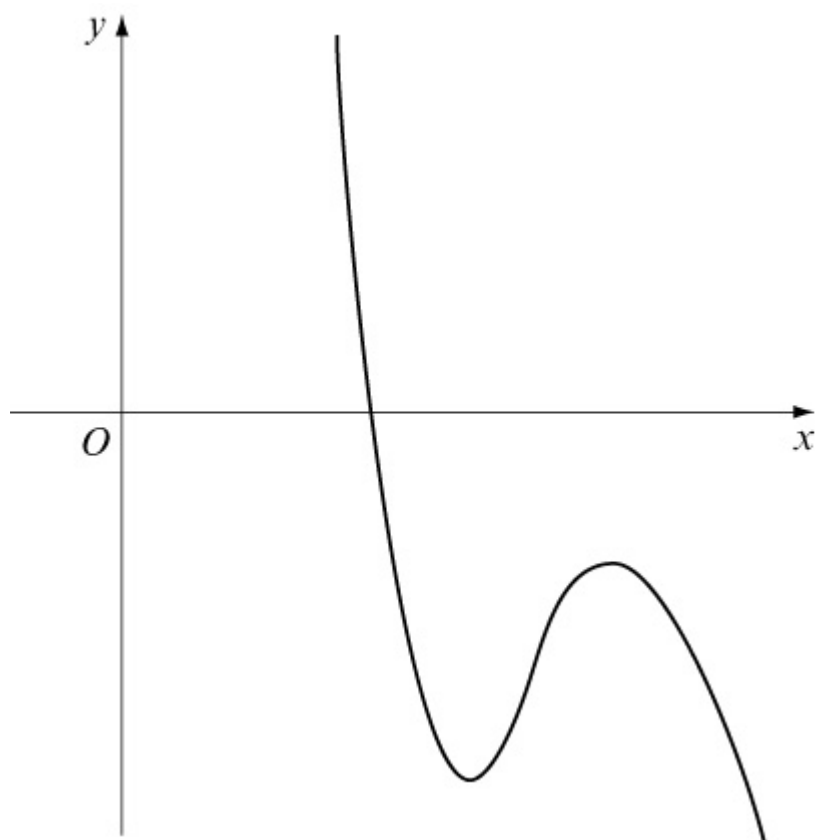
Point B: $x = \frac{5}{2}, f(x) = \frac{25}{4} - \frac{35}{2} + 5 \ln \frac{5}{2} + 8 = 5 \ln \frac{5}{2} - \frac{13}{4}$

B is $\left(\frac{5}{2}, 5 \ln \frac{5}{2} - \frac{13}{4} \right)$

(b) $y = f(x - 2)$. Horizontal translation of +2.



$y = -3f(x - 2)$. Reflection in the x-axis, and vertical stretch, scale factor 3.



(c) Using the transformations, point (X, Y) becomes $(X + 2, -3Y)$

$(1, 2) \rightarrow (3, -6)$ Minimum

$\left(\frac{5}{2}, 5 \ln \frac{5}{2} - \frac{13}{4}\right) \rightarrow \left(\frac{9}{2}, \frac{39}{4} - 15 \ln \frac{5}{2}\right)$ Maximum