Solutionbank
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Without using your calculator, write down the sign of the following trigonometric ratios:

(a) sec 300°

(b) cosec 190°

(c) cot 110°

(d) cot 200°

(e) sec 95°

Solution:

(a) 300° is in the 4th quadrant

$$\sec 300° = \frac{1}{\cos 300°}$$

In 4th quadrant cos is +ve, so sec 300° is +ve.

(b) 190° is in the 3rd quadrant

$$\text{cosec}\,190° = \frac{1}{\sin 190°}$$

In 3rd quadrant sin is −ve, so cosec 190° is −ve.

(c) 110° is in the 2nd quadrant

$$\cot 110° = \frac{1}{\tan 110°}$$

In the 2nd quadrant tan is −ve, so cot 110° is −ve.

(d) 200° is in the 3rd quadrant.
tan is +ve in the 3rd quadrant, so cot 200° is+ve.

(e) 95° is in the 2nd quadrant
cos is −ve in the 2nd quadrant, so sec 95° is −ve.

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Exercise A, Question 2

Question:

Use your calculator to find, to 3 significant figures, the values of

(a) $\sec 100°$

(b) $\operatorname{cosec} 260°$

(c) $\operatorname{cosec} 280°$

(d) $\cot 550°$

(e) $\cot \frac{4\pi}{3}$

(f) $\sec 2.4^{c}$

(g) $\operatorname{cosec} \frac{11\pi}{10}$

(h) $\sec 6^{c}$

Solution:

(a) $\sec 100° = \frac{1}{\cos 100°} = -5.76$

(b) $\operatorname{cosec} 260° = \frac{1}{\sin 260°} = -1.02$

(c) $\operatorname{cosec} 280° = \frac{1}{\sin 280°} = -1.02$

(d) $\cot 550° = \frac{1}{\tan 550°} = 5.67$

(e) $\cot \frac{4\pi}{3} = \frac{1}{\tan \frac{4\pi}{3}} = 0.577$

(f) $\sec 2.4^{c} = \dfrac{1}{\cos 2.4^{c}} = -1.36$

(g) $\operatorname{cosec} \dfrac{11\pi}{10} = \dfrac{1}{\sin \dfrac{11\pi}{10}} = -3.24$

(h) $\sec 6^{c} = \dfrac{1}{\cos 6^{c}} = 1.04$

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Exercise A, Question 3

Question:

Find the exact value (in surd form where appropriate) of the following:

(a) cosec 90°

(b) cot 135°

(c) sec 180°

(d) sec 240°

(e) cosec 300°

(f) $\cot(-45°)$

(g) sec 60°

(h) $\operatorname{cosec}(-210°)$

(i) sec 225°

(j) $\cot\frac{4\pi}{3}$

(k) $\sec\frac{11\pi}{6}$

(l) $\operatorname{cosec}\left(-\frac{3\pi}{4}\right)$

Solution:

(a) $\operatorname{cosec}90° = \frac{1}{\sin 90°} = \frac{1}{1} = 1$ (refer to graph of $y = \sin\theta$)

(b) $\cot 135° = \frac{1}{\tan 135°} = \frac{1}{-\tan 45°} = \frac{1}{-1} = -1$

(c) $\sec 180° = \frac{1}{\cos 180°} = \frac{1}{-1} = -1$ (refer to graph of $y = \cos\theta$)

(d) 240° is in 3rd quadrant

$$\sec 240° = \frac{1}{\cos 240°} = \frac{1}{-\cos 60°} = \frac{1}{-\frac{1}{2}} = -2$$

(e) $\operatorname{cosec} 300° = \frac{1}{\sin 300°} = \frac{1}{-\sin 60°} = -\frac{1}{\frac{1}{2}\sqrt{3}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

(f) $\cot(-45°) = \frac{1}{\tan(-45°)} = \frac{1}{-\tan 45°} = \frac{1}{-1} = -1$

(g) $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2$

(h) $-210°$ is in 2nd quadrant

$$\operatorname{cosec}(-210°) = \frac{1}{\sin(-210°)} = \frac{1}{\sin 30°} = \frac{1}{\frac{1}{2}} = 2$$

(i) 225° is in 3rd quadrant

$$\sec 225° = \frac{1}{\cos 225°} = \frac{1}{-\cos 45°}$$

$$= \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

(j) $\frac{4\pi}{3}$ is in 3rd quadrant

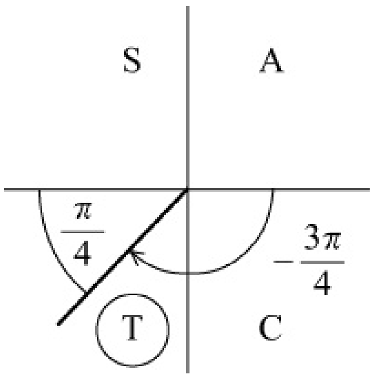
$$\cot\frac{4\pi}{3} = \frac{1}{\tan\frac{4\pi}{3}} = \frac{1}{+\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(k) $\frac{11\pi}{6} = 2\pi - \frac{\pi}{6}$ (in 4th quadrant)

$$\sec\frac{11\pi}{6} = \frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

(l)

$$\text{cosec} = \left(-\frac{3\pi}{4}\right) = \frac{1}{\sin\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\sin\frac{\pi}{4}} = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$



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Exercise A, Question 4

Question:

(a) Copy and complete the table, showing values (to 2 decimal places) of sec θ for selected values of θ.

θ	0°	30°	45°	60°	70°	80°	85°	95°
sec θ	1		1.41			5.76	11.47	

θ	100°	110°	120°	135°	150°	180°	210°
sec θ		−2.92		−1.41			−1.15

(b) Copy and complete the table, showing values (to 2 decimal places) of cosec θ for selected values of θ.

θ	10°	20°	30°	45°	60°	80°	90°	100°	120°	135°	150°	160°	170°
cosec θ				1.41			1		1.15	1.41			

θ	190°	200°	210°	225°	240°	270°	300°	315°	330°	340°	350°	390°
cosec θ					−1.15				−2			

(c) Copy and complete the table, showing values (to 2 decimal places) of cot θ for selected values of θ.

θ	−90°	−60°	−45°	−30°	−10°	10°	30°	45°	60°
cot θ	0	−0.58					1.73	1	0.58

θ	90°	120°	135°	150°	170°	210°	225°	240°	270°
cot θ			−1					0.58	

Solution:

(a) Change sec θ into $\frac{1}{\cos\theta}$ and use your calculator.

θ	0°	30°	45°	60°	70°	80°	85°	95°
sec θ	1	1.15	1.41	2	2.92	5.76	11.47	−11.47

θ	100°	110°	120°	135°	150°	180°	210°
sec θ	−5.76	−2.92	−2	−1.41	−1.15	−1	−1.15

(b) Change cosec θ to $\frac{1}{\sin\theta}$ and use your calculator.

θ	10°	20°	30°	45°	60°	80°	90°	100°	120°
cosec θ	5.76	2.92	2	1.41	1.15	1.02	1	1.02	1.15

θ	135°	150°	160°	170°	190°	200°	210°	225°	240°
cosec θ	1.41	2	2.92	5.76	−5.76	−2.92	−2	−1.41	−1.15

θ	270°	300°	315°	330°	340°	350°	390°
cosec θ	−1	−1.15	−1.41	−2	−2.92	−5.76	2

(c) Change cot θ to $\frac{1}{\tan\theta}$ and use your calculator.

θ	−90°	−60°	−45°	−30°	−10°	10°	30°	45°	60°
cot θ	0	−0.58	−1	−1.73	−5.67	5.67	1.73	1	0.58

θ	90°	120°	135°	150°	170°	210°	225°	240°	270°
cot θ	0	−0.58	−1	−1.73	−5.67	1.73	1	0.58	0

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Exercise B, Question 1

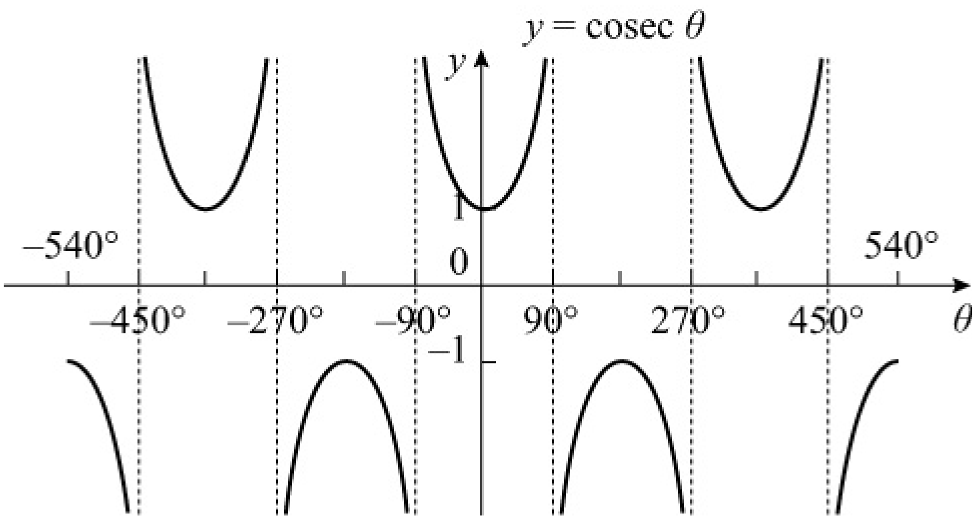
Question:

(a) Sketch, in the interval $-540° \leq \theta \leq 540°$, the graphs of:
(i) $\sec\theta$ (ii) $\operatorname{cosec}\theta$ (iii) $\cot\theta$

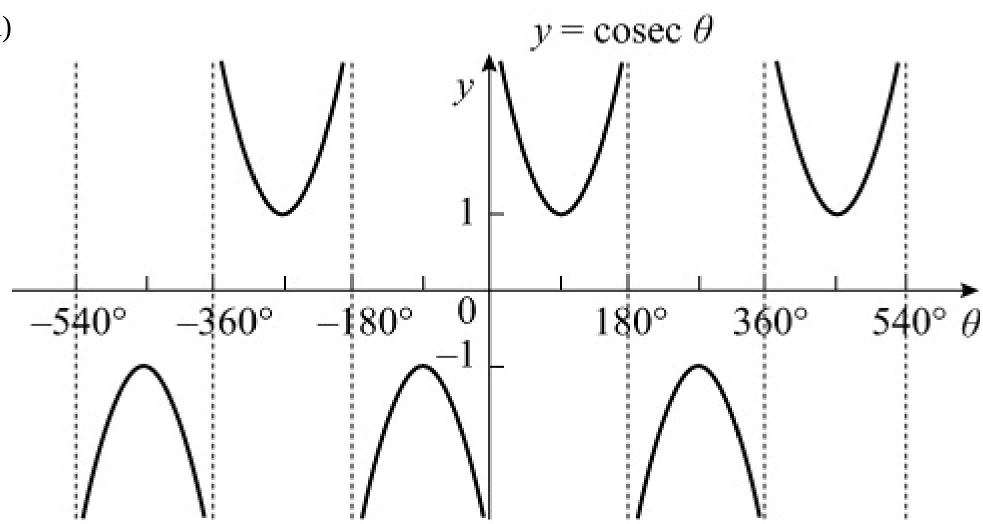
(b) Write down the range of
(i) $\sec\theta$ (ii) $\operatorname{cosec}\theta$ (iii) $\cot\theta$

Solution:

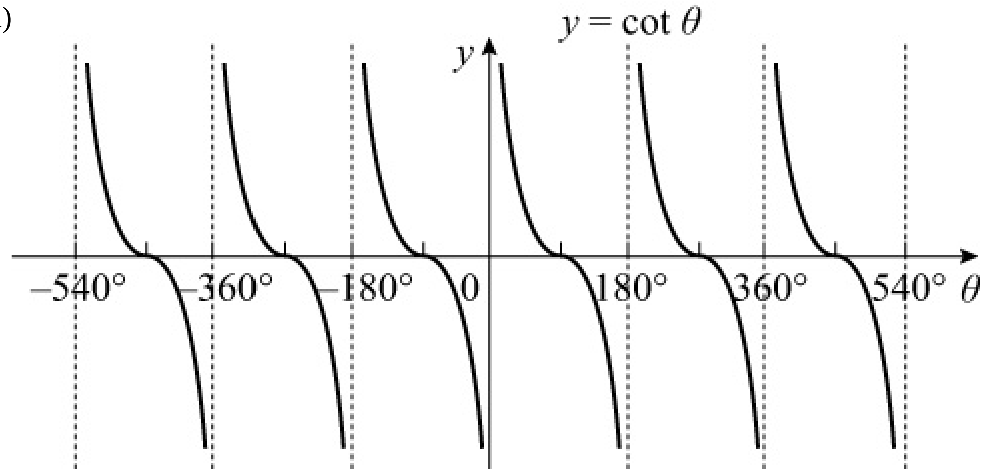
(a)(i)



(ii)



(iii)



(b)(i) (Note the gap in the range) $\sec\theta \leq -1, \sec\theta \geq 1$
(ii) ($\operatorname{cosec}\theta$ also has a gap in the range) $\operatorname{cosec}\theta \leq -1, \operatorname{cosec}\theta \geq 1$
(iii) $\cot\theta$ takes all real values, i.e. $\cot\theta \in \mathbb{R}$.

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Exercise B, Question 2

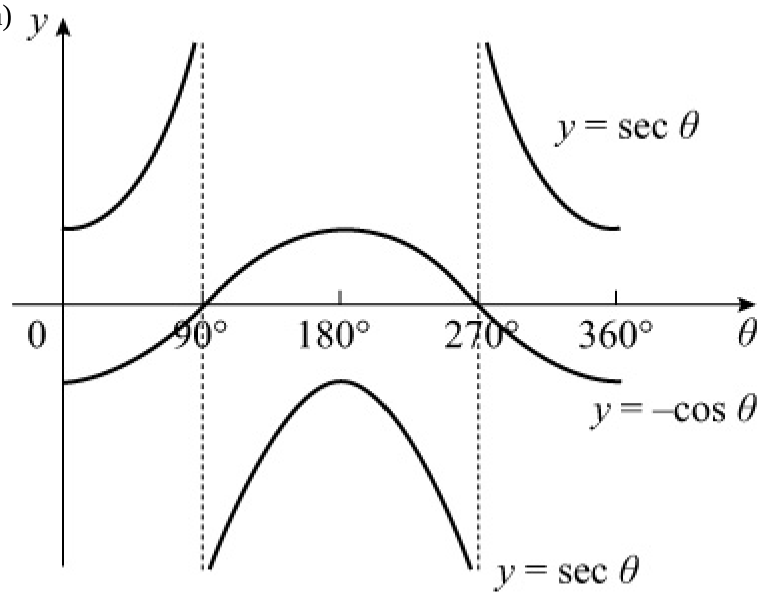
Question:

(a) Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360°$, the graphs of $y = \sec\theta$ and $y = -\cos\theta$.

(b) Explain how your graphs show that $\sec\theta = -\cos\theta$ has no solutions.

Solution:

(a)



(b) You can see that the graphs of $\sec\theta$ and $-\cos\theta$ do not meet, so $\sec\theta = -\cos\theta$ has no solutions.
Algebraically, the solutions of $\sec\theta = -\cos\theta$

are those of $\frac{1}{\cos\theta} = -\cos\theta$

This requires $\cos^2\theta = -1$, which is not possible for real θ.

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Exercise B, Question 3

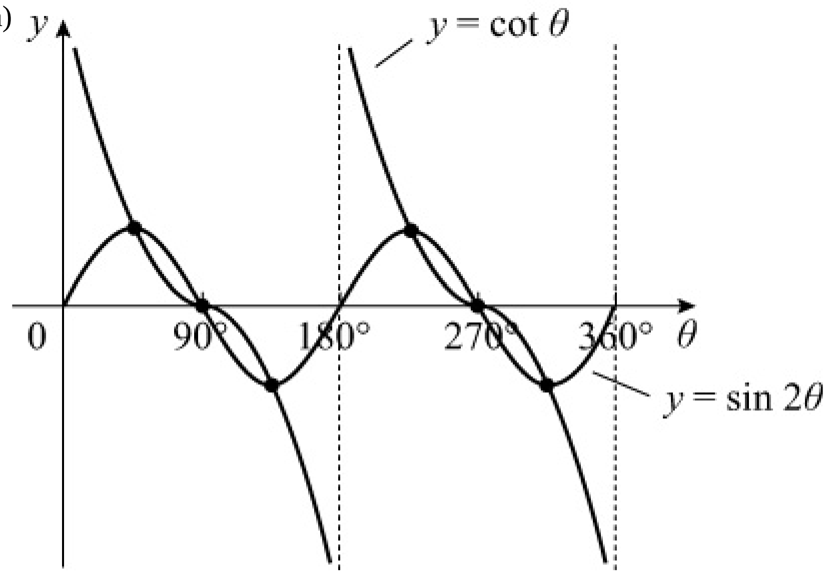
Question:

(a) Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360°$, the graphs of $y = \cot\theta$ and $y = \sin 2\theta$.

(b) Deduce the number of solutions of the equation $\cot\theta = \sin 2\theta$ in the interval $0 \leq \theta \leq 360°$.

Solution:

(a)



(b) The curves meet at the maxima and minima of $y = \sin 2\theta$, and on the θ-axis at odd integer multiples of 90°.
In the interval $0 \leq \theta \leq 360°$ there are 6 intersections.
So there are 6 solutions of $\cot\theta = \sin 2\theta$, is $0 \leq \theta \leq 360°$.

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Exercise B, Question 4

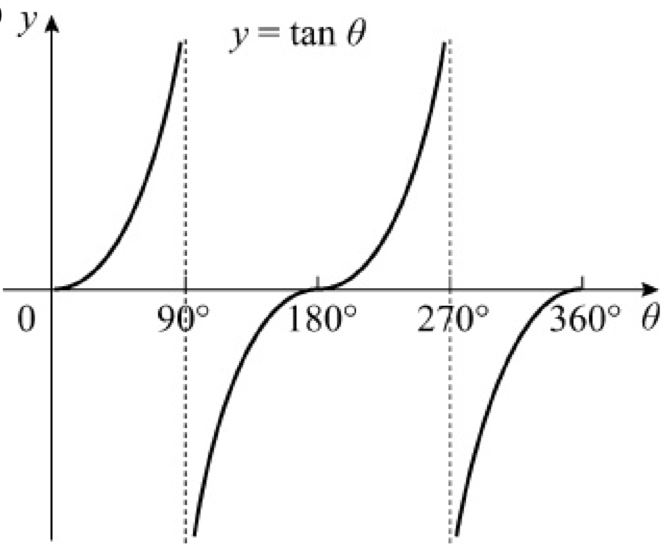
Question:

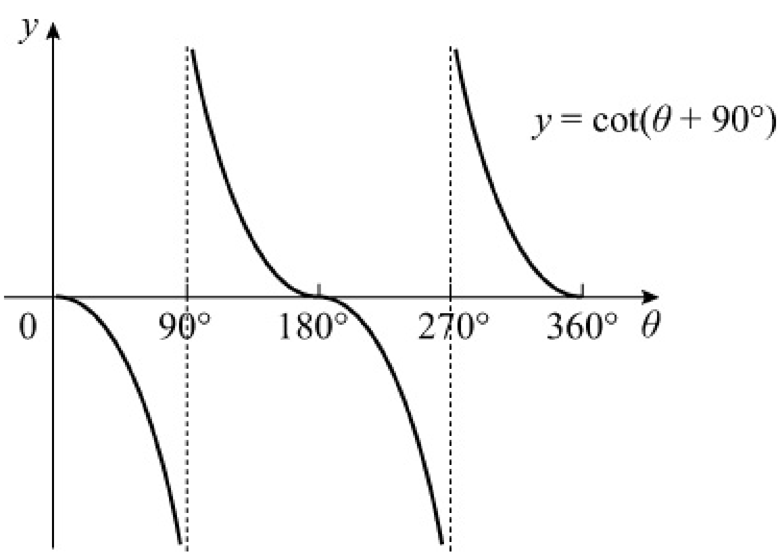
(a) Sketch on separate axes, in the interval $0 \le \theta \le 360°$, the graphs of $y = \tan\theta$ and $y = \cot(\theta + 90°)$.

(b) Hence, state a relationship between $\tan\theta$ and $\cot(\theta + 90°)$.

Solution:

(a)





(b) $y = \cot(\theta + 90°)$ is a reflection in the θ-axis of $y = \tan\theta$, so $\cot(\theta + 90°) = -\tan\theta$

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Exercise B, Question 5

Question:

(a) Describe the relationships between the graphs of

(i) $\tan\left(\theta + \frac{\pi}{2}\right)$ and $\tan\theta$

(ii) $\cot(-\theta)$ and $\cot\theta$

(iii) $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $\operatorname{cosec}\theta$

(iv) $\sec\left(\theta - \frac{\pi}{4}\right)$ and $\sec\theta$

(b) By considering the graphs of $\tan\left(\theta + \frac{\pi}{2}\right)$, $\cot(-\theta)$, $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ and $\sec\left(\theta - \frac{\pi}{4}\right)$, state which pairs of functions are equal.

Solution:

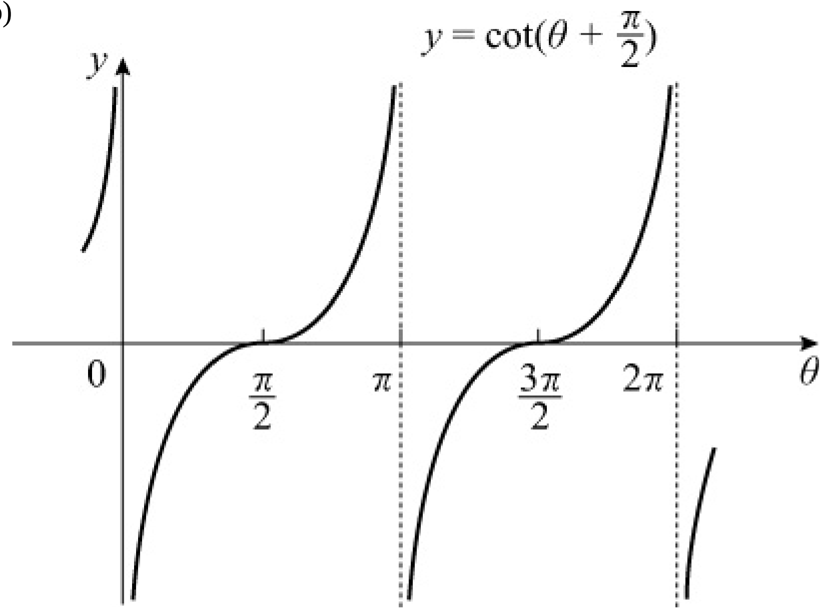
(a) (i) The graph of $\tan\left(\theta + \frac{\pi}{2}\right)$ is the same as that of $\tan\theta$ translated by $\frac{\pi}{2}$ to the left.

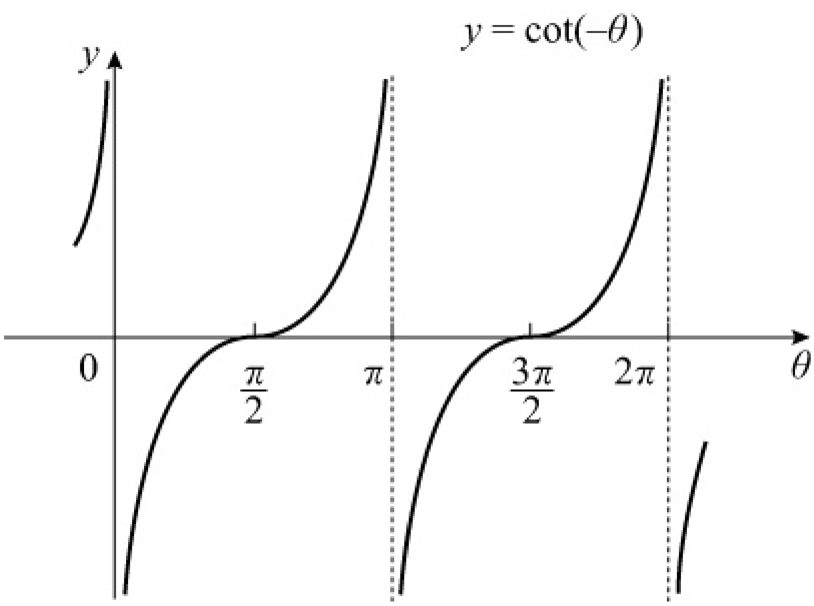
(ii) The graph of $\cot(-\theta)$ is the same as that of $\cot\theta$ reflected in the *y*-axis.

(iii) The graph of $\operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ is the same as that of $\operatorname{cosec}\theta$ translated by $\frac{\pi}{4}$ to the left.

(iv) The graph of $\sec\left(\theta - \frac{\pi}{4}\right)$ is the same as that of $\sec\theta$ translated by $\frac{\pi}{4}$ to the right.

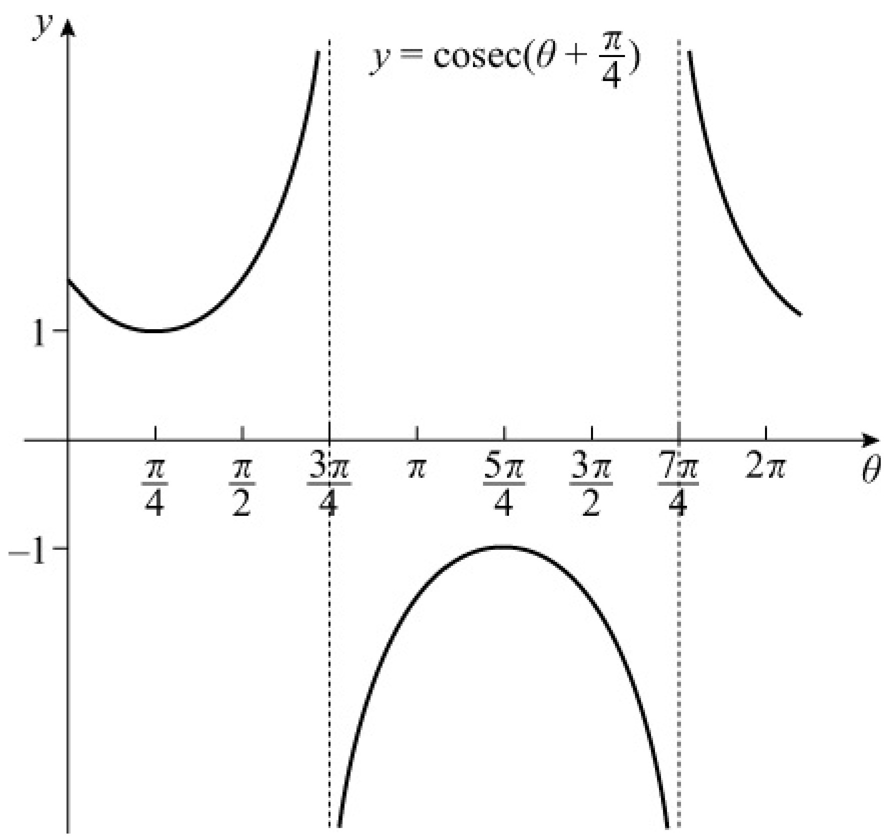
(b)

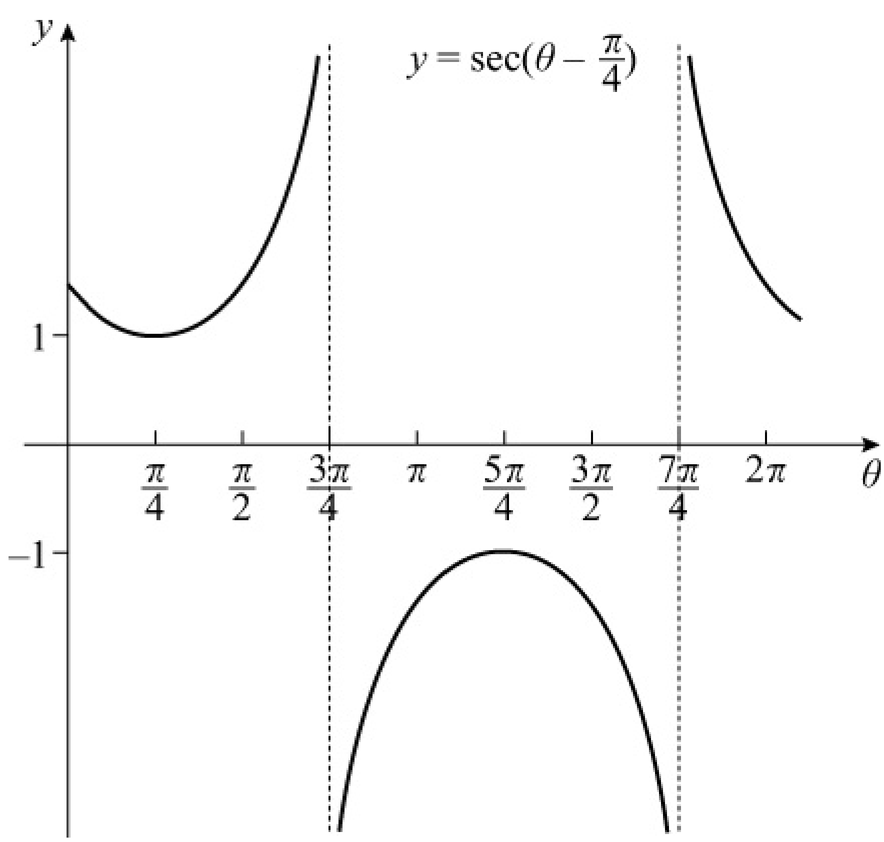




(reflect $y = \cot\theta$ in the y-axis)

$\tan\left(\theta + \frac{\pi}{2}\right) = \cot(-\theta)$





$$\text{cosec}\left(\theta + \frac{\pi}{4}\right) = \sec\left(\theta - \frac{\pi}{4}\right)$$

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Exercise B, Question 6

Question:

Sketch on separate axes, in the interval $0 \leq \theta \leq 360°$, the graphs of:

(a) $y = \sec 2\theta$

(b) $y = -\operatorname{cosec} \theta$

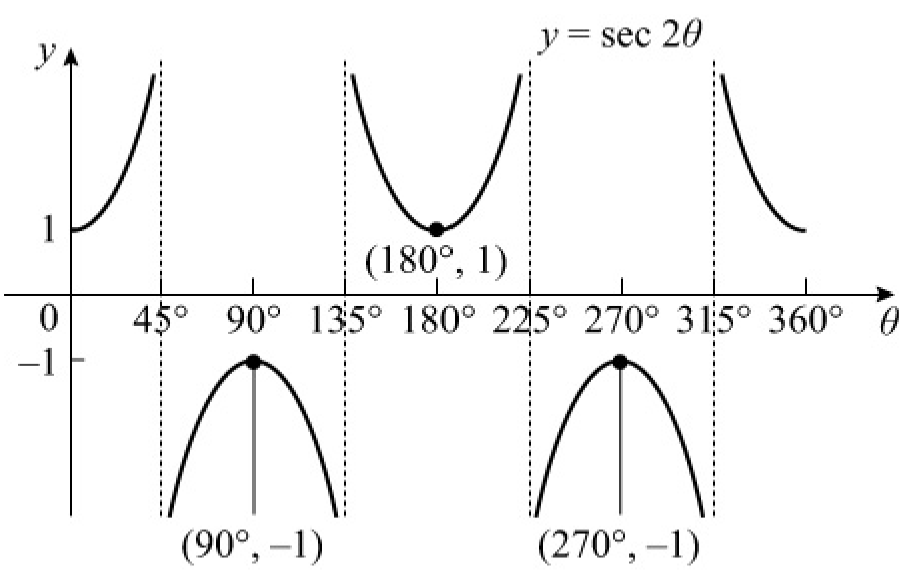
(c) $y = 1 + \sec \theta$

(d) $y = \operatorname{cosec}(\theta - 30°)$
In each case show the coordinates of any maximum and minimum points, and of any points at which the curve meets the axes.

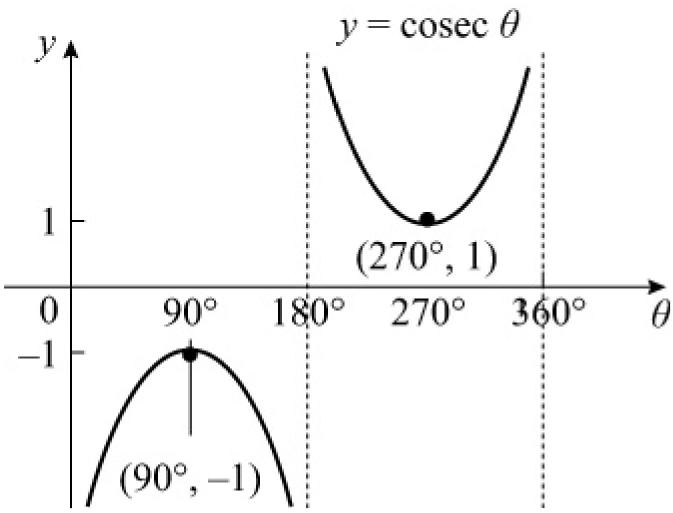
Solution:

(a) A stretch of $y = \sec \theta$ in the θ direction with scale factor $\frac{1}{2}$.

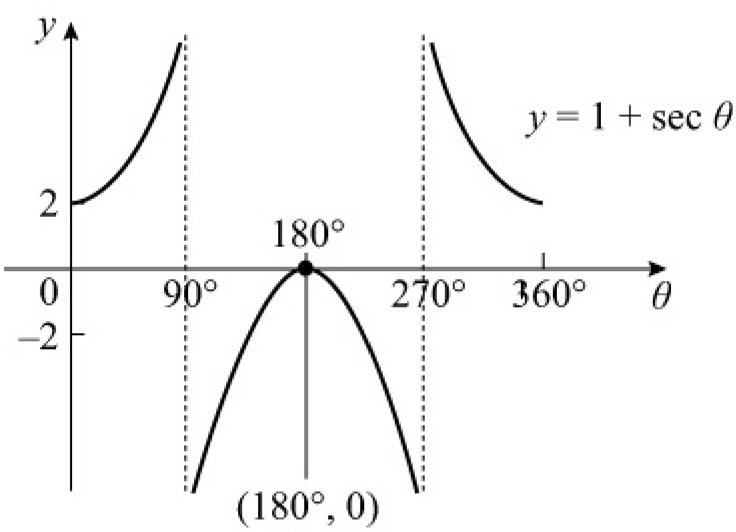
Minimum at (180°, 1)
Maxima at (90°, -1) and (270°, -1)



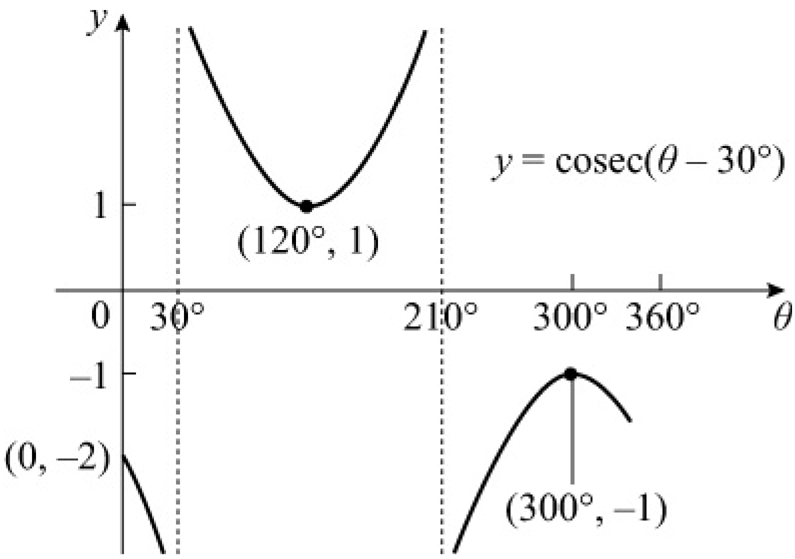
(b) Reflection in θ-axis of $y = \operatorname{cosec} \theta$.
Minimum at (270°, 1)
Maximum at (90°, -1)



(c) Translation of y = sec θ by + 1 in the y direction.
Maximum at (180°, 0)



(d) Translation of y = cosec θ by 30° to the right.
Minimum at (120°, 1)
Maximum at (300°, -1)


y
1
(120°, 1)
y = cosec(θ – 30°)
0
30°
210°
300°
360°
θ
–1
(0, –2)
(300°, –1)

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Exercise B, Question 7

Question:

Write down the periods of the following functions. Give your answer in terms of π.

(a) $\sec 3\theta$

(b) $\operatorname{cosec} \frac{1}{2}\theta$

(c) $2 \cot\theta$

(d) $\sec(-\theta)$

Solution:

(a) The period of $\sec\theta$ is 2π radians.

$y = \sec 3\theta$ is a stretch of $y = \sec\theta$ with scale factor $\frac{1}{3}$ in the θ direction.

So period of $\sec 3\theta$ is $\frac{2\pi}{3}$.

(b) $\operatorname{cosec}\theta$ has a period of 2π.

$\operatorname{cosec} \frac{1}{2}\theta$ is a stretch of $\operatorname{cosec}\theta$ in the θ direction with scale factor 2.

So period of $\operatorname{cosec} \frac{1}{2}\theta$ is 4π.

(c) $\cot\theta$ has a period of π.
$2\cot\theta$ is a stretch in the y direction by scale factor 2.
So the periodicity is not affected.
Period of $2\cot\theta$ is π.

(d) $\sec\theta$ has a period of 2π.
$\sec(-\theta)$ is a reflection of $\sec\theta$ in y-axis, so periodicity is unchanged.
Period of $\sec(-\theta)$ is 2π.

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Exercise B, Question 8

Question:

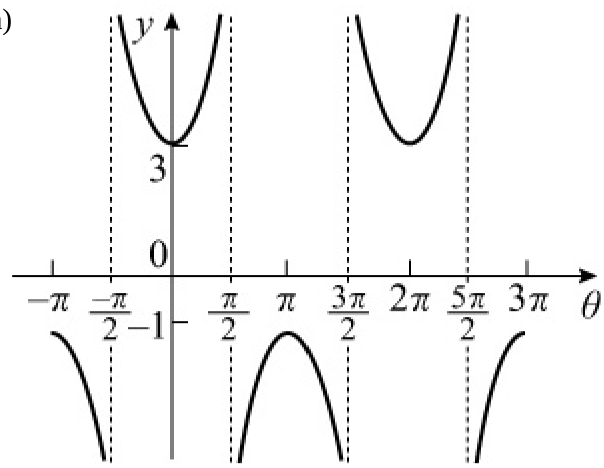
(a) Sketch the graph of $y = 1 + 2 \sec\theta$ in the interval $-\pi \leq \theta \leq 2\pi$.

(b) Write down the y-coordinate of points at which the gradient is zero.

(c) Deduce the maximum and minimum values of $\frac{1}{1 + 2 \sec\theta}$, and give the smallest positive values of θ at which they occur.

Solution:

(a)



(b) The y coordinates at stationary points are -1 and 3.

(c) Minimum value of $\frac{1}{1 + 2 \sec\theta}$ is where $1 + 2 \sec\theta$ is a maximum.

So minimum value of $\frac{1}{1 + 2 \sec\theta}$ is $\frac{1}{-1} = -1$

It occurs when $\theta = \pi$ (see diagram) (1st +ve value)

Maximum value of $\frac{1}{1 + 2 \sec\theta}$ is where $1 + 2 \sec\theta$ is a minimum.

So maximum value of $\frac{1}{1 + 2 \sec\theta}$ is $\frac{1}{3}$

It occurs when $\theta = 2\pi$ (1st +ve value)

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Exercise C, Question 1

Question:

Give solutions to these equations correct to 1 decimal place.

Rewrite the following as powers of sec θ, cosec θ or cot θ:

(a) $\frac{1}{\sin^3\theta}$

(b) $\sqrt{\frac{4}{\tan^6\theta}}$

(c) $\frac{1}{2\cos^2\theta}$

(d) $\frac{1-\sin^2\theta}{\sin^2\theta}$

(e) $\frac{\sec\theta}{\cos^4\theta}$

(f) $\sqrt{\operatorname{cosec}^3\theta\cot\theta\sec\theta}$

(g) $\frac{2}{\sqrt{\tan\theta}}$

(h) $\frac{\operatorname{cosec}^2\theta\tan^2\theta}{\cos\theta}$

Solution:

(a) $\frac{1}{\sin^3\theta} = \left(\frac{1}{\sin\theta}\right)^3 = \operatorname{cosec}^3\theta$

(b) $\sqrt{\frac{4}{\tan^6\theta}} = \frac{2}{\tan^3\theta} = 2\times\left(\frac{1}{\tan\theta}\right)^3 = 2\cot^3\theta$

(c) $\frac{1}{2\cos^2\theta} = \frac{1}{2} \times \left(\frac{1}{\cos\theta}\right)^2 = \frac{1}{2}\sec^2\theta$

(d) $\frac{1-\sin^2\theta}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$ (using $\sin^2\theta + \cos^2\theta \equiv 1$)

So $\frac{1-\sin^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \cot^2\theta$

(e) $\frac{\sec\theta}{\cos^4\theta} = \frac{1}{\cos\theta} \times \frac{1}{\cos^4\theta} = \frac{1}{\cos^5\theta} = \left(\frac{1}{\cos\theta}\right)^5 = \sec^5\theta$

(f) $\sqrt{\operatorname{cosec}^3\theta\cot\theta\sec\theta} = \sqrt{\frac{1}{\sin^3\theta} \times \frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos\theta}} = \sqrt{\frac{1}{\sin^4\theta}} =$

$\frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$

(g) $\frac{2}{\sqrt{\tan\theta}} = 2 \times \frac{1}{(\tan\theta)^{\frac{1}{2}}} = 2\cot^{\frac{1}{2}}\theta$

(h) $\frac{\operatorname{cosec}^2\theta\tan^2\theta}{\cos\theta} = \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} \times \frac{1}{\cos\theta} = \left(\frac{1}{\cos\theta}\right)^3 = \sec^3\theta$

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Exercise C, Question 2

Question:

Give solutions to these equations correct to 1 decimal place.

Write down the value(s) of cot x in each of the following equations:

(a) $5 \sin x = 4 \cos x$

(b) $\tan x = -2$

(c) $3 \frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

Solution:

(a) $5 \sin x = 4 \cos x$

$\Rightarrow \quad 5 = 4 \frac{\cos x}{\sin x}$ (divide by sin x)

$\Rightarrow \quad \frac{5}{4} = \cot x$ (divide by 4)

(b) $\tan x = -2$

$\Rightarrow \quad \frac{1}{\tan x} = \frac{1}{-2}$

$\Rightarrow \quad \cot x = -\frac{1}{2}$

(c) $3 \frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$

$\Rightarrow \quad 3 \sin^2 x = \cos^2 x$ (multiply by sin x cos x)

$\Rightarrow \quad 3 = \frac{\cos^2 x}{\sin^2 x}$ (divide by $\sin^2 x$)

$\Rightarrow \quad \left(\frac{\cos x}{\sin x} \right)^2 = 3$

$\Rightarrow \quad \cot^2 x = 3$

$\Rightarrow \quad \cot x = \pm \sqrt{3}$

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Exercise C, Question 3

Question:

Give solutions to these equations correct to 1 decimal place.

Using the definitions of **sec**, **cosec**, **cot** and **tan** simplify the following expressions:

(a) $\sin\theta \cot\theta$

(b) $\tan\theta \cot\theta$

(c) $\tan 2\theta \operatorname{cosec} 2\theta$

(d) $\cos\theta \sin\theta (\cot\theta + \tan\theta)$

(e) $\sin^3 x \operatorname{cosec} x + \cos^3 x \sec x$

(f) $\sec A - \sec A \sin^2 A$

(g) $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

Solution:

(a) $\sin\theta \ \cot\theta = \sin\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta$

(b) $\tan\theta \ \cot\theta = \tan\theta \times \frac{1}{\tan\theta} = 1$

(c) $\tan 2\theta \ \operatorname{cosec} 2\theta = \frac{\sin 2\theta}{\cos 2\theta} \times \frac{1}{\sin 2\theta} = \frac{1}{\cos 2\theta} = \sec 2\theta$

(d) $\cos\theta \ \sin\theta \ (\cot\theta + \tan\theta) = \cos\theta \ \sin\theta \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \right)$

$= \cos^2 \theta + \sin^2 \theta = 1$

(e) $\sin^3 x \ \operatorname{cosec} x + \cos^3 x \ \sec x = \sin^3 x \times \frac{1}{\sin x} + \cos^3 x \times$

$\frac{1}{\cos x} = \sin^2 x + \cos^2 x = 1$

(f) $\sec A - \sec A \sin^2 A$

$= \sec A (1 - \sin^2 A)$ (factorise)

$= \frac{1}{\cos A} \times \cos^2 A$ (using $\sin^2 A + \cos^2 A \equiv 1$)

$= \cos A$

(g) $\sec^2 x \cos^5 x + \cot x \operatorname{cosec} x \sin^4 x$

$= \frac{1}{\cos^2 x} \times \cos^5 x + \frac{\cos x}{\sin x} \times \frac{1}{\sin x} \times \sin^4 x$

$= \cos^3 x + \sin^2 x \cos x$

$= \cos x (\cos^2 x + \sin^2 x)$

$= \cos x$ (since $\cos^2 x + \sin^2 x \equiv 1$)

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Exercise C, Question 4

Question:

Show that

(a) $\cos\theta + \sin\theta\ \tan\theta \equiv \sec\theta$

(b) $\cot\theta + \tan\theta \equiv \operatorname{cosec}\theta\ \sec\theta$

(c) $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\ \cot\theta$

(d) $(1 - \cos x)(1 + \sec x) \equiv \sin x\ \tan x$

(e) $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} \equiv 2\ \sec x$

(f) $\frac{\cos\theta}{1 + \cot\theta} \equiv \frac{\sin\theta}{1 + \tan\theta}$

Solution:

(a) L.H.S. $\equiv \cos\theta + \sin\theta\ \tan\theta$

$$\equiv \cos\theta + \sin\theta\ \frac{\sin\theta}{\cos\theta}$$

$$\equiv \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$\equiv \frac{1}{\cos\theta} \quad (\text{using } \sin^2\theta + \cos^2\theta \equiv 1)$$

$$\equiv \sec\theta \equiv \text{R.H.S.}$$

(b) L.H.S. $\equiv \cot\theta + \tan\theta$

$$\equiv \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\equiv \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\ \cos\theta}$$

$$\equiv \frac{1}{\sin\theta\ \cos\theta}$$

$$\equiv \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$\equiv \operatorname{cosec}\theta\ \sec\theta \equiv \text{R.H.S.}$$

(c) L.H.S. $\equiv \operatorname{cosec}\theta - \sin\theta$

$$\equiv \frac{1}{\sin\theta} - \sin\theta$$

$$\equiv \frac{1 - \sin^2 \theta}{\sin\theta}$$

$$\equiv \frac{\cos^2 \theta}{\sin\theta}$$

$$\equiv \cos\theta \times \frac{\cos\theta}{\sin\theta}$$

$$\equiv \cos\theta \ \cot\theta \equiv \text{R.H.S.}$$

(d) L.H.S. $\equiv (1 - \cos x)(1 + \sec x)$

$\equiv 1 - \cos x + \sec x - \cos x \ \sec x$ (multiplying out)

$\equiv \sec x - \cos x$

$$\equiv \frac{1}{\cos x} - \cos x$$

$$\equiv \frac{1 - \cos^2 x}{\cos x}$$

$$\equiv \frac{\sin^2 x}{\cos x}$$

$$\equiv \sin x \times \frac{\sin x}{\cos x}$$

$$\equiv \sin x \ \tan x \equiv \text{R.H.S.}$$

(e) L.H.S. $\equiv \dfrac{\cos x}{1 - \sin x} + \dfrac{1 - \sin x}{\cos x}$

$$\equiv \frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x)\cos x}$$

$$\equiv \frac{\cos^2 x + (1 - 2\sin x + \sin^2 x)}{(1 - \sin x)\cos x}$$

$\equiv \dfrac{2 - 2\sin x}{(1 - \sin x)\cos x}$ (using $\sin^2 x + \cos^2 x \equiv 1$)

$\equiv \dfrac{2(1 - \sin x)}{(1 - \sin x)\cos x}$ (factorising)

$$= \frac{2}{\cos x}$$

$$\equiv 2 \ \sec x \equiv \text{R.H.S.}$$

(f)

$$
\begin{aligned}
\text{L.H.S.} &\equiv \frac{\cos\theta}{1+\cot\theta} \\
&\equiv \frac{\cos\theta}{1+\frac{1}{\tan\theta}} \\
&\equiv \frac{\cos\theta}{\frac{\tan\theta+1}{\tan\theta}} \\
&\equiv \frac{\cos\theta\tan\theta}{1+\tan\theta} \\
&\equiv \frac{\cos\theta\times\frac{\sin\theta}{\cos\theta}}{1+\tan\theta} \\
&\equiv \frac{\sin\theta}{1+\tan\theta} \equiv \text{R.H.S}
\end{aligned}
$$

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Exercise C, Question 5

Question:

Solve, for values of θ in the interval $0 \le \theta \le 360°$, the following equations. Give your answers to 3 significant figures where necessary.

(a) $\sec\theta = \sqrt{2}$

(b) $\operatorname{cosec}\theta = -3$

(c) $5\cot\theta = -2$

(d) $\operatorname{cosec}\theta = 2$

(e) $3\sec^2\theta - 4 = 0$

(f) $5\cos\theta = 3\cot\theta$

(g) $\cot^2\theta - 8\tan\theta = 0$

(h) $2\sin\theta = \operatorname{cosec}\theta$

Solution:

(a) $\sec\theta = \sqrt{2}$

$$\Rightarrow \frac{1}{\cos\theta} = \sqrt{2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$$

Calculator value is $\theta = 45°$

$\cos\theta$ is +ve $\Rightarrow$ θ in 1st and 4th quadrants

Solutions are 45°, 315°

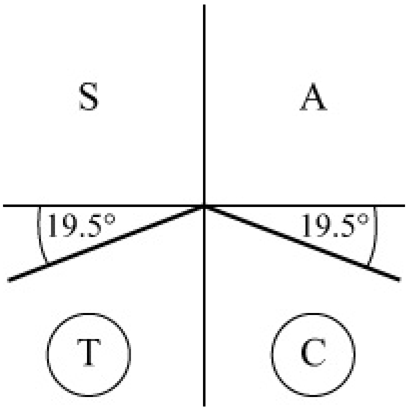
(b) $\operatorname{cosec}\theta = -3$

$$\Rightarrow \frac{1}{\sin\theta} = -3$$

$$\Rightarrow \sin\theta = -\frac{1}{3}$$

Calculator value is $-19.5°$

$\sin\theta$ is −ve $\Rightarrow$ θ is in 3rd and 4th quadrants



Solutions are 199°, 341° (3 s.f.)

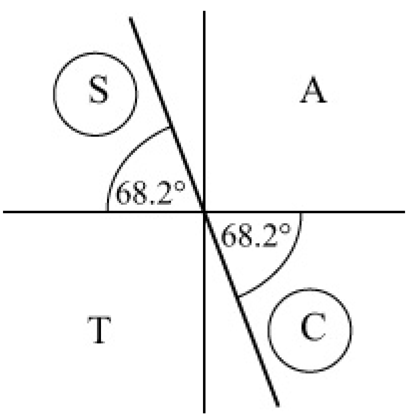
(c) $5 \cot\theta = -2$

$\Rightarrow \quad \cot\theta = -\frac{2}{5}$

$\Rightarrow \quad \tan\theta = -\frac{5}{2}$

Calculator value is $-68.2°$

$\tan\theta$ is −ve $\quad\Rightarrow\quad$ θ is in 2nd and 4th quadrants



Solutions are 112°, 292° (3 s.f.)

(d) $\operatorname{cosec}\theta = 2$

$\Rightarrow \quad \frac{1}{\sin\theta} = 2$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\sin\theta$ is +ve $\quad\Rightarrow\quad$ θ is in 1st and 2nd quadrants

Solutions are 30°, 150°

(e) $3 \sec^2 \theta = 4$

$\Rightarrow \quad \sec^2 \theta = \frac{4}{3}$

$\Rightarrow \quad \cos^2 \theta = \frac{3}{4}$

$\Rightarrow \quad \cos\theta = \pm \frac{\sqrt{3}}{2}$

Calculator value for $\cos\theta = \frac{\sqrt{3}}{2}$ is 30°

As $\cos\theta$ is $\pm$, θ is in all four quadrants
Solutions are 30°, 150°, 210°, 330°

(f) $5 \cos\theta = 3 \cot\theta$

$\Rightarrow \quad 5 \cos\theta = 3 \frac{\cos\theta}{\sin\theta}$

Note Do not cancel $\cos\theta$ on each side. Multiply through by $\sin\theta$.

$\Rightarrow \quad 5 \cos\theta \sin\theta = 3 \cos\theta$

$\Rightarrow \quad 5 \cos\theta \sin\theta - 3 \cos\theta = 0$

$\Rightarrow \quad \cos\theta (5 \sin\theta - 3) = 0$ (factorise)

So $\cos\theta = 0$ or $\sin\theta = \frac{3}{5}$

Solutions are (90°, 270°), (36.9°, 143°) = 36.9°, 90°, 143°, 270°.

(g) $\cot^2 \theta - 8 \tan\theta = 0$

$\Rightarrow \quad \frac{1}{\tan^2 \theta} - 8 \tan\theta = 0$

$\Rightarrow \quad 1 - 8 \tan^3 \theta = 0$

$\Rightarrow \quad 8 \tan^3 \theta = 1$

$\Rightarrow \quad \tan^3 \theta = \frac{1}{8}$

$\Rightarrow \quad \tan\theta = \frac{1}{2}$

$\tan\theta$ is +ve $\Rightarrow$ θ is in 1st and 3rd quadrants
Calculator value is 26.6°
Solutions are 26.6° and $(180° + 26.6°) = 26.6°$ and 207° (3 s.f.).

(h) $2 \sin\theta = \operatorname{cosec}\theta$

$\Rightarrow \quad 2 \sin\theta = \frac{1}{\sin\theta}$

$\Rightarrow \quad \sin^2 \theta = \frac{1}{2}$

$\Rightarrow \quad \sin\theta = \pm \frac{1}{\sqrt{2}}$

Calculator value for $\sin^{-1} \frac{1}{\sqrt{2}}$ is 45°

Solution are in all four quadrants
Solutions are 45°, 135°, 225°, 315°

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Exercise C, Question 6

Question:

Solve, for values of θ in the interval $-180° \le \theta \le 180°$, the following equations:

(a) $\text{cosec}\,\theta = 1$

(b) $\sec\theta = -3$

(c) $\cot\theta = 3.45$

(d) $2\,\text{cosec}^2\theta - 3\,\text{cosec}\,\theta = 0$

(e) $\sec\theta = 2\cos\theta$

(f) $3\cot\theta = 2\sin\theta$

(g) $\text{cosec}\,2\theta = 4$

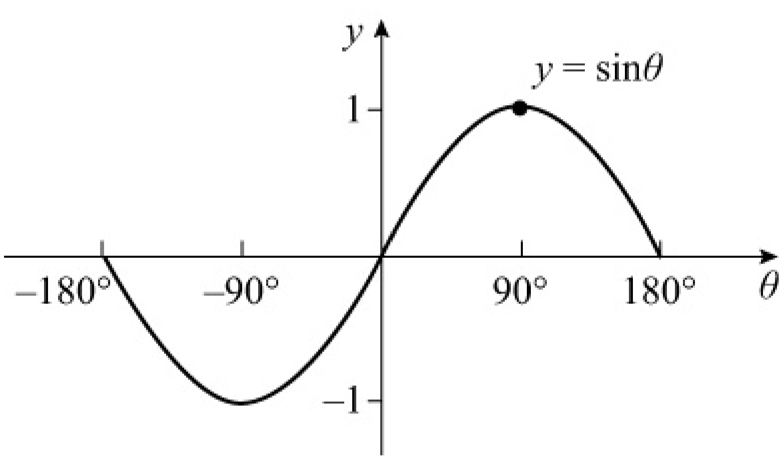
(h) $2\cot^2\theta - \cot\theta - 5 = 0$

Solution:

(a) $\text{cosec}\,\theta = 1$

$\Rightarrow \quad \sin\theta = 1$

$\Rightarrow \quad \theta = 90°$

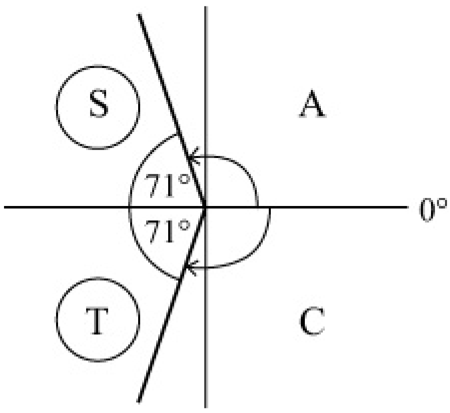


(b) $\sec\theta = -3$

$\Rightarrow \quad \cos\theta = -\frac{1}{3}$

Calculator value for $\cos^{-1}\left(-\frac{1}{3}\right)$ is 109° (3 s.f.)

$\cos\theta$ is −ve $\Rightarrow$ θ is in 2nd and 3rd quadrants



Solutions are 109° and $-109°$

[If you are not using the quadrant diagram, answer in this case would be $\cos^{-1}\left(-\frac{1}{3}\right)$ and $-360° + \cos^{-1}\left(-\frac{1}{3}\right)$. See key point on page 84.]

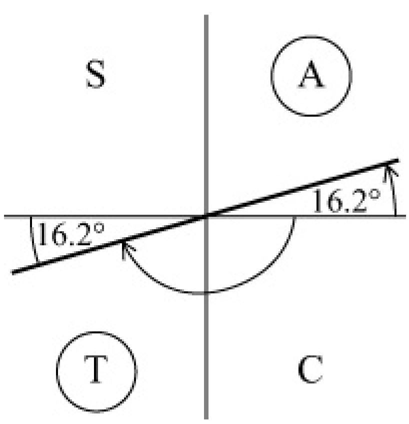
(c) $\cot\theta = 3.45$

$\Rightarrow \quad \frac{1}{\tan\theta} = 3.45$

$\Rightarrow \quad \tan\theta = \frac{1}{3.45} = 0.28985...$

Calculator value for $\tan^{-1}(0.28985...)$ is 16.16°

$\tan\theta$ is +ve $\Rightarrow$ θ is in 1st and 3rd quadrants



Solutions are $16.2°$, $-180° + 16.2° = 16.2°$, $-164°$ (3 s.f.)

(d) $2\ \text{cosec}^2\ \theta - 3\ \text{cosec}\,\theta = 0$

$\Rightarrow\quad \text{cosec}\,\theta\,(2\ \text{cosec}\,\theta - 3) = 0 \qquad \text{(factorise)}$

$\Rightarrow\quad \text{cosec}\,\theta = 0 \text{ or } \text{cosec}\,\theta = \frac{3}{2}$

$\Rightarrow\quad \sin\theta = \frac{2}{3} \qquad \text{cosec}\,\theta = 0 \text{ has no solutions}$

Calculator value for $\sin^{-1}\frac{2}{3}$ is 41.8°

θ is in 1st and 2nd quadrants
Solutions are 41.8°, $(180 - 41.8)° = 41.8°,\ 138°$ (3 s.f.)

(e) $\sec\theta = 2\ \cos\theta$

$\Rightarrow\quad \frac{1}{\cos\theta} = 2\ \cos\theta$

$\Rightarrow\quad \cos^2\ \theta = \frac{1}{2}$

$\Rightarrow\quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

Calculator value for $\cos^{-1}\frac{1}{\sqrt{2}}$ is 45°

θ is in all quadrants, but remember that $-180° \leq \theta \leq 180°$
Solutions are $\pm 45°,\ \pm 135°$

(f) $3\ \cot\theta = 2\ \sin\theta$

$\Rightarrow\quad 3\frac{\cos\theta}{\sin\theta} = 2\ \sin\theta$

$\Rightarrow\quad 3\ \cos\theta = 2\ \sin^2\ \theta$

$\Rightarrow\quad 3\ \cos\theta = 2(1 - \cos^2\ \theta) \qquad (\text{use } \sin^2\ \theta + \cos^2\ \theta \equiv 1)$

$\Rightarrow\quad 2\ \cos^2\ \theta + 3\ \cos\theta - 2 = 0$

$\Rightarrow\quad (2\ \cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow\quad \cos\theta = \frac{1}{2} \text{ or } \cos\theta = -2$

As $\cos\theta = -2$ has no solutions, $\cos\theta = \frac{1}{2}$

Solutions are $\pm 60°$

(g) $\text{cosec}\,2\theta = 4$

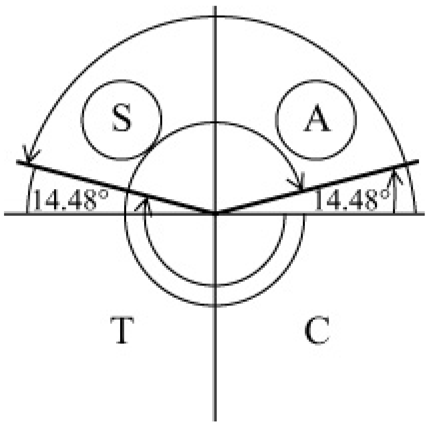
$\Rightarrow\quad \sin 2\theta = \frac{1}{4}$

Remember that $-180° \leq \theta \leq 180°$

So $-360° \leq 2\theta \leq 360°$

Calculator solution for 2θ is $\sin^{-1}\frac{1}{4} = 14.48°$

$\sin 2\theta$ is +ve $\Rightarrow$ 2θ is in 1st and 2nd quadrants



$2\theta = -194.48°, -345.52°, 14.48°, 165.52°$
$\theta = -97.2°, -172.8°, 7.24°, 82.76° = -173°, -97.2°, 7.24°, 82.8°$ (3 s.f.)

(h) $2\cot^2\theta - \cot\theta - 5 = 0$
As this quadratic in $\cot\theta$ does not factorise, use the quadratic formula

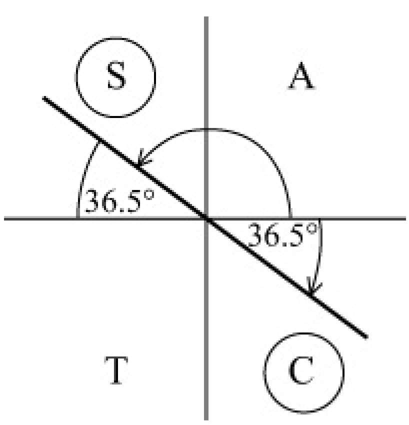
$\cot\theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(You could change $\cot\theta$ to $\frac{1}{\tan\theta}$ and work with the quadratic

$5\tan^2\theta + \tan\theta - 2 = 0$)

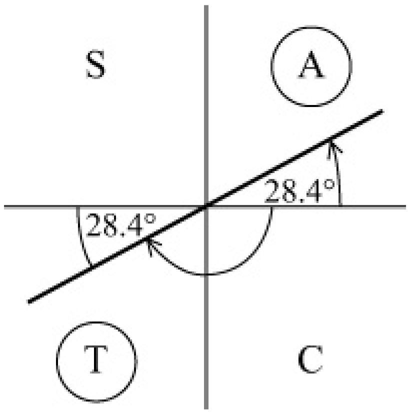
So $\cot\theta = \frac{1 \pm \sqrt{41}}{4} = -1.3507...$ or $1.8507 \quad \ldots$

So $\tan\theta = -0.7403...$ or $0.5403 \quad \ldots$
The calculator value for $\tan\theta = -0.7403...$ is $\theta = -36.51°$



Solution are $-36.5°, +143°$ (3 s.f.).

The calculator value for $\tan\theta = 0.5403...$ is $\theta = 28.38°$



Solution are 28.4°, $(-180 + 28.4)°$
Total set of solutions is $-152°$, $-36.5°$, 28.4°, 143° (3 s.f.)

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Exercise C, Question 7

Question:

Solve the following equations for values of θ in the interval $0 \leq \theta \leq 2\pi$. Give your answers in terms of π.

(a) $\sec\theta = -1$

(b) $\cot\theta = -\sqrt{3}$

(c) $\text{cosec}\ \frac{1}{2}\theta = \frac{2\sqrt{3}}{3}$

(d) $\sec\theta = \sqrt{2}\tan\theta \quad \left(\theta \neq \frac{\pi}{2},\ \theta \neq \frac{3\pi}{2}\right)$

Solution:

(a) $\sec\theta = -1$

$\Rightarrow \quad \cos\theta = -1$

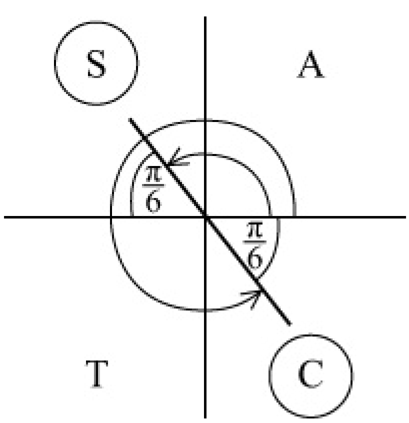
$\Rightarrow \quad \theta = \pi$ (refer to graph of $y = \cos\theta$)

(b) $\cot\theta = -\sqrt{3}$

$\Rightarrow \quad \tan\theta = -\frac{1}{\sqrt{3}}$

Calculator solution is $-\frac{\pi}{6}$ $\left(\text{you should know that } \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}\right)$

$-\frac{\pi}{6}$ is not in the interval



Solution are $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{5\pi}{6}, \frac{11\pi}{6}$

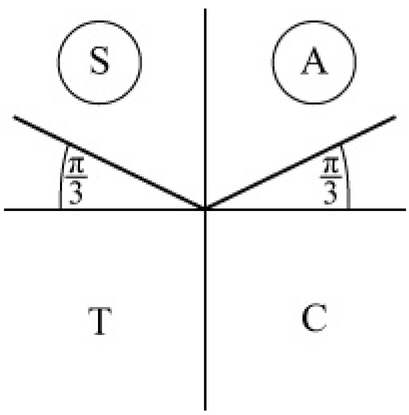
(c) $\text{cosec } \frac{1}{2}\theta = \frac{2\sqrt{3}}{3}$

$\Rightarrow \quad \sin \frac{1}{2}\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Remember that $0 \leq \theta \leq 2\pi$

so $0 \leq \frac{1}{2}\theta \leq \pi$

First solution for $\sin \frac{1}{2}\theta = \frac{\sqrt{3}}{2}$ is $\frac{1}{2}\theta = \frac{\pi}{3}$



So $\frac{1}{2}\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \quad \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

(d) $\sec\theta = \sqrt{2} \tan\theta$

$\Rightarrow \quad \frac{1}{\cos\theta} = \sqrt{2}\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad 1 = \sqrt{2} \sin\theta \qquad (\cos\theta \neq 0)$

$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$

Solutions are $\frac{\pi}{4}, \frac{3\pi}{4}$

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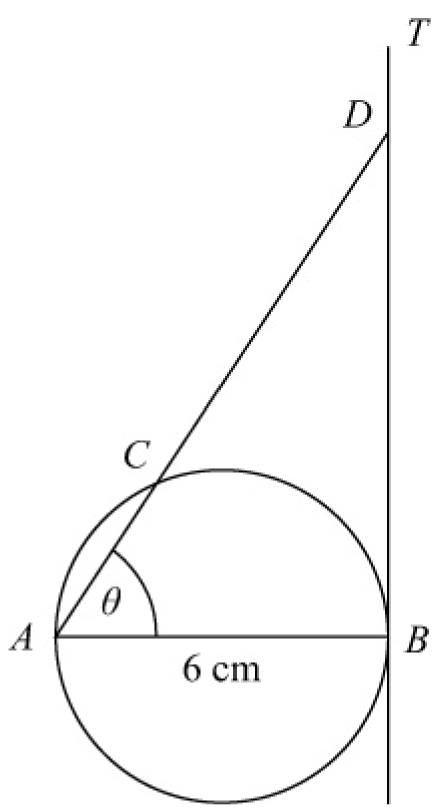
Exercise C, Question 8

Question:

In the diagram AB = 6 cm is the diameter of the circle and *BT* is the tangent to the circle at *B*. The chord *AC* is extended to meet this tangent at *D* and $\angle$ DAB = θ.

(a) Show that CD = 6 ($\sec\theta - \cos\theta$) .

(b) Given that CD = 16 cm, calculate the length of the chord *AC*.



Solution:

(a) In right-angled triangle *ABD*

$$\frac{AB}{AD} = \cos\theta$$

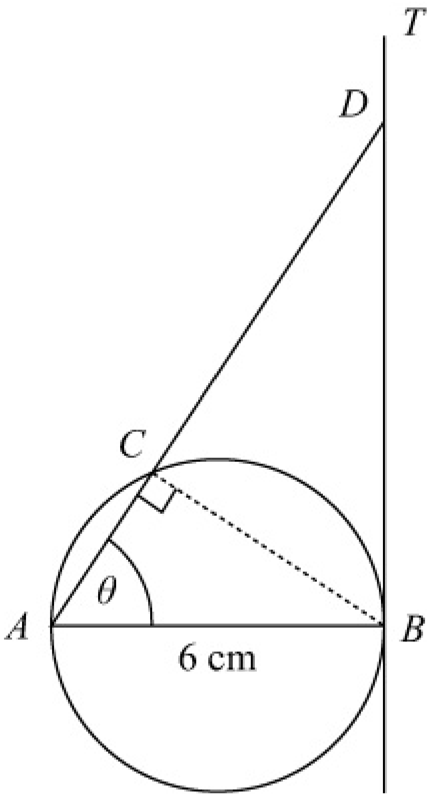
$$\Rightarrow \quad AD = \frac{6}{\cos\theta} = 6 \sec\theta$$

In right-angled triangle *ACB*

$$\frac{AC}{AB} = \cos\theta$$

$\Rightarrow \quad AC = 6 \cos\theta$

$DC = AD - AC = 6 \sec\theta - 6 \cos\theta = 6(\sec\theta - \cos\theta)$



(b) As $16 = 6 \sec\theta - 6 \cos\theta$

$\Rightarrow \quad 8 = \frac{3}{\cos\theta} - 3 \cos\theta$

$\Rightarrow \quad 8 \cos\theta = 3 - 3 \cos^2 \theta$

$\Rightarrow \quad 3 \cos^2 \theta + 8 \cos\theta - 3 = 0$

$\Rightarrow \quad (3 \cos\theta - 1)(\cos\theta + 3) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{3} \quad$ as $\cos\theta \neq -3$

From (a) $AC = 6 \cos\theta = 6 \times \frac{1}{3} = 2$ cm

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Exercise D, Question 1

Question:

Simplify each of the following expressions:

(a) $1 + \tan^2 \frac{1}{2}\theta$

(b) $(\sec\theta - 1) (\sec\theta + 1)$

(c) $\tan^2 \theta (\text{cosec}^2 \theta - 1)$

(d) $(\sec^2 \theta - 1) \cot\theta$

(e) $(\text{cosec}^2 \theta - \cot^2 \theta)^2$

(f) $2 - \tan^2 \theta + \sec^2 \theta$

(g) $\frac{\tan\theta \sec\theta}{1 + \tan^2 \theta}$

(h) $(1 - \sin^2 \theta) (1 + \tan^2 \theta)$

(i) $\frac{\text{cosec}\theta \cot\theta}{1 + \cot^2 \theta}$

(j) $(\sec^4 \theta - 2 \sec^2 \theta \tan^2 \theta + \tan^4 \theta)$

(k) $4 \text{cosec}^2 2\theta + 4 \text{cosec}^2 2\theta \cot^2 2\theta$

Solution:

(a) Use $1 + \tan^2 \theta = \sec^2 \theta$ with θ replaced with $\frac{1}{2}\theta$.

$$1 + \tan^2 \left(\frac{1}{2}\theta \right) = \sec^2 \left(\frac{1}{2}\theta \right)$$

(b) $(\sec\theta - 1) (\sec\theta + 1)$ (multiply out)

$= \sec^2 \theta - 1$

$= (1 + \tan^2 \theta) - 1$

$= \tan^2\theta$

(c) $\tan^2\theta(\operatorname{cosec}^2\theta - 1)$

$= \tan^2\theta[(1 + \cot^2\theta) - 1]$

$= \tan^2\theta\cot^2\theta$

$= \tan^2\theta \times \frac{1}{\tan^2\theta}$

$= 1$

(d) $(\sec^2\theta - 1)\cot\theta$

$= \tan^2\theta\cot\theta$

$= \tan^2\theta \times \frac{1}{\tan\theta}$

$= \tan\theta$

(e) $(\operatorname{cosec}^2\theta - \cot^2\theta)^2$

$= [(1 + \cot^2\theta) - \cot^2\theta]^2$

$= 1^2$

$= 1$

(f) $2 - \tan^2\theta + \sec^2\theta$

$= 2 - \tan^2\theta + (1 + \tan^2\theta)$

$= 2 - \tan^2\theta + 1 + \tan^2\theta$

$= 3$

(g) $\frac{\tan\theta\sec\theta}{1 + \tan^2\theta}$

$= \frac{\tan\theta\sec\theta}{\sec^2\theta}$

$= \frac{\tan\theta}{\sec\theta}$

$= \tan\theta\cos\theta$

$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$

$= \sin\theta$

(h) $(1 - \sin^2\theta)(1 + \tan^2\theta)$

$= \cos^2\theta \times \sec^2\theta$

$= \cos^2 \theta \times \frac{1}{\cos^2 \theta}$

$= 1$

(i) $\frac{\operatorname{cosec}\theta \cot\theta}{1 + \cot^2 \theta}$

$= \frac{\operatorname{cosec}\theta \cot\theta}{\operatorname{cosec}^2 \theta}$

$= \frac{1}{\operatorname{cosec}\theta} \times \cot\theta$

$= \frac{\sin\theta}{1} \times \frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

(j) $\sec^4 \theta - 2 \sec^2 \theta \tan^2 \theta + \tan^4 \theta$

$= (\sec^2 \theta - \tan^2 \theta)^2$ (factorise)

$= [(1 + \tan^2 \theta) - \tan^2 \theta]^2$

$= 1^2$

$= 1$

(k) $4 \operatorname{cosec}^2 2\theta + 4 \operatorname{cosec}^2 2\theta \cot^2 2\theta$

$= 4 \operatorname{cosec}^2 2\theta (1 + \cot^2 2\theta)$

$= 4 \operatorname{cosec}^2 2\theta \operatorname{cosec}^2 2\theta$

$= 4 \operatorname{cosec}^4 2\theta$

Solutionbank

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Exercise D, Question 2

Question:

Given that $\operatorname{cosec} x = \frac{k}{\operatorname{cosec} x}$, where $k > 1$, find, in terms of k, possible values of $\cot x$.

Solution:

$\operatorname{cosec} x = \frac{k}{\operatorname{cosec} x}$

$\Rightarrow \quad \operatorname{cosec}^2 x = k$

$\Rightarrow \quad 1 + \cot^2 x = k$

$\Rightarrow \quad \cot^2 x = k - 1$

$\Rightarrow \quad \cot x = \pm\sqrt{k - 1}$

Solutionbank
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Exercise D, Question 3

Question:

Given that $\cot\theta = -\sqrt{3}$, and that $90° < \theta < 180°$, find the exact value of

(a) $\sin\theta$

(b) $\cos\theta$

Solution:

(a) $\cot\theta = -\sqrt{3} \quad 90° < \theta < 180°$

$\Rightarrow \quad 1 + \cot^2\theta = 1 + 3 = 4$

$\Rightarrow \quad \text{cosec}^2\theta = 4$

$\Rightarrow \quad \sin^2\theta = \frac{1}{4}$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$ (as θ is in 2nd quadrant, $\sin\theta$ is +ve)

(b) Using $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \quad \cos\theta = -\frac{\sqrt{3}}{2}$ (as θ is in 2nd quadrant, $\cos\theta$ is −ve)

Solutionbank
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Exercise D, Question 4

Question:

Given that $\tan\theta = \frac{3}{4}$, and that $180° < \theta < 270°$, find the exact value of

(a) $\sec\theta$

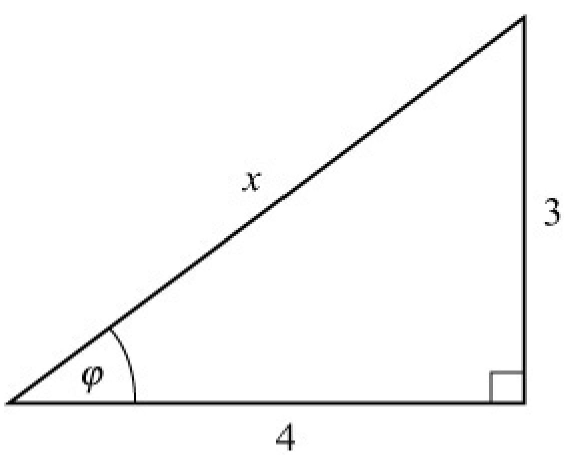
(b) $\cos\theta$

(c) $\sin\theta$

Solution:

$\tan\theta = \frac{3}{4} \quad 180° < \theta < 270°$

Draw right-angled triangle where $\tan\theta = \frac{3}{4}$



Using Pythagoras' theorem, $x = 5$

So $\cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{3}{5}$

As θ is in 3rd quadrant, both $\sin\theta$ and $\cos\theta$ are −ve.

So $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$

(a) $\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{4}$

(b) $\cos\theta = -\frac{4}{5}$

(c) $\sin\theta = -\frac{3}{5}$

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Exercise D, Question 5

Question:

Given that $\cos\theta = \frac{24}{25}$, and that θ is a reflex angle, find the exact value of

(a) $\tan\theta$

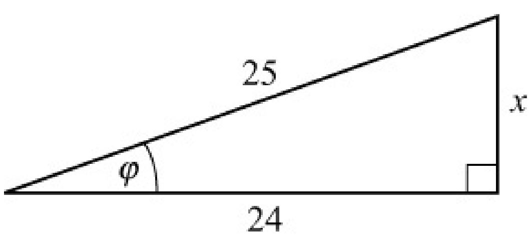
(b) $\text{cosec}\,\theta$

Solution:

$\cos\theta = \frac{24}{25}$, θ reflex

As $\cos\theta$ is +ve and θ reflex, θ is in the 4th quadrant.

Use right-angled triangle where $\cos\theta = \frac{24}{25}$



Using Pythagoras' theorem,

$25^2 = x^2 + 24^2$

$\Rightarrow \quad x^2 = 25^2 - 24^2 = 49$

$\Rightarrow \quad x = 7$

So $\tan\phi = \frac{7}{24}$ and $\sin\phi = \frac{7}{25}$

As θ is in 4th quadrant,

(a) $\tan\theta = -\frac{7}{24}$

(b) $\text{cosec}\,\theta = \frac{1}{\sin\theta} = -\frac{1}{\frac{7}{25}} = -\frac{25}{7}$

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Exercise D, Question 6

Question:

Prove the following identities:

(a) $\sec^4\theta - \tan^4\theta \equiv \sec^2\theta + \tan^2\theta$

(b) $\text{cosec}^2 x - \sin^2 x \equiv \cot^2 x + \cos^2 x$

(c) $\sec^2 A(\cot^2 A - \cos^2 A) \equiv \cot^2 A$

(d) $1 - \cos^2\theta \equiv (\sec^2\theta - 1)(1 - \sin^2\theta)$

(e) $\frac{1 - \tan^2 A}{1 + \tan^2 A} \equiv 1 - 2\sin^2 A$

(f) $\sec^2\theta + \text{cosec}^2\theta \equiv \sec^2\theta\,\text{cosec}^2\theta$

(g) $\text{cosec} A\sec^2 A \equiv \text{cosec} A + \tan A\sec A$

(h) $(\sec\theta - \sin\theta)(\sec\theta + \sin\theta) \equiv \tan^2\theta + \cos^2\theta$

Solution:

(a) L.H.S. $\equiv \sec^4\theta - \tan^4\theta$

$\equiv (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$ (difference of two squares)

$\equiv (1)(\sec^2\theta + \tan^2\theta)$ (as $1 + \tan^2\theta \equiv \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta \equiv 1$)

$\equiv \sec^2\theta + \tan^2\theta \equiv$ R.H.S.

(b) L.H.S. $\equiv \text{cosec}^2 x - \sin^2 x$

$\equiv (1 + \cot^2 x) - (1 - \cos^2 x)$

$\equiv 1 + \cot^2 x - 1 + \cos^2 x$

$\equiv \cot^2 x + \cos^2 x \equiv$ R.H.S.

(c) L.H.S. $\equiv \sec^2 A(\cot^2 A - \cos^2 A)$

$$\equiv \frac{1}{\cos^2 A}\left(\frac{\cos^2 A}{\sin^2 A} - \cos^2 A\right)$$

$$\equiv \frac{1}{\sin^2 A} - 1$$

$\equiv \text{cosec}^2 A - 1$ (use $1 + \cot^2 \theta = \text{cosec}^2 \theta$)

$\equiv 1 + \cot^2 A - 1$

$\equiv \cot^2 A \equiv$ R.H.S.

(d) R.H.S. $\equiv (\sec^2 \theta - 1)(1 - \sin^2 \theta)$

$\equiv \tan^2 \theta \times \cos^2 \theta$ (use $1 + \tan^2 \theta \equiv \sec^2 \theta$ and $\cos^2 \theta + \sin^2 \theta \equiv 1$)

$$\equiv \frac{\sin^2 \theta}{\cos^2 \theta} \times \cos^2 \theta$$

$\equiv \sin^2 \theta$

$\equiv 1 - \cos^2 \theta \equiv$ L.H.S.

(e) L.H.S. $\equiv \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$$\equiv \frac{1 - \tan^2 A}{\sec^2 A}$$

$$\equiv \frac{1}{\sec^2 A}(1 - \tan^2 A)$$

$$\equiv \cos^2 A\left(1 - \frac{\sin^2 A}{\cos^2 A}\right)$$

$\equiv \cos^2 A - \sin^2 A$

$\equiv (1 - \sin^2 A) - \sin^2 A$

$\equiv 1 - 2\sin^2 A \equiv$ R.H.S.

(f) R.H.S. $\equiv \sec^2 \theta \,\text{cosec}^2 \theta$

$\equiv \sec^2 \theta (1 + \cot^2 \theta)$

$$\equiv \sec^2 \theta + \frac{1}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\equiv \sec^2 \theta + \frac{1}{\sin^2 \theta}$$

$\equiv \sec^2 \theta + \text{cosec}^2 \theta \equiv$ L.H.S.

(g) L.H.S. $\equiv \text{cosec} A \sec^2 A$

$$\equiv \operatorname{cosec} A\,(1+\tan^2 A)$$
$$\equiv \operatorname{cosec} A + \frac{1}{\sin A} \times \frac{\sin^2 A}{\cos^2 A}$$
$$\equiv \operatorname{cosec} A + \frac{\sin A}{\cos^2 A}$$
$$\equiv \operatorname{cosec} A + \frac{\sin A}{\cos A} \times \frac{1}{\cos A}$$
$$\equiv \operatorname{cosec} A + \tan A \sec A \equiv \text{R.H.S.}$$

(h) L.H.S. $\equiv (\sec\theta - \sin\theta)(\sec\theta + \sin\theta)$
$$\equiv \sec^2\theta - \sin^2\theta$$
$$\equiv (1+\tan^2\theta) - (1-\cos^2\theta)$$
$$\equiv 1+\tan^2\theta - 1 + \cos^2\theta$$
$$\equiv \tan^2\theta + \cos^2\theta \equiv \text{R.H.S.}$$

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Exercise D, Question 7

Question:

Given that $3\ \tan^2\ \theta + 4\ \sec^2\ \theta = 5$, and that θ is obtuse, find the exact value of $\sin \theta$.

Solution:

$3\ \tan^2\ \theta + 4\ \sec^2\ \theta = 5$

$\Rightarrow\quad 3\ \tan^2\ \theta + 4\left(1 + \tan^2\ \theta\right) = 5$

$\Rightarrow\quad 3\ \tan^2\ \theta + 4 + 4\ \tan^2\ \theta = 5$

$\Rightarrow\quad 7\ \tan^2\ \theta = 1$

$\Rightarrow\quad \tan^2\ \theta = \frac{1}{7}$

$\Rightarrow\quad \cot^2\ \theta = 7$

$\Rightarrow\quad \text{cosec}^2\ \theta - 1 = 7$

$\Rightarrow\quad \text{cosec}^2\ \theta = 8$

$\Rightarrow\quad \sin^2\ \theta = \frac{1}{8}$

As θ is obtuse (2nd quadrant), so $\sin \theta$ is +ve.

So $\sin \theta = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

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Exercise D, Question 8

Question:

Giving answers to 3 significant figures where necessary, solve the following equations in the given intervals:

(a) $\sec^2\theta = 3\tan\theta, 0 \le \theta \le 360°$

(b) $\tan^2\theta - 2\sec\theta + 1 = 0, -\pi \le \theta \le \pi$

(c) $\text{cosec}^2\theta + 1 = 3\cot\theta, -180° \le \theta \le 180°$

(d) $\cot\theta = 1 - \text{cosec}^2\theta, 0 \le \theta \le 2\pi$

(e) $3\sec\frac{1}{2}\theta = 2\tan^2\frac{1}{2}\theta, 0 \le \theta \le 360°$

(f) $(\sec\theta - \cos\theta)^2 = \tan\theta - \sin^2\theta, 0 \le \theta \le \pi$

(g) $\tan^2 2\theta = \sec 2\theta - 1, 0 \le \theta \le 180°$

(h) $\sec^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 1, 0 \le \theta \le 2\pi$

Solution:

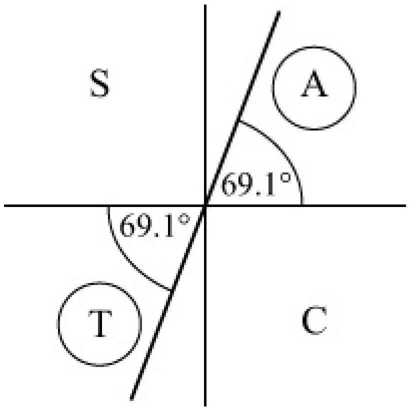
(a) $\sec^2\theta = 3\tan\theta \quad 0 \le \theta \le 360°$

$\Rightarrow \quad 1 + \tan^2\theta = 3\tan\theta$

$\Rightarrow \quad \tan^2\theta - 3\tan\theta + 1 = 0$

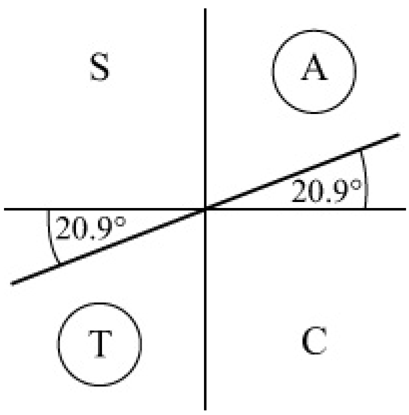
$\tan\theta = \frac{3 \pm \sqrt{5}}{2}$ (equation does not factorise).

For $\tan\theta = \frac{3 + \sqrt{5}}{2}$, calculator value is 69.1°



Solutions are 69.1°, 249°

For $\tan\theta = \frac{3-\sqrt{5}}{2}$, calculator value is 20.9°



Solutions are 20.9°, 201°

Set of solutions: 20.9°, 69.1°, 201°, 249° (3 s.f.)

(b) $\tan^2\theta - 2\sec\theta + 1 = 0 \quad -\pi \le \theta \le \pi$

$\Rightarrow \quad (\sec^2\theta - 1) - 2\sec\theta + 1 = 0$

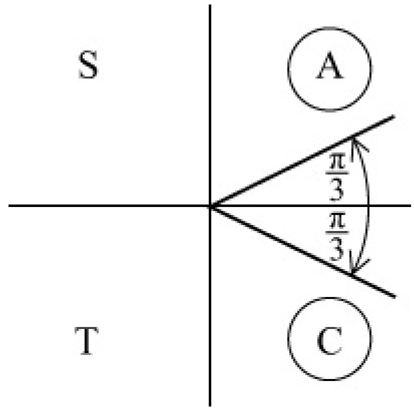
$\Rightarrow \quad \sec^2\theta - 2\sec\theta = 0$

$\Rightarrow \quad \sec\theta(\sec\theta - 2) = 0$

$\Rightarrow \quad \sec\theta = 2$ (as $\sec\theta$ cannot be 0)

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = -\frac{\pi}{3}, \frac{\pi}{3}$



(c) $\text{cosec}^2\theta + 1 = 3\cot\theta \quad -180° \le \theta \le 180°$

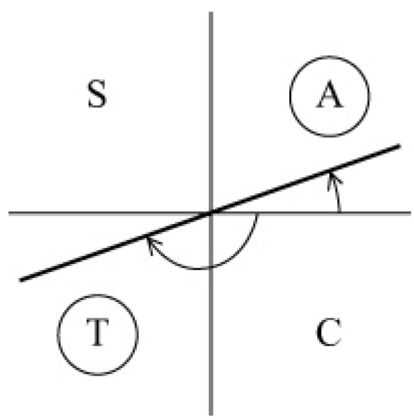
$\Rightarrow (1 + \cot^2\theta) + 1 = 3\cot\theta$

$\Rightarrow \cot^2\theta - 3\cot\theta + 2 = 0$

$\Rightarrow (\cot\theta - 1)(\cot\theta - 2) = 0$

$\Rightarrow \cot\theta = 1$ or $\cot\theta = 2$

$\Rightarrow \tan\theta = 1$ or $\tan\theta = \frac{1}{2}$



$\tan\theta = 1 \Rightarrow \theta = -135°, 45°$

$\tan\theta = \frac{1}{2} \Rightarrow \theta = -153°, 26.6°$

(d) $\cot\theta = 1 - \text{cosec}^2\theta \quad 0 \le \theta \le 2\pi$

$\Rightarrow \cot\theta = 1 - (1 + \cot^2\theta)$

$\Rightarrow \cot\theta = -\cot^2\theta$

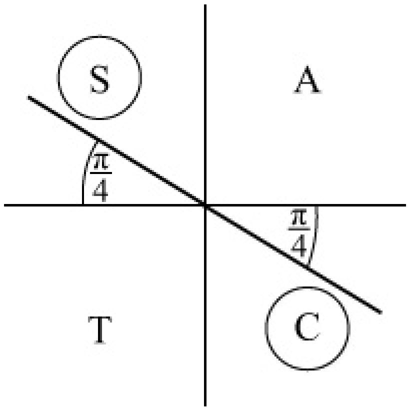
$\Rightarrow \cot^2\theta + \cot\theta = 0$

$\Rightarrow \cot\theta(\cot\theta + 1) = 0$

$\Rightarrow \cot\theta = 0$ or $\cot\theta = -1$

For $\cot\theta = 0$ refer to graph: $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

For $\cot\theta = -1$, $\tan\theta = -1$



So $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

Set of solutions: $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

(e) $3\sec\frac{1}{2}\theta = 2\tan^2\frac{1}{2}\theta \quad 0 \le \theta \le 360°$

$\Rightarrow \quad 3\sec\frac{1}{2}\theta = 2\left(\sec^2\frac{1}{2}\theta - 1\right)$ (use $1 + \tan^2 A \equiv \sec^2 A$ with $A = \frac{1}{2}\theta$)

$\Rightarrow \quad 2\sec^2\frac{1}{2}\theta - 3\sec\frac{1}{2}\theta - 2 = 0$

$\Rightarrow \quad \left(2\sec\frac{1}{2}\theta + 1\right)\left(\sec\frac{1}{2}\theta - 2\right) = 0$

$\Rightarrow \quad \sec\frac{1}{2}\theta = -\frac{1}{2}$ or $\sec\frac{1}{2}\theta = 2$

Only $\sec\frac{1}{2}\theta = 2$ applies as $\sec A \le -1$ or $\sec A \ge 1$

$\Rightarrow \quad \cos\frac{1}{2}\theta = \frac{1}{2}$

As $0 \le \theta \le 360°$

so $0 \le \frac{1}{2}\theta \le 180°$

Calculator value is 60°

This is the only value in the interval.

So $\frac{1}{2}\theta = 60°$

$\Rightarrow \quad \theta = 120°$

(f) $(\sec\theta - \cos\theta)^2 = \tan\theta - \sin^2\theta \quad 0 \le \theta \le \pi$

$\Rightarrow \quad \sec^2\theta - 2\sec\theta\cos\theta + \cos^2\theta = \tan\theta - \sin^2\theta$

$\Rightarrow \quad \sec^2\theta - 2 + \cos^2\theta = \tan\theta - \sin^2\theta \quad \left(\sec\theta\cos\theta = \frac{1}{\cos\theta} \times \cos\theta = 1\right)$

$\Rightarrow \quad (1 + \tan^2\theta) - 2 + (\cos^2\theta + \sin^2\theta) = \tan\theta$

$\Rightarrow \quad 1 + \tan^2\theta - 2 + 1 = \tan\theta$

$\Rightarrow \quad \tan^2\theta - \tan\theta = 0$

$\Rightarrow \quad \tan\theta(\tan\theta - 1) = 0$

$\Rightarrow \quad \tan\theta = 0$ or $\tan\theta = 1$

$\tan\theta = 0 \quad \Rightarrow \quad \theta = 0, \pi$

$\tan\theta = 1 \quad \Rightarrow \quad \theta = \frac{\pi}{4}$

Set of solutions: $0, \frac{\pi}{4}, \pi$

(g) $\tan^2 2\theta = \sec 2\theta - 1 \quad 0 \le \theta \le 180°$

$\Rightarrow \quad \sec^2 2\theta - 1 = \sec 2\theta - 1$

$\Rightarrow \quad \sec^2 2\theta - \sec 2\theta = 0$

$\Rightarrow \quad \sec 2\theta(\sec 2\theta - 1) = 0$

$\Rightarrow \quad \sec 2\theta = 0$ (not possible) or $\sec 2\theta = 1$

$\Rightarrow \quad \cos 2\theta = 1 \quad 0 \le 2\theta \le 360°$

Refer to graph of $y = \cos\theta$

$\Rightarrow \quad 2\theta = 0°, 360°$

$\Rightarrow \quad \theta = 0°, 180°$

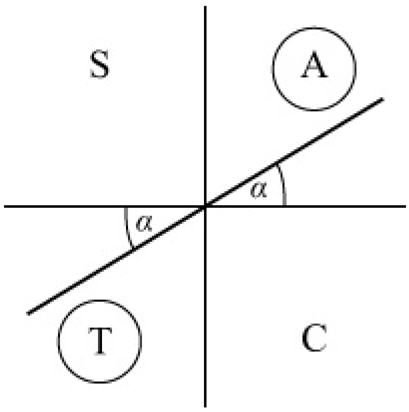
(h) $\sec^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 1 \quad 0 \le \theta \le 2\pi$

$\Rightarrow \quad (1 + \tan^2\theta) - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 1$

$\Rightarrow \quad \tan^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 0$

$\Rightarrow \quad (\tan\theta - \sqrt{3})(\tan\theta - 1) = 0$

$\Rightarrow \quad \tan\theta = \sqrt{3}$ or $\tan\theta = 1$



First answer (α) for $\tan\theta = \sqrt{3}$ is $\frac{\pi}{3}$

Second solution is $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

First answer for $\tan\theta = 1$ is $\frac{\pi}{4}$

Second solution is $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Set of solutions: $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$

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Exercise D, Question 9

Question:

Given that $\tan^2 k = 2\sec k$,

(a) find the value of $\sec k$.

(b) deduce that $\cos k = \sqrt{2} - 1$

(c) hence solve, in the interval $0 \leq k \leq 360°$, $\tan^2 k = 2\sec k$, giving your answers to 1 decimal place.

Solution:

(a) $\tan^2 k = 2\sec k$

$\Rightarrow \quad (\sec^2 k - 1) = 2\sec k$

$\Rightarrow \quad \sec^2 k - 2\sec k - 1 = 0$

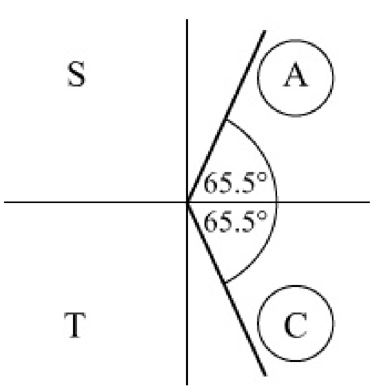
$\Rightarrow \quad \sec k = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

As sec k has no values between -1 and 1
$\sec k = 1 + \sqrt{2}$

(b) $\cos k = \frac{1}{1 + \sqrt{2}} = \frac{\sqrt{2} - 1}{(1 + \sqrt{2})(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$

(c) Solutions of $\tan^2 k = 2\sec k$, $0 \leq k \leq 360°$
are solutions of $\cos k = \sqrt{2} - 1$
Calculator solution is 65.5°

$\Rightarrow \quad k = 65.5°, 360° - 65.5° = 65.5°, 294.5°$ (1 d.p.)



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Exercise D, Question 10

Question:

Given that $a = 4 \sec x$, $b = \cos x$ and $c = \cot x$,

(a) express b in terms of a

(b) show that $c^2 = \frac{16}{a^2 - 16}$

Solution:

(a) As $a = 4 \sec x$

$\Rightarrow \quad \sec x = \frac{a}{4}$

$\Rightarrow \quad \cos x = \frac{4}{a}$

As $\cos x = b$

$\Rightarrow \quad b = \frac{4}{a}$

(b) $c = \cot x$

$\Rightarrow \quad c^2 = \cot^2 x$

$\Rightarrow \quad \frac{1}{c^2} = \tan^2 x$

$\Rightarrow \quad \frac{1}{c^2} = \sec^2 x - 1 \qquad (\text{use } 1 + \tan^2 x \equiv \sec^2 x)$

$\Rightarrow \quad \frac{1}{c^2} = \frac{a^2}{16} - 1 \qquad \left(\sec x = \frac{a}{4} \right)$

$\Rightarrow \quad 16 = a^2c^2 - 16c^2 \qquad (\text{multiply by } 16c^2)$

$\Rightarrow \quad c^2 (a^2 - 16) = 16$

$\Rightarrow \quad c^2 = \frac{16}{a^2 - 16}$

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Exercise D, Question 11

Question:

Given that $x = \sec\theta + \tan\theta$,

(a) show that $\frac{1}{x} = \sec\theta - \tan\theta$.

(b) Hence express $x^2 + \frac{1}{x^2} + 2$ in terms of θ, in its simplest form.

Solution:

(a) $x = \sec\theta + \tan\theta$

$$\frac{1}{x} = \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta - \tan\theta}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$= \sec\theta - \tan\theta$ (as $1 + \tan^2\theta \equiv \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta \equiv 1$)

(b) $x + \frac{1}{x} = \sec\theta + \tan\theta + \sec\theta - \tan\theta = 2\sec\theta$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 4\sec^2\theta$$

$$\Rightarrow x^2 + 2x \times \frac{1}{x} + \frac{1}{x^2} = 4\sec^2\theta$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\sec^2\theta$$

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Exercise D, Question 12

Question:

Given that $2\sec^2\theta - \tan^2\theta = p$ show that $\operatorname{cosec}^2\theta = \frac{p-1}{p-2}$, $p \neq 2$.

Solution:

$2\sec^2\theta - \tan^2\theta = p$

$\Rightarrow\quad 2(1+\tan^2\theta) - \tan^2\theta = p$

$\Rightarrow\quad 2 + 2\tan^2\theta - \tan^2\theta = p$

$\Rightarrow\quad \tan^2\theta = p - 2$

$\Rightarrow\quad \cot^2\theta = \frac{1}{p-2} \qquad \left(\cot\theta = \frac{1}{\tan\theta}\right)$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{1}{p-2} = \frac{(p-2)+1}{p-2} = \frac{p-1}{p-2}$

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Exercise E, Question 1

Question:

Without using a calculator, work out, giving your answer in terms of π, the value of:

(a) $\arccos 0$

(b) $\arcsin(1)$

(c) $\arctan(-1)$

(d) $\arcsin\left(-\frac{1}{2}\right)$

(e) $\arccos\left(-\frac{1}{\sqrt{2}}\right)$

(f) $\arctan\left(-\frac{1}{\sqrt{3}}\right)$

(g) $\arcsin\left(\sin\frac{\pi}{3}\right)$

(h) $\arcsin\left(\sin\frac{2\pi}{3}\right)$

Solution:

(a) $\arccos 0$ is the angle α in $0 \leq \alpha \leq \pi$ for which $\cos\alpha = 0$

Refer to graph of $y = \cos\theta \Rightarrow \alpha = \frac{\pi}{2}$

So $\arccos 0 = \frac{\pi}{2}$

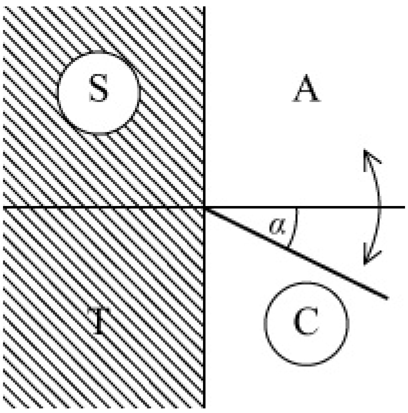
(b) $\arcsin 1$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which $\sin\alpha = 1$

Refer to graph of $y = \sin\theta \Rightarrow \alpha = \frac{\pi}{2}$

So $\arcsin 1 = \frac{\pi}{2}$

(c) $\arctan(-1)$ is the angle α in $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ for which $\tan\alpha = -1$

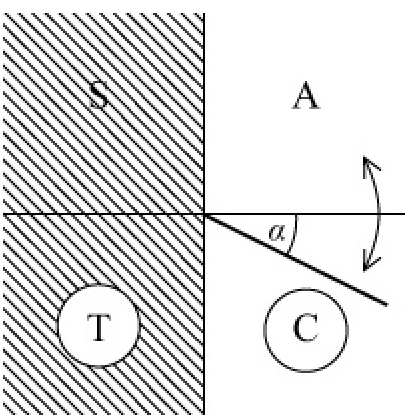
So $\arctan(-1) = -\frac{\pi}{4}$



(d) $\arcsin\left(-\frac{1}{2}\right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

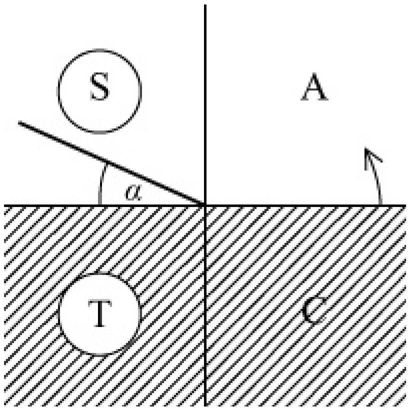
$\sin\alpha = -\frac{1}{2}$

So $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$



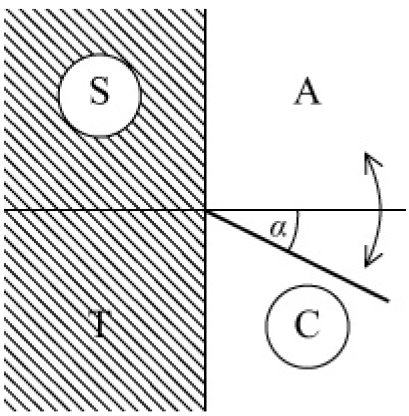
(e) $\arccos\left(-\frac{1}{\sqrt{2}}\right)$ is the angle α in $0 \leq \alpha \leq \pi$ for which $\cos\alpha = -\frac{1}{\sqrt{2}}$

So $\arccos\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$



(f) $\arctan\left(-\frac{1}{\sqrt{3}}\right)$ is the angle α in $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ for which $\tan\alpha = -\frac{1}{\sqrt{3}}$

So $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$



(g) $\arcsin\left(\sin\frac{\pi}{3}\right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

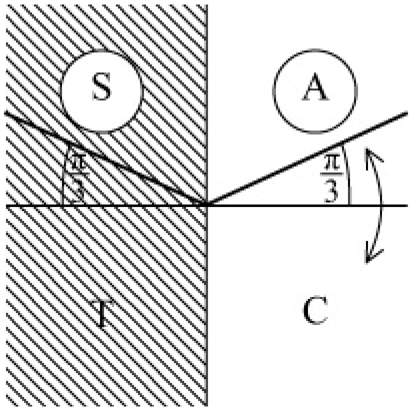
$\sin\alpha = \sin\frac{\pi}{3}$

So $\arcsin\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

(h) $\arcsin\left(\sin\frac{2\pi}{3}\right)$ is the angle α in $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ for which

$\sin\alpha = \sin\frac{2\pi}{3}$

So $\arcsin\left(\sin\frac{2\pi}{3}\right) = \frac{\pi}{3}$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Exercise E, Question 2

Question:

Find the value of:

(a) $\arcsin\left(\frac{1}{2}\right)+\arcsin\left(-\frac{1}{2}\right)$

(b) $\arccos\left(\frac{1}{2}\right)-\arccos\left(-\frac{1}{2}\right)$

(c) $\arctan(1)-\arctan(-1)$

Solution:

(a) $\arcsin\left(\frac{1}{2}\right)+\arcsin\left(-\frac{1}{2}\right)=\frac{\pi}{6}+\left(-\frac{\pi}{6}\right)=0$

(b) $\arccos\left(\frac{1}{2}\right)-\arccos\left(-\frac{1}{2}\right)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

(c) $\arctan(1)-\arctan(-1)=\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)=\frac{\pi}{2}$

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Exercise E, Question 3

Question:

Without using a calculator, work out the values of:

(a) $\sin\left(\arcsin\frac{1}{2}\right)$

(b) $\sin\left[\arcsin\left(-\frac{1}{2}\right)\right]$

(c) $\tan\left[\arctan\left(-1\right)\right]$

(d) $\cos(\arccos 0)$

Solution:

(a) $\sin\left(\arcsin\frac{1}{2}\right)$

$\arcsin\frac{1}{2}=\alpha$ where $\sin\alpha=\frac{1}{2}, -\frac{\pi}{2}\le\alpha\le\frac{\pi}{2}$

So $\arcsin\frac{1}{2}=\frac{\pi}{6}$

$\Rightarrow\quad \sin\left(\arcsin\frac{1}{2}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

(b) $\sin\left[\arcsin\left(-\frac{1}{2}\right)\right]$

$\arcsin\left(-\frac{1}{2}\right)=\alpha$ where $\sin\alpha=-\frac{1}{2}, -\frac{\pi}{2}\le\alpha\le\frac{\pi}{2}$

So $\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\Rightarrow\quad \sin\left[\arcsin\left(-\frac{1}{2}\right)\right]=\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}$

(c) $\tan\left[\arctan\left(-1\right)\right]$

$\arctan\left(-1\right)=\alpha$ where $\tan\alpha=-1, -\frac{\pi}{2}<\alpha<\frac{\pi}{2}$

So $\arctan(-1) = -\frac{\pi}{4}$

$$\Rightarrow \quad \tan[\arctan(-1)] = \tan\left(-\frac{\pi}{4}\right) = -1$$

(d) $\cos(\arccos 0)$

$\arccos 0 = \alpha$ where $\cos\alpha = 0$, $0 \leq \alpha \leq \pi$

So $\arccos 0 = \frac{\pi}{2}$

$$\Rightarrow \quad \cos(\arccos 0) = \cos\frac{\pi}{2} = 0$$

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Exercise E, Question 4

Question:

Without using a calculator, work out the exact values of:

(a) $\sin \left[\arccos \left(\frac{1}{2} \right) \right]$

(b) $\cos \left[\arcsin \left(-\frac{1}{2} \right) \right]$

(c) $\tan \left[\arccos \left(-\frac{\sqrt{2}}{2} \right) \right]$

(d) $\sec [\arctan (\sqrt{3})]$

(e) $\operatorname{cosec} [\arcsin (-1)]$

(f) $\sin \left[2 \arcsin \left(\frac{\sqrt{2}}{2} \right) \right]$

Solution:

(a) $\sin \left(\arccos \frac{1}{2} \right)$

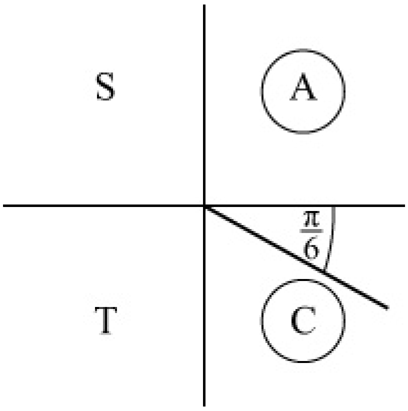
$\arccos \frac{1}{2} = \frac{\pi}{3}$

$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(b) $\cos \left[\arcsin \left(-\frac{1}{2} \right) \right]$

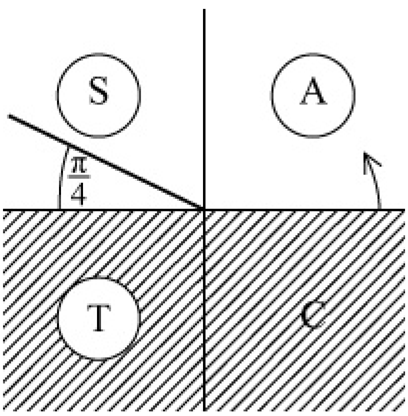
$\arcsin \left(-\frac{1}{2} \right) = -\frac{\pi}{6}$

$\cos \left(-\frac{\pi}{6} \right) = +\frac{\sqrt{3}}{2}$



(c) $\tan\left[\arccos\left(-\frac{\sqrt{2}}{2}\right)\right]$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \alpha$ where $\cos\alpha = -\frac{\sqrt{2}}{2}, 0 \le \alpha \le \pi$



So $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

$\tan\frac{3\pi}{4} = -1$

(d) $\sec(\arctan\sqrt{3})$

$\arctan\sqrt{3} = \frac{\pi}{3}$ (the angle whose tan is $\sqrt{3}$)

$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

(e) $\text{cosec}[\arcsin(-1)]$

$\arcsin(-1) = \alpha$ where $\sin\alpha = -1, -\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

So $\arcsin(-1) = -\frac{\pi}{2}$

$$\Rightarrow \quad \text{cosec}\left[\arcsin(-1)\right] = \frac{1}{\sin\left(-\frac{\pi}{2}\right)} = \frac{1}{-1} = -1$$

(f) $\sin\left[2\arcsin\left(\frac{\sqrt{2}}{2}\right)\right]$

$\arcsin\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

So $\sin\left[2\arcsin\left(\frac{\sqrt{2}}{2}\right)\right] = \sin\frac{\pi}{2} = 1$

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Exercise E, Question 5

Question:

Given that $\arcsin k = \alpha$, where $0 < k < 1$ and α is in radians, write down, in terms of α, the first two positive values of x satisfying the equation $\sin x = k$.

Solution:

As k is positive, the first two positive solutions of $\sin x = k$ are
$\arcsin k$ and $\pi - \arcsin k$
i.e. α and $\pi - \alpha$
(Try a few examples, taking specific values for k).

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Exercise E, Question 6

Question:

Given that x satisfies $\arcsin x = k$, where $0 < k < \frac{\pi}{2}$,

(a) state the range of possible values of x

(b) express, in terms of x,
(i) $\cos k$ (ii) $\tan k$

Given, instead, that $-\frac{\pi}{2} < k < 0$,

(c) how, if at all, would it affect your answers to (b)?

Solution:

(a) $\arcsin x$ is the angle α in $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$ such that $\sin \alpha = x$

In this case $x = \sin k$ where $0 < k < \frac{\pi}{2}$

As sin is an increasing function

$\sin 0 < x < \sin \frac{\pi}{2}$

i.e. $0 < x < 1$

(b) (i) $\cos k = \pm \sqrt{1 - \sin^2 k} = \pm \sqrt{1 - x^2}$
k is in the 1st quadrant $\Rightarrow \cos k > 0$
So $\cos k = \sqrt{1 - x^2}$
(ii) $\tan k = \frac{\sin k}{\cos k} = \frac{x}{\sqrt{1 - x^2}}$

(c) k is now in the 4th quadrant, where $\cos k$ is positive.
So the value of $\cos k$ remains the same and there is no change to $\tan k$.

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Exercise E, Question 7

Question:

The function f is defined as $f : x \to \arcsin x$, $-1 \leq x \leq 1$, and the function g is such that $g(x) = f(2x)$.

(a) Sketch the graph of $y = f(x)$ and state the range of f.

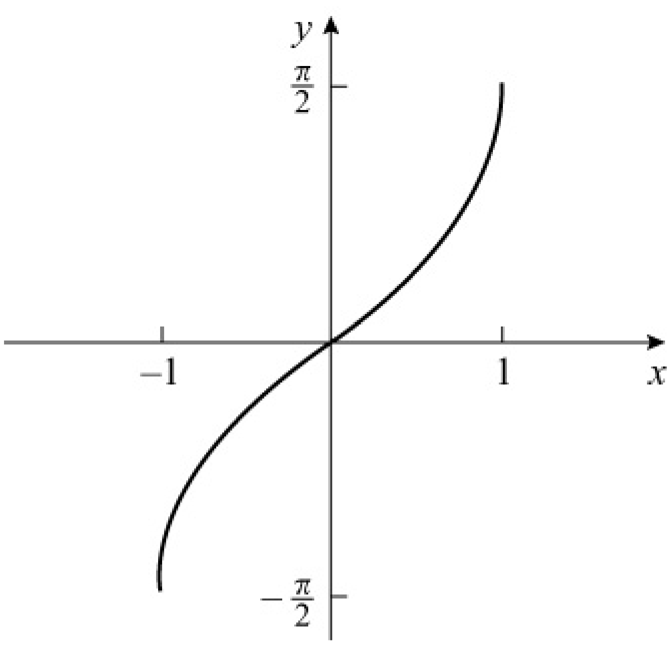
(b) Sketch the graph of $y = g(x)$.

(c) Define g in the form $g : x \to \ldots$ and give the domain of g.

(d) Define g^{-1} in the form $g^{-1} : x \to \ldots$

Solution:

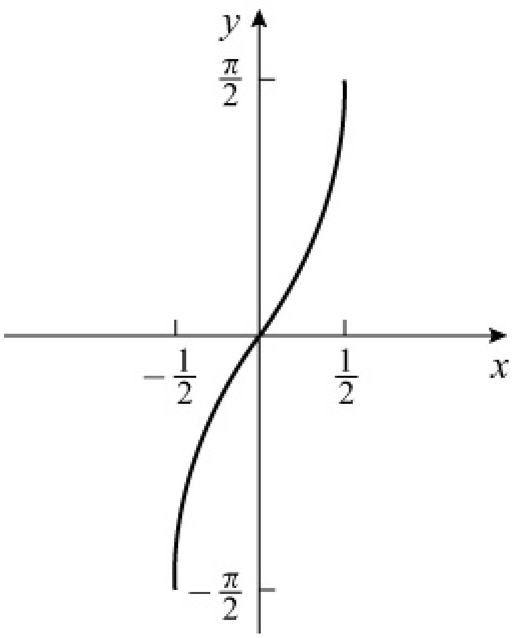
(a) $y = \arcsin x$



Range: $-\frac{\pi}{2} \leq f(x) \leq \frac{\pi}{2}$

(b) Using the transformation work, the graph of $y = f(2x)$ is the graph of $y = f(x)$ stretched in the x direction by scale factor $\frac{1}{2}$.

$y = g(x)$



(c) $g : x \to \arcsin 2x, \quad -\frac{1}{2} \leq x \leq \frac{1}{2}$

(d) Let $y = \arcsin 2x$

$\Rightarrow \quad 2x = \sin y$

$\Rightarrow \quad x = \frac{1}{2}\sin y$

So $g^{-1} : x \to \frac{1}{2}\sin x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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Exercise E, Question 8

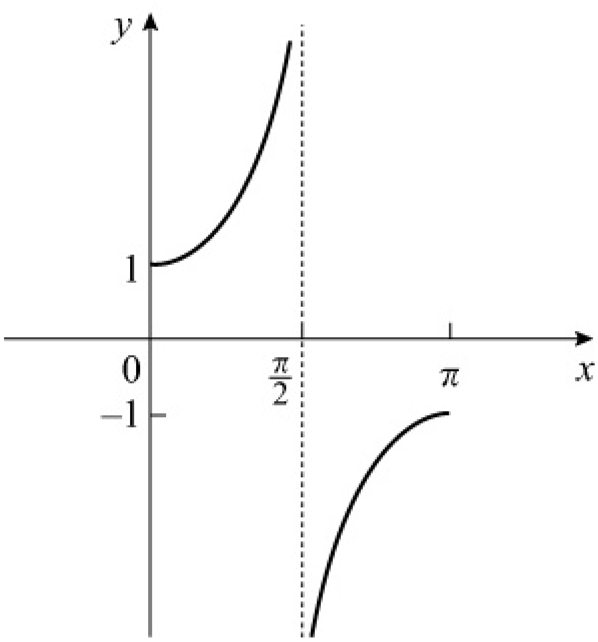
Question:

(a) Sketch the graph of $y = \sec x$, with the restricted domain $0 \leq x \leq \pi,\ x \neq \frac{\pi}{2}$.

(b) Given that arcsec x is the inverse function of $\sec x$, $0 \leq x \leq \pi,\ x \neq \frac{\pi}{2}$, sketch the graph of $y = \text{arcsec } x$ and state the range of arcsec x.

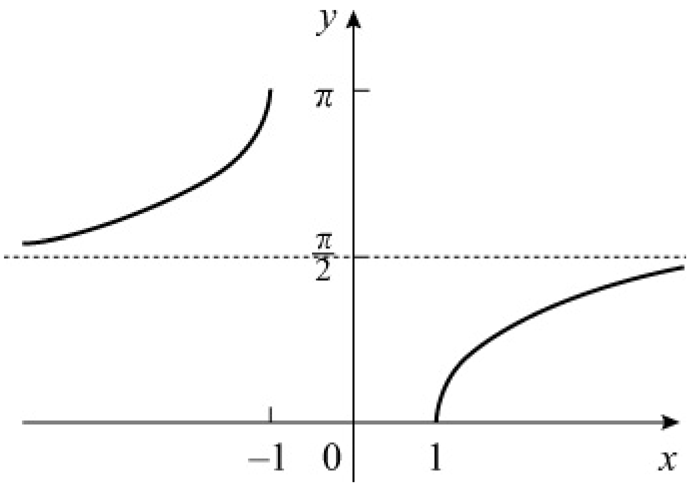
Solution:

(a) $y = \sec x$



(b) Reflect the above graph in the line $y = x$

$y = \text{arcsec } x,\ x \leq -1,\ x \geq 1$



Range: $0 \leq \text{arcsec } x \leq \pi$, $\text{arcsec } x \neq \frac{\pi}{2}$

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Exercise F, Question 1

Question:

Solve $\tan x = 2\cot x$, in the interval $-180° \leq x \leq 90°$. Give any non-exact answers to 1 decimal place.

Solution:

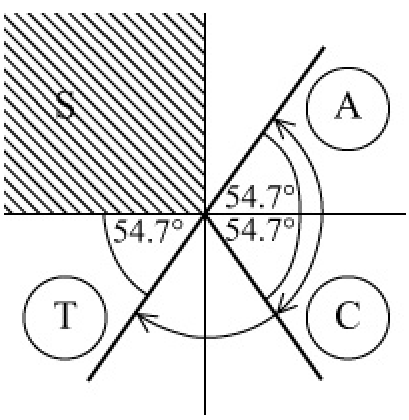
$\tan x = 2\cot x$, $-180° \leq x \leq 90°$

$\Rightarrow \quad \tan x = \frac{2}{\tan x}$

$\Rightarrow \quad \tan^2 x = 2$

$\Rightarrow \quad \tan x = \pm\sqrt{2}$

Calculator value for $\tan x = +\sqrt{2}$ is 54.7°



Solutions are required in the 1st, 3rd and 4th quadrants.
Solution set: $-125.3°$, $-54.7°$, $+54.7°$

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Exercise F, Question 2

Question:

Given that $p = 2\sec\theta$ and $q = 4\cos\theta$, express p in terms of q.

Solution:

$$p = 2\sec\theta \quad \Rightarrow \quad \sec\theta = \frac{p}{2}$$

$$q = 4\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{q}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} \quad \Rightarrow \quad \frac{p}{2} = \frac{1}{\frac{q}{4}} = \frac{4}{q} \quad \Rightarrow \quad p = \frac{8}{q}$$

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Exercise F, Question 3

Question:

Given that $p = \sin\theta$ and $q = 4\ \cot\theta$, show that $p^2q^2 = 16\ (\ 1 - p^2\ )$.

Solution:

$p = \sin\theta \quad \Rightarrow \quad \frac{1}{p} = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

$q = 4\ \cot\theta \quad \Rightarrow \quad \cot\theta = \frac{q}{4}$

Using $1 + \cot^2\ \theta \equiv \operatorname{cosec}^2\ \theta$

$\Rightarrow \quad 1 + \frac{q^2}{16} = \frac{1}{p^2}$ (multiply by $16p^2$)

$\Rightarrow \quad 16p^2 + p^2q^2 = 16$

$\Rightarrow \quad p^2q^2 = 16 - 16p^2 = 16\ (\ 1 - p^2\ )$

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Exercise F, Question 4

Question:

Give any non-exact answers to 1 decimal place.

(a) Solve, in the interval $0 < \theta < 180°$,
(i) $\operatorname{cosec}\theta = 2\cot\theta$
(ii) $2\cot^2\theta = 7\operatorname{cosec}\theta - 8$

(b) Solve, in the interval $0 \leq \theta \leq 360°$,
(i) $\sec(2\theta - 15°) = \operatorname{cosec}135°$
(ii) $\sec^2\theta + \tan\theta = 3$

(c) Solve, in the interval $0 \leq x \leq 2\pi$,
(i) $\operatorname{cosec}\left(x + \frac{\pi}{15}\right) = -\sqrt{2}$
(ii) $\sec^2 x = \frac{4}{3}$

Solution:

(a) (i) $\operatorname{cosec}\theta = 2\cot\theta$, $0 < \theta < 180°$

$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{2\cos\theta}{\sin\theta}$

$\Rightarrow \quad 2\cos\theta = 1$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

(ii) $2\cot^2\theta = 7\operatorname{cosec}\theta - 8$, $0 < \theta < 180°$

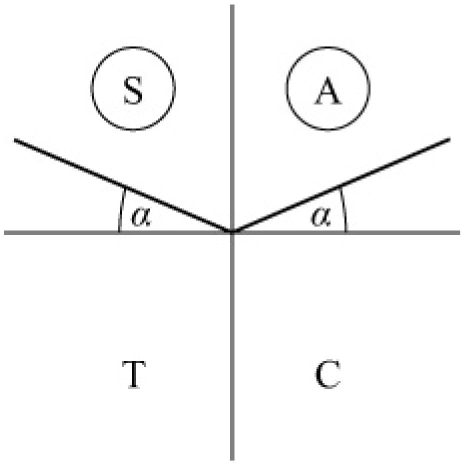
$\Rightarrow \quad 2(\operatorname{cosec}^2\theta - 1) = 7\operatorname{cosec}\theta - 8$

$\Rightarrow \quad 2\operatorname{cosec}^2\theta - 7\operatorname{cosec}\theta + 6 = 0$

$\Rightarrow \quad (2\operatorname{cosec}\theta - 3)(\operatorname{cosec}\theta - 2) = 0$

$\Rightarrow \quad \operatorname{cosec}\theta = \frac{3}{2}$ or $\operatorname{cosec}\theta = 2$

So $\sin\theta = \frac{2}{3}$ or $\sin\theta = \frac{1}{2}$



Solutions are $\alpha°$ and $(180 - \alpha)°$ where α is the calculator value.
Solutions set: 41.8°, 138.2°, 30°, 150°
i.e. 30°, 41.8°, 138.2°, 150°

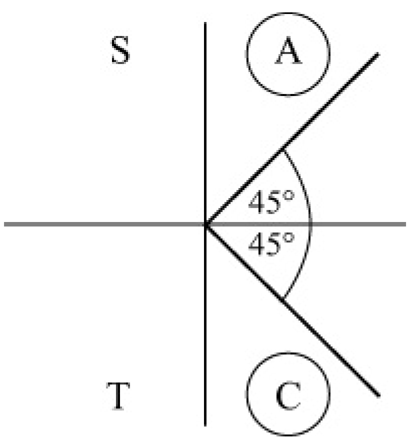
(b) (i) $\sec(2\theta - 15°) = \text{cosec}\,135°$, $0 \leq \theta \leq 360°$

$\Rightarrow \quad \cos(2\theta - 15°) = \dfrac{1}{\text{cosec}\,135°} = \sin 135° = \dfrac{\sqrt{2}}{2}$

Solve $\cos(2\theta - 15°) = \dfrac{\sqrt{2}}{2}$, $-15° \leq 2\theta - 15° \leq 705°$

The calculator value is $\cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right) = 45°$

cos is positive, so $(2\theta - 15°)$ is in the 1st and 4th quadrants.



So $(2\theta - 15°) = 45°, 315°, 405°, 675°$

$\Rightarrow \quad 2\theta = 60°, 330°, 420°, 690°$

$\Rightarrow \quad \theta = 30°, 165°, 210°, 345°$

(ii) $\sec^2\theta + \tan\theta = 3$, $0 \leq \theta \leq 360°$

$\Rightarrow \quad 1 + \tan^2\theta + \tan\theta = 3$

$\Rightarrow \quad \tan^2\theta + \tan\theta - 2 = 0$

$\Rightarrow \quad (\tan\theta - 1)(\tan\theta + 2) = 0$

$\Rightarrow \quad \tan\theta = 1$ or $\tan\theta = -2$

$\tan\theta = 1 \quad \Rightarrow \quad \theta = 45°, 180° + 45°$, i.e. 45°, 225°

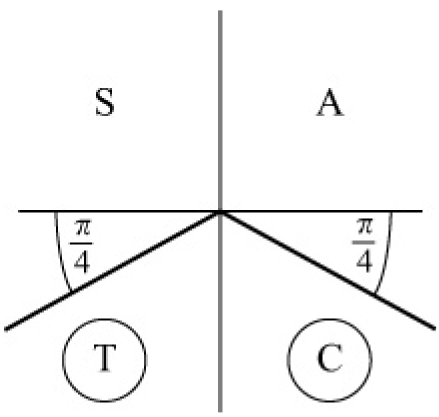
$\tan\theta = -2 \quad \Rightarrow \quad \theta = 180° + (-63.4)°, 360° + (-63.4°)$, i.e. 116.6°, 296.6°

(c) (i) $\text{cosec}\left(x + \frac{\pi}{15}\right) = -\sqrt{2}, 0 \leq x \leq 2\pi$

$\Rightarrow \quad \sin\left(x + \frac{\pi}{15}\right) = -\frac{1}{\sqrt{2}}$

Calculator value is $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

$\sin\left(x + \frac{\pi}{15}\right)$ is negative, so $x + \frac{\pi}{15}$ is in 3rd and 4th quadrants.



So $x + \frac{\pi}{15} = \frac{5\pi}{4}, \frac{7\pi}{4}$

$\Rightarrow \quad x = \frac{5\pi}{4} - \frac{\pi}{15}, \frac{7\pi}{4} - \frac{\pi}{15} = \frac{75\pi - 4\pi}{60}, \frac{105\pi - 4\pi}{60} = \frac{71\pi}{60}, \frac{101\pi}{60}$

(ii) $\sec^2 x = \frac{4}{3}, 0 \leq x \leq 2\pi$

$\Rightarrow \quad \cos^2 x = \frac{3}{4}$

$\Rightarrow \quad \cos x = \pm\frac{\sqrt{3}}{2}$

Calculator value for $\cos x = +\frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$

As $\cos x$ is $\pm$, x is in all four quadrants.

Solutions set: $x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

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Exercise F, Question 5

Question:

Given that $5 \sin x \cos y + 4 \cos x \sin y = 0$, and that $\cot x = 2$, find the value of $\cot y$.

Solution:

$5 \sin x \cos y + 4 \cos x \sin y = 0$

$$\Rightarrow \frac{5 \sin x \cos y}{\sin x \sin y} + \frac{4 \cos x \sin y}{\sin x \sin y} = 0 \qquad \text{(divide by } \sin x \sin y\text{)}$$

$$\Rightarrow \frac{5 \cos y}{\sin y} + \frac{4 \cos x}{\sin x} = 0$$

So $5 \cot y + 4 \cot x = 0$

As $\cot x = 2$

$5 \cot y + 8 = 0$

$5 \cot y = -8$

$\cot y = -\frac{8}{5}$

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Exercise F, Question 6

Question:

Show that:

(a) $(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) \equiv \sec\theta + \operatorname{cosec}\theta$

(b) $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$

(c) $(1 - \sin x)(1 + \operatorname{cosec} x) \equiv \cos x \cot x$

(d) $\dfrac{\cot x}{\operatorname{cosec} x - 1} - \dfrac{\cos x}{1 + \sin x} \equiv 2\tan x$

(e) $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} \equiv 2\sec\theta\tan\theta$

(f) $\dfrac{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{1 + \tan^2\theta} \equiv \cos^2\theta$

Solution:

(a) L.H.S. $\equiv (\tan\theta + \cot\theta)(\sin\theta + \cos\theta)$

$$\equiv \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\sin\theta + \cos\theta)$$

$$\equiv \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)(\sin\theta + \cos\theta)$$

$$\equiv \left(\frac{1}{\cos\theta\sin\theta}\right)(\sin\theta + \cos\theta)$$

$$\equiv \frac{\sin\theta}{\cos\theta\sin\theta} + \frac{\cos\theta}{\cos\theta\sin\theta}$$

$$\equiv \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

$\equiv \sec\theta + \operatorname{cosec}\theta \equiv$ R.H.S.

(b)

$$\begin{aligned}
\text{L.H.S.} &\equiv \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \\
&\equiv \frac{\frac{1}{\sin x}}{\frac{1}{\sin x} - \sin x} \\
&\equiv \frac{\frac{1}{\sin x}}{\frac{1-\sin^2 x}{\sin x}} \\
&\equiv \frac{1}{\sin x} \times \frac{\sin x}{1-\sin^2 x} \\
&\equiv \frac{1}{1-\sin^2 x} \\
&\equiv \frac{1}{\cos^2 x} \quad (\text{using } \sin^2 x + \cos^2 x \equiv 1) \\
&\equiv \sec^2 x \equiv \text{R.H.S.}
\end{aligned}$$

(c)
$$\begin{aligned}
\text{L.H.S.} &\equiv (1 - \sin x)(1 + \operatorname{cosec} x) \\
&\equiv 1 - \sin x + \operatorname{cosec} x - \sin x \operatorname{cosec} x \\
&\equiv 1 - \sin x + \operatorname{cosec} x - 1 \qquad \left(\text{as } \operatorname{cosec} x = \frac{1}{\sin x}\right) \\
&\equiv \operatorname{cosec} x - \sin x \\
&\equiv \frac{1}{\sin x} - \sin x \\
&\equiv \frac{1 - \sin^2 x}{\sin x} \\
&\equiv \frac{\cos^2 x}{\sin x} \\
&\equiv \frac{\cos x}{\sin x} \times \cos x \\
&\equiv \cos x \cot x \equiv \text{R.H.S.}
\end{aligned}$$

(d)

$$\text{L.H.S.} \equiv \frac{\cot x}{\operatorname{cosec} x - 1} - \frac{\cos x}{1+\sin x}$$

$$\equiv \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - 1} - \frac{\cos x}{1+\sin x}$$

$$\equiv \frac{\frac{\cos x}{\sin x}}{\frac{1-\sin x}{\sin x}} - \frac{\cos x}{1+\sin x}$$

$$\equiv \frac{\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x}$$

$$\equiv \frac{\cos x(1+\sin x) - \cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\equiv \frac{2\cos x \sin x}{1-\sin^2 x}$$

$$\equiv \frac{2\cos x \sin x}{\cos^2 x}$$

$$\equiv 2\frac{\sin x}{\cos x}$$

$$\equiv 2\tan x \equiv \text{R.H.S.}$$

(e) $$\text{L.H.S.} \equiv \frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}$$

$$\equiv \frac{(\operatorname{cosec}\theta + 1) + (\operatorname{cosec}\theta - 1)}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$$

$$\equiv \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$$

$$\equiv \frac{2\operatorname{cosec}\theta}{\cot^2\theta} \qquad (1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta)$$

$$\equiv \frac{2}{\sin\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$\equiv 2 \times \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$$

$$\equiv 2\sec\theta\tan\theta \equiv \text{R.H.S.}$$

(f) $$\text{L.H.S.} \equiv \frac{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{1 + \tan^2\theta}$$

$$\equiv \frac{\sec^2\theta - \tan^2\theta}{\sec^2\theta}$$

$$\equiv \frac{(1 + \tan^2\theta) - \tan^2\theta}{\sec^2\theta}$$

$$\equiv \frac{1}{\sec^2\theta}$$

$$\equiv \cos^2\theta \equiv \text{R.H.S.}$$

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Exercise F, Question 7

Question:

(a) Show that $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}\equiv 2\operatorname{cosec} x$.

(b) Hence solve, in the interval $-2\pi \le x \le 2\pi$, $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=-\frac{4}{\sqrt{3}}$.

Solution:

(a) L.H.S. $\equiv \frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}$

$$\equiv \frac{\sin^2 x+(1+\cos x)^2}{(1+\cos x)\sin x}$$

$$\equiv \frac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$$

$$\equiv \frac{2+2\cos x}{(1+\cos x)\sin x} \quad (\sin^2 x+\cos^2 x\equiv 1)$$

$$\equiv \frac{2(1+\cos x)}{(1+\cos x)\sin x}$$

$$\equiv \frac{2}{\sin x}$$

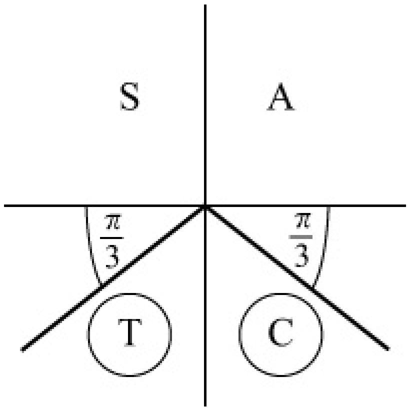
$$\equiv 2\operatorname{cosec} x \equiv \text{R.H.S.}$$

(b) Solve $2\operatorname{cosec} x=-\frac{4}{\sqrt{3}}$, $-2\pi \le x \le 2\pi$

$\Rightarrow \quad \operatorname{cosec} x=-\frac{2}{\sqrt{3}}$

$\Rightarrow \quad \sin x=-\frac{\sqrt{3}}{2}$

Calculator value is $-\frac{\pi}{3}$



Solutions in $-2\pi \leq x \leq 2\pi$ are

$-\frac{\pi}{3}, \quad -\pi + \frac{\pi}{3}, \quad \pi + \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3},$

i.e. $-\frac{\pi}{3}, \quad -\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

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Exercise F, Question 8

Question:

Prove that $\frac{1+\cos\theta}{1-\cos\theta} \equiv (\operatorname{cosec}\theta + \cot\theta)^2$.

Solution:

$$\text{R.H.S.} \equiv (\operatorname{cosec}\theta + \cot\theta)^2$$
$$\equiv \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$
$$\equiv \frac{(1+\cos\theta)^2}{\sin^2\theta}$$
$$\equiv \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$
$$\equiv \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$
$$\equiv \frac{1+\cos\theta}{1-\cos\theta} \equiv \text{L.H.S.}$$

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Exercise F, Question 9

Question:

Given that $\sec A = -3$, where $\frac{\pi}{2} < A < \pi$,

(a) calculate the exact value of $\tan A$.

(b) Show that $\operatorname{cosec} A = \frac{3\sqrt{2}}{4}$.

Solution:

(a) $\sec A = -3$, $\frac{\pi}{2} < A < \pi$, i.e. A is in 2nd quadrant.

As $1 + \tan^2 A = \sec^2 A$
$1 + \tan^2 A = 9$
$\tan^2 A = 8$
$\tan A = \pm\sqrt{8} = \pm 2\sqrt{2}$
As A is in 2nd quadrant, $\tan A$ is $-$ ve.
So $\tan A = -2\sqrt{2}$

(b) $\sec A = -3$, so $\cos A = -\frac{1}{3}$

As $\tan A = \frac{\sin A}{\cos A}$

$\sin A = \cos A \times \tan A = -\frac{1}{3} \times -2\sqrt{2} = \frac{2\sqrt{2}}{3}$

So $\operatorname{cosec} A = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{2 \times 2} = \frac{3\sqrt{2}}{4}$

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Exercise F, Question 10

Question:

Given that $\sec\theta = k$, $|k| \geq 1$, and that θ is obtuse, express in terms of k:

(a) $\cos\theta$

(b) $\tan^2\theta$

(c) $\cot\theta$

(d) $\operatorname{cosec}\theta$

Solution:

$\sec\theta = k$, $|k| \geq |$
θ is in the 2nd quadrant $\Rightarrow$ k is negative

(a) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{k}$

(b) Using $1 + \tan^2\theta = \sec^2\theta$
$\tan^2\theta = k^2 - 1$

(c) $\tan\theta = \pm\sqrt{k^2 - 1}$
In the 2nd quadrant, $\tan\theta$ is $-$ ve.
So $\tan\theta = -\sqrt{k^2 - 1}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\sqrt{k^2 - 1}} = -\frac{1}{\sqrt{k^2 - 1}}$

(d) Using $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta = 1 + \frac{1}{k^2 - 1} = \frac{k^2 - 1 + 1}{k^2 - 1} = \frac{k^2}{k^2 - 1}$

So $\operatorname{cosec}\theta = \pm\frac{k}{\sqrt{k^2 - 1}}$
In the 2nd quadrant, $\operatorname{cosec}\theta$ is +ve.

As k is $-$ ve, $\operatorname{cosec}\theta = \frac{-k}{\sqrt{k^2 - 1}}$

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Exercise F, Question 11

Question:

Solve, in the interval $0 \le x \le 2\pi$, the equation $\sec\left(x + \frac{\pi}{4}\right) = 2$, giving your answers in terms of π.

Solution:

$\sec\left(x + \frac{\pi}{4}\right) = 2,\ 0 \le x \le 2\pi$

$\Rightarrow \quad \cos\left(x + \frac{\pi}{4}\right) = \frac{1}{2},\ 0 \le x \le 2\pi$

$\Rightarrow \quad x + \frac{\pi}{4} = \cos^{-1}\frac{1}{2},\ 2\pi - \cos^{-1}\frac{1}{2} = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$

So $x = \frac{\pi}{3} - \frac{\pi}{4},\ \frac{5\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12},\ \frac{20\pi - 3\pi}{12} = \frac{\pi}{12},\ \frac{17\pi}{12}$

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Exercise F, Question 12

Question:

Find, in terms of π, the value of $\arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right)$.

Solution:

$\arcsin\left(\frac{1}{2}\right)$ is the angle in the interval $-\frac{\pi}{2} \leq \text{angle} \leq \frac{\pi}{2}$ whose sine is $\frac{1}{2}$.

So $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Similarly, $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

So $\arcsin\left(\frac{1}{2}\right) - \arcsin\left(-\frac{1}{2}\right) = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$

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Exercise F, Question 13

Question:

Solve, in the interval $0 \leq x \leq 2\pi$, the equation $\sec^2 x - \frac{2\sqrt{3}}{3}\tan x - 2 = 0$, giving your answers in terms of π.

Solution:

$$\sec^2 x - \frac{2\sqrt{3}}{3}\tan x - 2 = 0, \ 0 \leq x \leq 2\pi$$

$$\Rightarrow \quad (1 + \tan^2 x) - \frac{2\sqrt{3}}{3}\tan x - 2 = 0$$

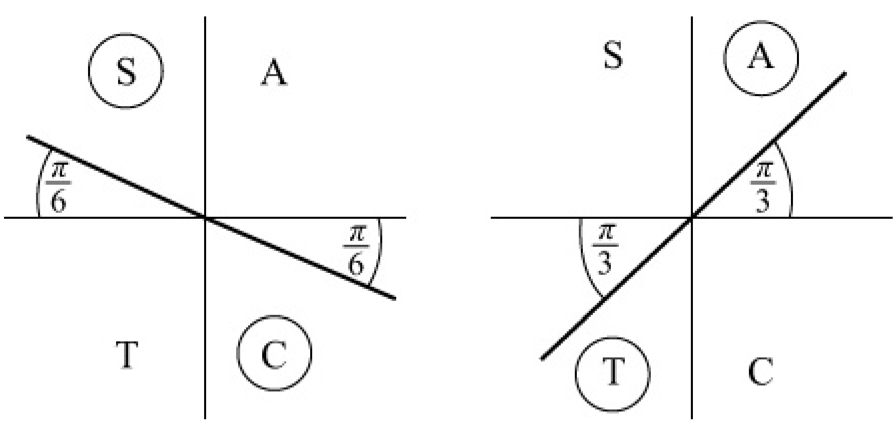
$$\tan^2 x - \frac{2\sqrt{3}}{3}\tan x - 1 = 0$$

(This does factorise but you may not have noticed!)

$$\left(\tan x + \frac{\sqrt{3}}{3}\right)(\tan x - \sqrt{3}) = 0$$

$$\Rightarrow \quad \tan x = -\frac{\sqrt{3}}{3} \text{ or } \tan x = \sqrt{3}$$

Calculator values are $-\frac{\pi}{6}$ and $\frac{\pi}{3}$.



Solution set: $\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}$

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Exercise F, Question 14

Question:

(a) Factorise $\sec x \ \mathrm{cosec}\, x - 2 \ \sec x - \mathrm{cosec}\, x + 2$.

(b) Hence solve $\sec x \ \mathrm{cosec}\, x - 2 \ \sec x - \mathrm{cosec}\, x + 2 = 0$, in the interval $0 \le x \le 360°$.

Solution:

(a) $\sec x \ \mathrm{cosec}\, x - 2 \ \sec x - \mathrm{cosec}\, x + 2$

$= \sec x (\mathrm{cosec}\, x - 2) - (\mathrm{cosec}\, x - 2)$

$= (\mathrm{cosec}\, x - 2) (\sec x - 1)$

(b) So $\sec x \ \mathrm{cosec}\, x - 2 \ \sec x - \mathrm{cosec}\, x + 2 = 0$

$\Rightarrow \quad (\mathrm{cosec}\, x - 2) (\sec x - 1) = 0$

$\Rightarrow \quad \mathrm{cosec}\, x = 2$ or $\sec x = 1$

$\Rightarrow \quad \sin x = \frac{1}{2}$ or $\cos x = 1$

$\sin x = \frac{1}{2}, \quad 0 \le x \le 360°$

$\Rightarrow \quad x = 30°, \quad (180 - 30)°$

$\cos x = 1, \quad 0 \le x \le 360°,$

$\Rightarrow \quad x = 0°, \quad 360°$ (from the graph)

Full set of solutions: 0°, 30°, 150°, 360°

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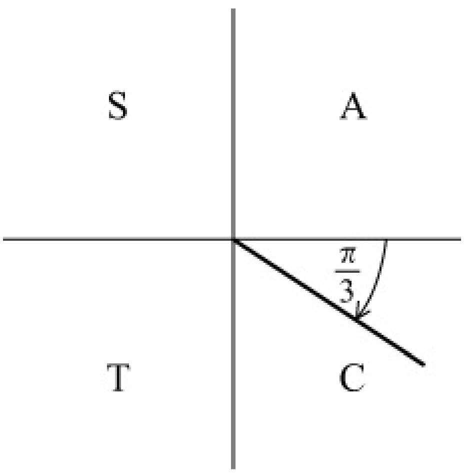
Exercise F, Question 15

Question:

Given that $\arctan (x - 2) = - \frac{\pi}{3}$, find the value of x.

Solution:

$$\arctan \left(x - 2 \right) = - \frac{\pi}{3}$$



$\Rightarrow \quad x - 2 = \tan \left(- \frac{\pi}{3} \right)$

$\Rightarrow \quad x - 2 = - \sqrt{3}$

$\Rightarrow \quad x = 2 - \sqrt{3}$

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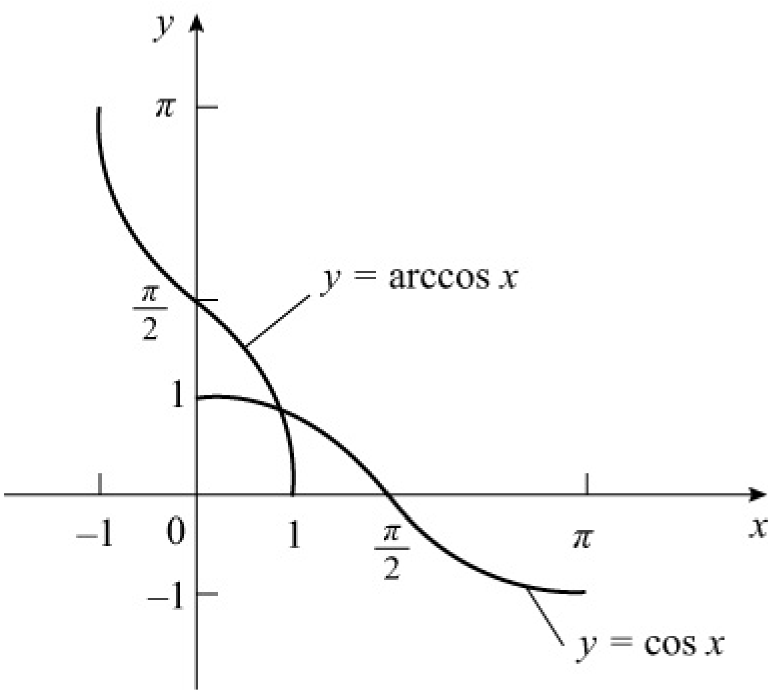
Exercise F, Question 16

Question:

On the same set of axes sketch the graphs of $y = \cos x$, $0 \leq x \leq \pi$, and $y = \arccos x$, $-1 \leq x \leq 1$, showing the coordinates of points in which the curves meet the axes.

Solution:



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Exercise F, Question 17

Question:

(a) Given that $\sec x + \tan x = -3$, use the identity $1 + \tan^2 x \equiv \sec^2 x$ to find the value of $\sec x - \tan x$.

(b) Deduce the value of
(i) $\sec x$
(ii) $\tan x$

(c) Hence solve, in the interval $-180° \leq x \leq 180°$, $\sec x + \tan x = -3$. (Give answer to 1 decimal place).

Solution:

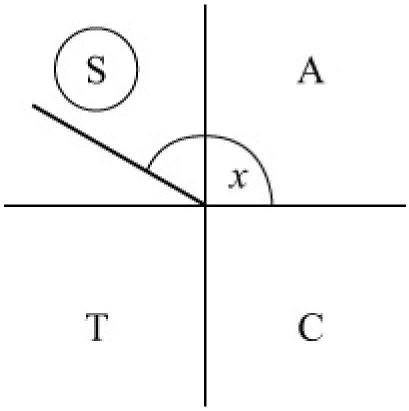
(a) As $1 + \tan^2 x \equiv \sec^2 x$
$\sec^2 x - \tan^2 x \equiv 1$
$\Rightarrow \ (\sec x - \tan x)(\sec x + \tan x) \equiv 1$ (difference of two squares)
As $\tan x + \sec x = -3$ is given,
so $-3(\sec x - \tan x) = 1$
$\Rightarrow \ \sec x - \tan x = -\frac{1}{3}$

(b) $\sec x + \tan x = -3$
and $\sec x - \tan x = -\frac{1}{3}$

(i) Add the equations $\Rightarrow \ 2\sec x = -\frac{10}{3} \ \Rightarrow \ \sec x = -\frac{5}{3}$

(ii) Subtract the equation $\Rightarrow \ 2\tan x = -3 + \frac{1}{3} = -\frac{8}{3} \ \Rightarrow \ \tan x = -\frac{4}{3}$

(c) As $\sec x$ and $\tan x$ are both −ve, $\cos x$ and $\tan x$ are both −ve.
So x must be in the 2nd quadrant.



Solving $\tan x = -\frac{4}{3}$, where x is in the 2nd quadrant, gives $180° + (-53.1°) = 126.9°$

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Exercise F, Question 18

Question:

Given that $p = \sec\theta - \tan\theta$ and $q = \sec\theta + \tan\theta$, show that $p = \frac{1}{q}$.

Solution:

$p = \sec\theta - \tan\theta$, $q = \sec\theta + \tan\theta$
Multiply together:
$pq = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$ (since $1 + \tan^2\theta = \sec^2\theta$)

$\Rightarrow \quad p = \frac{1}{q}$

(There are several ways of solving this problem).

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Exercise F, Question 19

Question:

(a) Prove that $\sec^4\theta - \tan^4\theta \equiv \sec^2\theta + \tan^2\theta$.

(b) Hence solve, in the interval $-180° \leq \theta \leq 180°$, $\sec^4\theta = \tan^4\theta + 3\tan\theta$. (Give answers to 1 decimal place).

Solution:

(a) L.H.S. $\equiv \sec^4\theta - \tan^4\theta$

$\equiv (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$

$\equiv (\sec^2\theta + \tan^2\theta)(1)$

$\equiv \sec^2\theta + \tan^2\theta \equiv$ R.H.S.

(b) $\sec^4\theta = \tan^4\theta + 3\tan\theta$

$\Rightarrow \sec^4\theta - \tan^4\theta = 3\tan\theta$

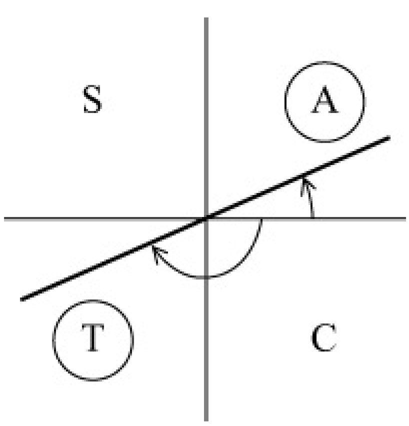
$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$ [using part (a)]

$\Rightarrow (1 + \tan^2\theta) + \tan^2\theta = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = \frac{1}{2}$ or $\tan\theta = 1$



In the interval $-180° \leq \theta \leq 180°$

$\tan\theta = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\frac{1}{2}, \ -180° + \tan^{-1}$

$\frac{1}{2} = 26.6°\ , \quad -153.4°$

$\tan\theta = 1 \quad \Rightarrow \quad \theta = \tan^{-1} 1\ , \quad -180° + \tan^{-1} 1 = 45°\ , \quad -135°$

Set of solutions: $-153.4°$, $-135°$, 26.6°, 45° (3 s.f.)

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Exercise F, Question 20

Question:

(Although integration is not in the specification for C3, this question only requires you to know that the area under a curve can be represented by an integral.)

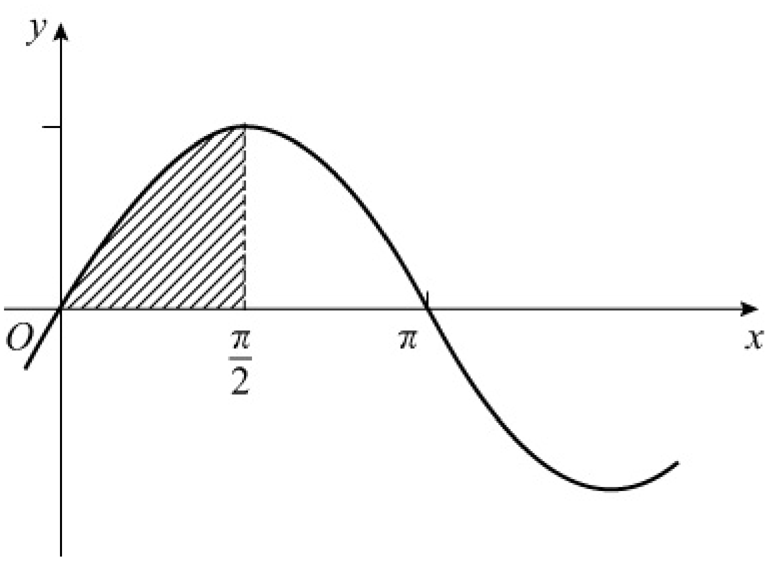
(a) Sketch the graph of $y = \sin x$ and shade in the area representing $\int_0^{\frac{\pi}{2}} \sin x \, dx$.

(b) Sketch the graph of $y = \arcsin x$ and shade in the area representing $\int_0^1 \arcsin x \, dx$.

(c) By considering the shaded areas explain why $\int_0^{\frac{\pi}{2}} \sin x \, dx + \int_0^1 \arcsin x \, dx = \frac{\pi}{2}$.

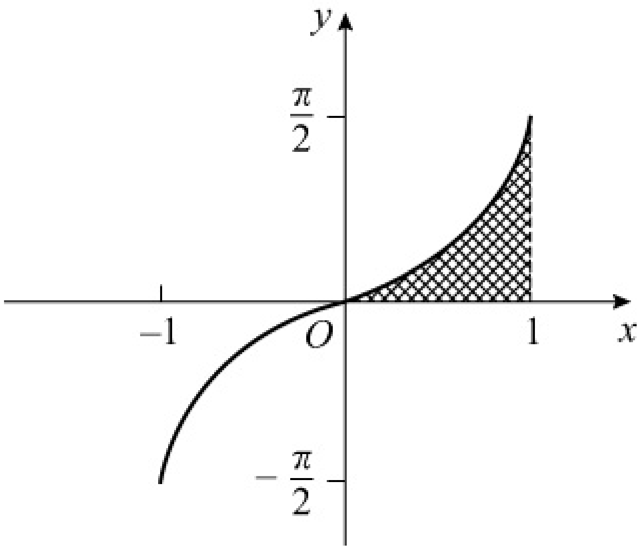
Solution:

(a) $y = \sin x$



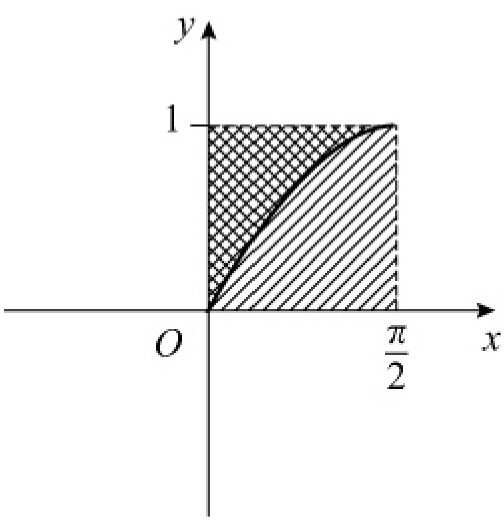
$\int_0^{\frac{\pi}{2}} \sin x \, dx$ represents the area between $y = \sin x$, x-axis and $x = \frac{\pi}{2}$.

(b) $y = \arcsin x$, $-1 \le x \le 1$



$\int_0^1 \arcsin x \, dx$ represents the area between the curve, x-axis and $x = 1$.

(c) The curves are the same with the axes interchanged.
The shaded area in (b) could be added to the graph in (a) to form a rectangle with sides 1 and $\frac{\pi}{2}$, as in the diagram.



Area of rectangle $= \frac{\pi}{2}$

So $\int_0^{\frac{\pi}{2}} \sin x \, dx + \int_0^1 \arcsin x \, dx = \frac{\pi}{2}$