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Solutionbank
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Differentiate:

(a) $(1+2x)^4$

(b) $(3-2x^2)^{-5}$

(c) $(3+4x)^{\frac{1}{2}}$

(d) $(6x+x^2)^7$

(e) $\frac{1}{3+2x}$

(f) $\sqrt{7-x}$

(g) $4(2+8x)^4$

(h) $3(8-x)^{-6}$

Solution:

(a) Let $u = 1+2x$, then $y = u^4$

$\frac{du}{dx} = 2$ and $\frac{dy}{du} = 4u^3$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 4u^3 \times 2 = 8u^3 = 8(1+2x)^3$

(b) Let $u = 3-2x^2$ then $y = u^{-5}$

$\frac{du}{dx} = -4x$ and $\frac{dy}{du} = -5u^{-6}$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -5u^{-6} \times -4x = 20xu^{-6} = 20x(3-2x^2)^{-6}$

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(c) Let $u = 3 + 4x$, then $y = u^{\frac{1}{2}}$

$\frac{du}{dx} = 4$ and $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times 4 = 2u^{-\frac{1}{2}} = 2(3 + 4x)^{-\frac{1}{2}}$

(d) Let $u = 6x + x^2$, then $y = u^7$

$\frac{du}{dx} = 6 + 2x$ and $\frac{dy}{du} = 7u^6$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 7u^6 \times (6 + 2x) = 7(6 + 2x)(6x + x^2)^6$

(e) Let $u = 3 + 2x$, then $y = \frac{1}{u} = u^{-1}$

$\frac{du}{dx} = 2$ and $\frac{dy}{du} = -u^{-2}$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -u^{-2} \times 2 = -2u^{-2} = \frac{-2}{(3 + 2x)^2}$

(f) Let $u = 7 - x$, then $y = u^{\frac{1}{2}}$

$\frac{du}{dx} = -1$ and $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times -1 = -\frac{1}{2}(7 - x)^{-\frac{1}{2}}$

(g) Let $u = 2 + 8x$, then $y = 4u^4$

$\frac{du}{dx} = 8$ and $\frac{dy}{du} = 16u^3$

Using the chain rule,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 16u^3 \times 8 = 128(2 + 8x)^3$

(h) Let $u = 8 - x$, then $y = 3u^{-6}$

$\frac{du}{dx} = -1$ and $\frac{dy}{du} = -18u^{-7}$

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Using the chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -18u^{-7} \times -1 = 18(8-x)^{-7}$$

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Exercise A, Question 2

Question:

Given that $y = \frac{1}{(4x+1)^2}$ find the value of $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.

Solution:

Let $u = 4x + 1$, then $y = u^{-2}$

$\frac{du}{dx} = 4 \qquad \frac{dy}{du} = -2u^{-3}$

$\therefore \frac{dy}{dx} = -8u^{-3} = \frac{-8}{(4x+1)^3}$

When $x = \frac{1}{4}$, $\frac{dy}{dx} = -1$

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Exercise A, Question 3

Question:

Given that $y = (5 - 2x)^3$ find the value of $\frac{dy}{dx}$ at (1, 27).

Solution:

Let $u = 5 - 2x$, then $y = u^3$

$\frac{du}{dx} = -2 \qquad \frac{dy}{du} = 3u^2$

$\therefore \frac{dy}{dx} = -6u^2 = -6(5 - 2x)^2$

When $x = 1$, $\frac{dy}{dx} = -54$

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Exercise A, Question 4

Question:

Find the value of $\frac{dy}{dx}$ at the point (8, 2) on the curve with equation $3y^2 - 2y = x$.

Solution:

$x = 3y^2 - 2y$

$\frac{dx}{dy} = 6y - 2$

$\frac{dy}{dx} = \frac{1}{6y - 2}$

At (8, 2) the value of y is 2.

$\therefore \frac{dy}{dx} = \frac{1}{12 - 2} = \frac{1}{10}$

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Exercise A, Question 5

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(2\frac{1}{2} , 4 \right)$ on the curve with equation $y^{\frac{1}{2}} + y^{-\frac{1}{2}} = x$.

Solution:

$$x = y^{\frac{1}{2}} + y^{-\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{2}y^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{1}{2}y^{-\frac{1}{2}} - \frac{1}{2}y^{-\frac{3}{2}}}$$

At the point $\left(2\frac{1}{2} , 4 \right)$ the value of y is 4.

$$\therefore \frac{dy}{dx} = \frac{1}{\frac{1}{2}(4)^{-\frac{1}{2}} - \frac{1}{2}(4)^{-\frac{3}{2}}} = \frac{1}{\frac{1}{4} - \frac{1}{16}} = \frac{1}{\frac{3}{16}} = \frac{16}{3} = 5\frac{1}{3}$$

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Exercise B, Question 1

Question:

Differentiate:

(a) $x(1+3x)^5$

(b) $2x(1+3x^2)^3$

(c) $x^3(2x+6)^4$

(d) $3x^2(5x-1)^{-1}$

Solution:

(a) Let $y = x(1+3x)^5$

Let $u = x$ and $v = (1+3x)^5$

Then $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = 5 \times 3(1+3x)^4$ (using the chain rule)

Now use the product rule.

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = x \times 15(1+3x)^4 + (1+3x)^5 \times 1$$

$$= (1+3x)^4(15x+1+3x)$$

$$= (1+3x)^4(1+18x)$$

(b) Let $y = 2x(1+3x^2)^3$

Let $u = 2x$ and $v = (1+3x^2)^3$

Then $\frac{du}{dx} = 2$ and $\frac{dv}{dx} = 18x(1+3x^2)^2$

Using the product rule,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

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$$\therefore \frac{dy}{dx} = 2x \times 18x(1+3x^2)^2 + 2(1+3x^2)^3$$
$$= (1+3x^2)^2 [36x^2 + 2(1+3x^2)]$$
$$= (1+3x^2)^2 (42x^2 + 2)$$
$$= 2(1+3x^2)^2 (1+21x^2)$$

(c) Let $y = x^3(2x+6)^4$

Let $u = x^3$ and $v = (2x+6)^4$

Then $\frac{du}{dx} = 3x^2$ and $\frac{dv}{dx} = 8(2x+6)^3$

Using the product rule,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = x^3 \times 8(2x+6)^3 + (2x+6)^4 \times 3x^2$$
$$= x^2(2x+6)^3 [8x + 3(2x+6)]$$
$$= x^2(2x+6)^3 (14x+18)$$
$$= 2x^2(2x+6)^3 (7x+9)$$

(d) Let $y = 3x^2(5x-1)^{-1}$

Let $u = 3x^2$ and $v = (5x-1)^{-1}$

Then $\frac{du}{dx} = 6x$ and $\frac{dv}{dx} = -5(5x-1)^{-2}$

Using the product rule,

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = 3x^2 \times -5(5x-1)^{-2} + (5x-1)^{-1} \times 6x$$
$$= -15x^2(5x-1)^{-2} + 6x(5x-1)^{-1}$$
$$= 3x(5x-1)^{-2} [-5x + 2(5x-1)]$$
$$= 3x(5x-2)(5x-1)^{-2}$$

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Exercise B, Question 2

Question:

(a) Find the value of $\frac{dy}{dx}$ at the point (1, 8) on the curve with equation $y = x^2 (3x - 1) ^3$.

(b) Find the value of $\frac{dy}{dx}$ at the point (4, 36) on the curve with equation $y = 3x (2x + 1) ^{\frac{1}{2}}$.

(c) Find the value of $\frac{dy}{dx}$ at the point $\left(2 , \frac{1}{5} \right)$ on the curve with equation $y = (x - 1) (2x + 1) ^{-1}$.

Solution:

(a) $y = x^2 (3x - 1) ^3$

Let $u = x^2$, $v = (3x - 1) ^3$

Then $\frac{du}{dx} = 2x$, $\frac{dv}{dx} = 9 (3x - 1) ^2$

Use the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ to give

$$\frac{dy}{dx} = x^2 \times 9 (3x - 1) ^2 + (3x - 1) ^3 \times 2x$$

$$= x (3x - 1) ^2 [9x + 2 (3x - 1)]$$

$$= x (3x - 1) ^2 (15x - 2) \qquad *$$

At the point (1, 8), $x = 1$.

Substitute $x = 1$ into the expression *.

Then $\frac{dy}{dx} = 1 \times 2^2 \times 13 = 52$

(b) $y = 3x (2x + 1) ^{\frac{1}{2}}$

Let $u = 3x$ and $v = (2x + 1) ^{\frac{1}{2}}$

Then $\frac{du}{dx} = 3$ and $\frac{dv}{dx} = \frac{1}{2} \times 2 (2x + 1) ^{-\frac{1}{2}} = (2x + 1) ^{-\frac{1}{2}}$

Use the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ to give

$$\frac{dy}{dx} = 3x(2x+1)^{-\frac{1}{2}} + 3(2x+1)^{\frac{1}{2}}$$

$$= 3(2x+1)^{-\frac{1}{2}}[x + (2x+1)]$$

$$= 3(3x+1)(2x+1)^{-\frac{1}{2}} \quad *$$

At the point (4, 36), $x = 4$.

Substitute $x = 4$ into *.

Then $\frac{dy}{dx} = 3 \times 13 \times 9^{-\frac{1}{2}} = 3 \times 13 \times \frac{1}{3} = 13$

(c) $y = (x-1)(2x+1)^{-1}$

Let $u = x - 1$ and $v = (2x+1)^{-1}$

Then $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = -2(2x+1)^{-2}$

Use the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ to give

$$\frac{dy}{dx} = -2(x-1)(2x+1)^{-2} + (2x+1)^{-1} \times 1$$

$$= (2x+1)^{-2}[-2x + 2 + (2x+1)]$$

$$= 3(2x+1)^{-2} \quad *$$

At the point $\left(2, \frac{1}{5}\right)$, $x = 2$.

Substitute $x = 2$ into *

Then $\frac{dy}{dx} = 3 \times 5^{-2} = \frac{3}{25}$

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Exercise B, Question 3

Question:

Find the points where the gradient is zero on the curve with equation $y = (x-2)^2 \left(2x+3\right)$.

Solution:

$y = (x-2)^2 (2x+3)$
$u = (x-2)^2$ and $v = (2x+3)$
$\frac{du}{dx} = 2\left(x-2\right)$ and $\frac{dv}{dx} = 2$

Use the product rule $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ to give

$$\frac{dy}{dx} = (x-2)^2 \times 2 + 2(x-2)(2x+3)$$
$$= 2(x-2)\left[(x-2) + 2x + 3\right]$$
$$= 2(x-2)(3x+1)$$

When the gradient is zero, $\frac{dy}{dx} = 0$

$\therefore 2(x-2)(3x+1) = 0$

$\therefore x = 2$ or $x = -\frac{1}{3}$

Substitute values for x into $y = (x-2)^2(2x+3)$.

When $x = 2$, $y = 0$; when $x = -\frac{1}{3}$, $y = 12\frac{19}{27}$.

So points of zero gradient are (2, 0) and $\left(-\frac{1}{3}, 12\frac{19}{27}\right)$.

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Exercise C, Question 1

Question:

Differentiate:

(a) $\frac{5x}{x+1}$

(b) $\frac{2x}{3x-2}$

(c) $\frac{x+3}{2x+1}$

(d) $\frac{3x^2}{(2x-1)^2}$

(e) $\frac{6x}{(5x+3)^{\frac{1}{2}}}$

Solution:

(a) Let $u = 5x$ and $v = x + 1$

$\therefore \frac{du}{dx} = 5$ and $\frac{dv}{dx} = 1$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(x+1) \times 5 - 5x \times 1}{(x+1)^2} = \frac{5}{(x+1)^2}$$

(b) Let $u = 2x$ and $v = 3x - 2$

$\therefore \frac{du}{dx} = 2$ and $\frac{dv}{dx} = 3$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(3x-2) \times 2 - 2x \times 3}{(3x-2)^2} = \frac{6x - 4 - 6x}{(3x-2)^2} = \frac{-4}{(3x-2)^2}$$

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(c) Let $u = x + 3$ and $v = 2x + 1$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = 2$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(2x+1) \times 1 - (x+3) \times 2}{(2x+1)^2} = \frac{2x+1-2x-6}{(2x+1)^2} = \frac{-5}{(2x+1)^2}$$

(d) Let $u = 3x^2$ and $v = (2x-1)^2$

$\therefore \frac{du}{dx} = 6x$ and $\frac{dv}{dx} = 4\left(2x-1\right)$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(2x-1)^2 \times 6x - 3x^2 \times 4(2x-1)}{(2x-1)^4}$$

$$= \frac{6x(2x-1)[(2x-1) - 2x]}{(2x-1)^4}$$

$$= \frac{-6x}{(2x-1)^3}$$

(e) Let $u = 6x$ and $v = (5x+3)^{\frac{1}{2}}$

$\therefore \frac{du}{dx} = 6$ and $\frac{dv}{dx} = \frac{5}{2}(5x+3)^{-\frac{1}{2}}$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(5x+3)^{\frac{1}{2}} \times 6 - 6x \times \frac{5}{2}(5x+3)^{-\frac{1}{2}}}{[(5x+3)^{\frac{1}{2}}]^2}$$

$$= \frac{3(5x+3)^{-\frac{1}{2}}[2(5x+3) - 5x]}{(5x+3)}$$

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$$= \frac{3(5x+3)^{-\frac{1}{2}}(10x+6-5x)}{(5x+3)}$$

$$= \frac{3(5x+6)}{(5x+3)^{\frac{3}{2}}}$$

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Exercise C, Question 2

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(1 , \frac{1}{4} \right)$ on the curve with equation $y = \frac{x}{3x + 1}$.

Solution:

Let $u = x$ and $v = 3x + 1$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = 3$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(3x + 1) \times 1 - x \times 3}{(3x + 1)^2} = \frac{1}{(3x + 1)^2} \quad *$$

At the point $\left(1 , \frac{1}{4} \right)$, $x = 1$. Substitute $x = 1$ into *.

Then $\frac{dy}{dx} = \frac{1}{16}$

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Exercise C, Question 3

Question:

Find the value of $\frac{dy}{dx}$ at the point (12, 3) on the curve with equation $y = \frac{x+3}{(2x+1)^{\frac{1}{2}}}$.

Solution:

Let $u = x + 3$ and $v = (2x+1)^{\frac{1}{2}}$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = (2x+1)^{-\frac{1}{2}}$

Use the quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

to give

$$\frac{dy}{dx} = \frac{(2x+1)^{\frac{1}{2}} \times 1 - (x+3)(2x+1)^{-\frac{1}{2}}}{(2x+1)^{+1}}$$

$$= \frac{(2x+1)^{-\frac{1}{2}}[(2x+1) - (x+3)]}{(2x+1)^{+1}}$$

$$= (2x+1)^{-\frac{3}{2}}(x-2)$$

At the point (12, 3), $x = 12$ and

$$\frac{dy}{dx} = (25)^{-\frac{3}{2}}(10) = \frac{10}{125} = \frac{2}{25}$$

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Exercise D, Question 1

Question:

Differentiate:

(a) e^{2x}

(b) e^{-6x}

(c) e^{x+3}

(d) $4e^{3x^2}$

(e) $9e^{3-x}$

(f) xe^{2x}

(g) $(x^2+3)e^{-x}$

(h) $(3x-5)e^{x^2}$

(i) $2x^4e^{1+x}$

(j) $(9x-1)e^{3x}$

(k) $\frac{x}{e^{2x}}$

(l) $\frac{e^{x^2}}{x}$

(m) $\frac{e^x}{x+1}$

(n) $\frac{e^{-2x}}{\sqrt{x+1}}$

Solution:

(a) Let $y = e^{2x}$, then $y = e^t$ where $t = 2x$

$\therefore \frac{dy}{dt} = e^t$ and $\frac{dt}{dx} = 2$

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By the chain rule

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2e^{t} = 2e^{2x}$

(b) Let $y = e^{-6x}$

If $y = e^{f(x)}$ then $\frac{dy}{dx} = f'(x) e^{f(x)}$

Let $f(x) = -6x$, then $f'(x) = -6$

$\therefore \frac{dy}{dx} = -6e^{-6x}$

(c) Let $y = e^{x+3}$

Let $f(x) = x + 3$, then $f'(x) = 1$

If $y = e^{f(x)}$ then $\frac{dy}{dx} = f'(x) e^{f(x)}$

$\therefore \frac{dy}{dx} = 1 \times e^{x+3} = e^{x+3}$

(d) Let $y = 4e^{3x^2}$

Let $f(x) = 3x^2$, then $f'(x) = 6x$

$\therefore \frac{dy}{dx} = 4 \times 6xe^{3x^2} = 24xe^{3x^2}$

(e) Let $y = 9e^{3-x}$

Let $f(x) = 3 - x$, then $f'(x) = -1$

$\therefore \frac{dy}{dx} = 9 \times -1 \times e^{3-x} = -9e^{3-x}$

(f) Let $y = xe^{2x}$

Let $u = x$ and $v = e^{2x}$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = 2e^{2x}$

Use product formula.

$\frac{dy}{dx} = x \times 2e^{2x} + e^{2x} \times 1 = e^{2x}(2x + 1)$

(g) Let $y = (x^2 + 3) e^{-x}$

Let $u = x^2 + 3$ and $v = e^{-x}$

$\therefore \frac{du}{dx} = 2x$ and $\frac{dv}{dx} = -e^{-x}$

Use product formula.

$\frac{dy}{dx} = (x^2 + 3)(-e^{-x}) + e^{-x} \times 2x = -e^{-x}(x^2 - 2x + 3)$

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(h) Let $y = (3x - 5)\,e^{x^2}$

Let $u = 3x - 5$ and $v = e^{x^2}$

$\therefore \frac{du}{dx} = 3$ and $\frac{dv}{dx} = 2xe^{x^2}$

Use product formula.

$$\frac{dy}{dx} = (3x - 5) \times 2xe^{x^2} + e^{x^2} \times 3 = e^{x^2}\left(6x^2 - 10x + 3\right)$$

(i) Let $y = 2x^4e^{1+x}$

Let $u = 2x^4$ and $v = e^{1+x}$

$\therefore \frac{du}{dx} = 8x^3$ and $\frac{dv}{dx} = e^{1+x}$

Use product formula.

$$\frac{dy}{dx} = 2x^4e^{1+x} + e^{1+x} \times 8x^3 = 2x^3e^{1+x}(x + 4)$$

(j) Let $y = (9x - 1)\,e^{3x}$

Let $u = 9x - 1$ and $v = e^{3x}$

$\therefore \frac{du}{dx} = 9$ and $\frac{dv}{dx} = 3e^{3x}$

Use product formula.

$$\frac{dy}{dx} = (9x - 1) \times 3e^{3x} + e^{3x} \times 9 = 3e^{3x}(9x + 2)$$

(k) Let $y = \frac{x}{e^{2x}}$

Let $u = x$ and $v = e^{2x}$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = 2e^{2x}$

Use quotient rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{e^{2x} \times 1 - x \times 2e^{2x}}{e^{4x}} = e^{-2x}(1 - 2x)$$

(l) Let $y = \frac{e^{x^2}}{x}$

Let $u = e^{x^2}$ and $v = x$

$\therefore \frac{du}{dx} = 2xe^{x^2}$ and $\frac{dv}{dx} = 1$

Use quotient rule.

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$$\frac{dy}{dx} = \frac{x \times 2xe^{x^2} - e^{x^2} \times 1}{x^2} = \frac{e^{x^2}(2x^2 - 1)}{x^2}$$

(m) Let $y = \frac{e^x}{x+1}$

Let $u = e^x$ and $v = x + 1$

$\therefore \frac{du}{dx} = e^x$ and $\frac{dv}{dx} = 1$

Use quotient rule.

$$\frac{dy}{dx} = \frac{(x+1)e^x - e^x \times 1}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

(n) Let $y = \frac{e^{-2x}}{\sqrt{x+1}}$

Let $u = e^{-2x}$ and $v = (x+1)^{\frac{1}{2}}$

$\therefore \frac{du}{dx} = -2e^{-2x}$ and $\frac{dv}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

Use quotient rule.

$$\frac{dy}{dx} = \frac{(x+1)^{\frac{1}{2}}(-2e^{-2x}) - e^{-2x}\left[\frac{1}{2}(x+1)^{-\frac{1}{2}}\right]}{\left[(x+1)^{\frac{1}{2}}\right]^2}$$

$$= \frac{\left[-2(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}\right]e^{-2x}}{x+1}$$

$$= \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}\left[-4(x+1) - 1\right]e^{-2x}}{x+1}$$

$$= \frac{-(4x+5)e^{-2x}}{2(x+1)^{\frac{3}{2}}}$$

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Exercise D, Question 2

Question:

Find the value of $\frac{dy}{dx}$ at the point $\left(1 , \frac{1}{e} \right)$ on the curve with equation

$y = xe^{-x}$.

Solution:

$y = xe^{-x}$

Let $u = x$ and $v = e^{-x}$

$\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = -e^{-x}$

Use the product rule to give

$$\frac{dy}{dx} = x\left(-e^{-x} \right) + e^{-x} \times 1 = e^{-x}\left(1 - x \right)$$

At $\left(1 , \frac{1}{e} \right)$, $x = 1$.

$\therefore \frac{dy}{dx} = 0$

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Exercise D, Question 3

Question:

Find the value of $\frac{dy}{dx}$ at the point (0, 3) on the curve with equation $y = (2x + 3) e^{2x}$.

Solution:

$y = (2x + 3) e^{2x}$

Let $u = 2x + 3$ and $v = e^{2x}$

$\therefore \frac{du}{dx} = 2$ and $\frac{dv}{dx} = 2e^{2x}$

Use the product rule.

$\frac{dy}{dx} = (2x + 3) 2e^{2x} + e^{2x} \times 2 = 2e^{2x} (2x + 3 + 1) = 2e^{2x} (2x + 4)$

$= 4e^{2x} \left(x + 2 \right)$

At the point (0, 3), $x = 0$.

$\therefore \frac{dy}{dx} = 4 \times 2 = 8$

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Exercise D, Question 4

Question:

Find the equation of the tangent to the curve $y = xe^{2x}$ at the point $\left(\frac{1}{2}, \frac{1}{2}e\right)$.

Solution:

$y = xe^{2x}$

$\therefore \frac{dy}{dx} = x\left(2e^{2x}\right) + e^{2x}(1)$ (From the product rule)

At the point $\left(\frac{1}{2}, \frac{1}{2}e\right)$, $x = \frac{1}{2}$.

$\therefore \frac{dy}{dx} = e + e = 2e$

The tangent at $\left(\frac{1}{2}, \frac{1}{2}e\right)$ has gradient 2e.

Its equation is

$y - \frac{1}{2}e = 2e\left(x - \frac{1}{2}\right)$

$y - \frac{1}{2}e = 2ex - e$

$y = 2ex - \frac{1}{2}e$

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Exercise D, Question 5

Question:

Find the equation of the tangent to the curve $y = \frac{e^{\frac{x}{3}}}{x}$ at the point $\left(3 , \frac{1}{3}e \right)$.

Solution:

$$y = \frac{e^{\frac{x}{3}}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{x \left(\frac{1}{3} \right) e^{\frac{x}{3}} - e^{\frac{x}{3}} \times 1}{x^2}$$ (From the quotient rule)

At the point $\left(3 , \frac{1}{3}e \right)$, $x = 3$.

$$\therefore \frac{dy}{dx} = \frac{e - e}{9} = 0$$

The tangent at $\left(3 , \frac{1}{3}e \right)$ has gradient 0.

Its equation is

$$y - \frac{1}{3}e = 0 (x - 3)$$

$$y = \frac{1}{3}e$$

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Exercise D, Question 6

Question:

Find the coordinates of the turning points on the curve $y = x^2 e^{-x}$, and determine whether these points are maximum or minimum points.

Solution:

$y = x^2 e^{-x}$

$\frac{dy}{dx} = x^2 \left(-e^{-x} \right) + e^{-x} (2x) = xe^{-x}(2 - x)$ (From the product rule)

At a turning point on the curve $\frac{dy}{dx} = 0$.

$\therefore xe^{-x}(2 - x) = 0$

$\therefore x = 0$ or $x = 2$

Substitute these values of x into the equation $y = x^2 e^{-x}$.

When $x = 0$, $y = 0$

When $x = 2$, $y = 4e^{-2}$

The turning points are at (0, 0) and $\left(2, \frac{4}{e^2} \right)$.

To establish the nature of the points find $\frac{d^2y}{dx^2}$.

As $\frac{dy}{dx} = xe^{-x}(2 - x) = e^{-x}\left(2x - x^2 \right)$

$\frac{d^2y}{dx^2} = e^{-x}(2 - 2x) + (2x - x^2)(-e^{-x}) = e^{-x}(2 - 4x + x^2)$

(From the product rule)

When $x = 0$, $\frac{d^2y}{dx^2} = 2 > 0$ $\therefore$ (0, 0) is a **minimum** point

When $x = 2$, $\frac{d^2y}{dx^2} = -2e^{-2} < 0$ $\therefore$ $\left(2, \frac{4}{e^2} \right)$ is a **maximum** point

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Exercise D, Question 7

Question:

Given that $y = \frac{e^{3x}}{x}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, simplifying your answers.

Use these answers to find the coordinates of the turning point on the curve with equation $y = \frac{e^{3x}}{x}$, $x > 0$, and determine the nature of this turning point.

Solution:

$$y = \frac{e^{3x}}{x}$$

$$\frac{dy}{dx} = \frac{x\left(3e^{3x}\right) - e^{3x} \times 1}{x^2} = \frac{\left(3x - 1\right)e^{3x}}{x^2} \quad \text{(From the quotient rule)}$$

To determine $\frac{d^2y}{dx^2}$ use the quotient rule again with

$u = 3xe^{3x} - e^{3x}$ and $v = x^2$

$$\frac{d^2y}{dx^2} = \frac{x^2\left(9xe^{3x} + 3e^{3x} - 3e^{3x}\right) - \left(3xe^{3x} - e^{3x}\right)\left(2x\right)}{x^4}$$

$$= \frac{9x^3e^{3x} - 6x^2e^{3x} + 2xe^{3x}}{x^4}$$

$$= \frac{xe^{3x}\left(9x^2 - 6x + 2\right)}{x^4}$$

$$= \frac{e^{3x}\left(9x^2 - 6x + 2\right)}{x^3}$$

At the turning point $\frac{dy}{dx} = 0$.

$$\therefore \frac{\left(3x - 1\right)e^{3x}}{x^2} = 0$$

$$\therefore x = \frac{1}{3}$$

Substitute $x = \frac{1}{3}$ into $y = \frac{e^{3x}}{x}$.

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$\therefore y = \dfrac{e}{\frac{1}{3}} = 3e$

So $\left(\frac{1}{3}, 3e \right)$ are the coordinates of the point with zero gradient.

To determine the nature of this point, substitute $x = \frac{1}{3}$ into

$$\frac{d^2y}{dx^2} = \frac{e^{3x}(9x^2 - 6x + 2)}{x^3}$$

When $x = \frac{1}{3}$, $\dfrac{d^2y}{dx^2} = \dfrac{e(1 - 2 + 2)}{\frac{1}{27}} = 27e > 0$

So there is a **minimum** point at $\left(\frac{1}{3}, 3e \right)$.

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Exercise E, Question 1

Question:

Find the function $f'(x)$ where f(x) is

(a) $\ln(x+1)$

(b) $\ln 2x$

(c) $\ln 3x$

(d) $\ln(5x-4)$

(e) $3\ln x$

(f) $4\ln 2x$

(g) $5\ln(x+4)$

(h) $x\ln x$

(i) $\frac{\ln x}{x+1}$

(j) $\ln(x^2-5)$

(k) $(3+x)\ln x$

(l) $e^x \ln x$

Solution:

(a) $f(x) = \ln(x+1)$

$F(x) = x+1, f'(x) = 1$

$\therefore f'(x) = \frac{1}{x+1}$

(b) $f(x) = \ln 2x$

$F(x) = 2x, f'(x) = 2$

$\therefore f'(x) = \frac{2}{2x} = \frac{1}{x}$

(c) $f(x) = \ln 3x$

$f'(x) = \frac{3}{3x} = \frac{1}{x}$

(d) $f(x) = \ln(5x - 4)$

$f'(x) = \frac{5}{5x - 4}$

(e) $f(x) = 3\ln x$

$f'(x) = 3 \times \frac{1}{x} = \frac{3}{x}$

(f) $f(x) = 4\ln 2x$

$f'(x) = 4 \times \frac{2}{2x} = \frac{4}{x}$

(g) $f(x) = 5\ln(x + 4)$

$f'(x) = 5 \times \frac{1}{x + 4} = \frac{5}{x + 4}$

(h) $f(x) = x\ln x$

Use the product rule with $u = x$ and $v = \ln x$.

Then $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = \frac{1}{x}$

$\therefore f'(x) = x \times \frac{1}{x} + \ln x \times 1 = 1 + \ln x$

(i) $f(x) = \frac{\ln x}{x + 1}$

Use the quotient rule with $u = \ln x$ and $v = x + 1$.

Then $\frac{du}{dx} = \frac{1}{x}$ and $\frac{dv}{dx} = 1$

$$\therefore f'(x) = \frac{(x + 1)\left(\frac{1}{x}\right) - \ln x \times 1}{(x + 1)^2} = \frac{x + 1 - x\ln x}{x(x + 1)^2}$$

(j) $f(x) = \ln(x^2 - 5)$

$F(x) = x^2 - 5, \quad f'(x) = 2x$

If $f(x) = \ln F(x)$ then $f'(x) = \frac{f'(x)}{F(x)}$

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$\therefore f'(x) = \frac{2x}{x^2 - 5}$

(k) $f(x) = (3 + x)\ln x$

Use the product rule with $u = 3 + x$ and $v = \ln x$.

Then $\frac{du}{dx} = 1$ and $\frac{dv}{dx} = \frac{1}{x}$

$\therefore f'(x) = (3 + x)\frac{1}{x} + \ln x = \frac{3 + x}{x} + \ln x$

(l) $f(x) = e^x \ln x$

Use the product rule with $u = e^x$ and $v = \ln x$.

Then $\frac{du}{dx} = e^x$ and $\frac{dv}{dx} = \frac{1}{x}$

$\therefore f'(x) = e^x \times \frac{1}{x} + \ln x \times e^x = e^x \ln x + \frac{e^x}{x}$

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Exercise F, Question 1

Question:

Differentiate:

(a) $y = \sin 5x$

(b) $y = 2\sin \frac{1}{2}x$

(c) $y = 3\sin^2 x$

(d) $y = \sin (2x + 1)$

(e) $y = \sin 8x$

(f) $y = 6\sin \frac{2}{3}x$

(g) $y = \sin^3 x$

(h) $y = \sin^5 x$

Solution:

(a) $y = \sin 5x$

If $y = \sin f (x)$, then $\frac{dy}{dx} = f'(x) \cos f (x)$.

Let $f (x) = 5x$, then $f'(x) = 5$

$\therefore \frac{dy}{dx} = 5\cos 5x$

(b) $y = 2\sin \frac{1}{2}x$

$\frac{dy}{dx} = 2 \times \left(\frac{1}{2}\cos \frac{1}{2}x \right) = \cos \frac{1}{2}x$

(c) $y = 3\sin^2 x = 3 (\sin x)^2$

Let $u = \sin x$, then $y = 3u^2$

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$\frac{du}{dx} = \cos x$ and $\frac{dy}{du} = 6u$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$= 6u \cos x$
$= 6 \sin x \cos x$
$= 3 (2 \sin x \cos x)$
$= 3 \sin 2x$ (From the double angle formula)

(d) $y = \sin (2x + 1)$

Let $f (x) = 2x + 1$, then $f' (x) = 2$

$\therefore \frac{dy}{dx} = 2 \cos (2x + 1)$

(e) $y = \sin 8x$

$\therefore \frac{dy}{dx} = 8 \cos 8x$

(f) $y = 6 \sin \frac{2}{3}x$

$\therefore \frac{dy}{dx} = 6 \times \frac{2}{3} \cos \frac{2}{3}x = 4 \cos \frac{2}{3}x$

(g) $y = \sin^3 x$

Let $u = \sin x$, then $y = u^3$

$\frac{du}{dx} = \cos x$ and $\frac{dy}{du} = 3u^2$

$\therefore \frac{dy}{dx} = 3u^2 \cos x = 3 \sin^2 x \cos x$ (From the chain rule)

(h) $y = \sin^5 x$

Let $u = \sin x$, then $y = u^5$

$\frac{du}{dx} = \cos x$ and $\frac{dy}{du} = 5u^4$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times \cos x = 5 \sin^4 x \cos x$$

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Exercise G, Question 1

Question:

Differentiate:

(a) $y = 2\cos x$

(b) $y = \cos^5 x$

(c) $y = 6\cos \frac{5}{6}x$

(d) $y = 4\cos (3x + 2)$

(e) $y = \cos 4x$

(f) $y = 3\cos^2 x$

(g) $y = 4\cos \frac{1}{2}x$

(h) $y = 3\cos 2x$

Solution:

(a) $y = 2\cos x$

$\therefore \frac{dy}{dx} = -2\sin x$

(b) $y = \cos^5 x$

Let $u = \cos x$, then $y = u^5$

$\frac{du}{dx} = -\sin x$ and $\frac{dy}{du} = 5u^4$

From the chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times (-\sin x) = -5\cos^4 x \sin x$

(c) $y = 6\cos \frac{5}{6}x$

$\therefore \frac{dy}{dx} = 6 \times -\frac{5}{6}\sin \frac{5}{6}x = -5\sin \frac{5}{6}x$

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(d) $y = 4\cos(3x + 2)$

$\therefore \frac{dy}{dx} = 4 \times -3\sin(3x + 2) = -12\sin(3x + 2)$

(e) $y = \cos 4x$

$\therefore \frac{dy}{dx} = -4\sin 4x$

(f) $y = 3\cos^2 x$

Let $u = \cos x$, then $y = 3u^2$

$\frac{du}{dx} = -\sin x$ and $\frac{dy}{du} = 6u$

From the chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 6u(-\sin x)$

$= -6\cos x\sin x$

$= -3(2\sin x\cos x)$

$= -3\sin 2x$ (From the double angle formula)

(g) $y = 4\cos\frac{1}{2}x$

$\therefore \frac{dy}{dx} = 4\left(-\frac{1}{2}\sin\frac{1}{2}x\right) = -2\sin\frac{1}{2}x$

(h) $y = 3\cos 2x$

$\therefore \frac{dy}{dx} = 3(-2\sin 2x) = -6\sin 2x$

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Exercise H, Question 1

Question:

Differentiate:

(a) $y = \tan 3x$

(b) $y = 4\tan^3 x$

(c) $y = \tan (x - 1)$

(d) $y = x^2 \tan \frac{1}{2}x + \tan \left(x - \frac{1}{2} \right)$

Solution:

(a) $y = \tan 3x$

$\therefore \frac{dy}{dx} = 3\sec^2 3x$

(b) $y = 4\tan^3 x$

Let $u = \tan x$, then $y = 4u^3$

$\frac{du}{dx} = \sec^2 x$ and $\frac{dy}{du} = 12u^2$

From the chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 12u^2 \sec^2 x = 12\tan^2 x \sec^2 x$

(c) $y = \tan (x - 1)$

$\therefore \frac{dy}{dx} = \sec^2 (x - 1)$

(d) $y = x^2 \tan \frac{1}{2}x + \tan \left(x - \frac{1}{2} \right)$

The first term is a product with $u = x^2$ and $v = \tan \frac{1}{2}x$

$\therefore \frac{du}{dx} = 2x$ and $\frac{dv}{dx} = \frac{1}{2}\sec^2 \frac{1}{2}x$

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$$\frac{dy}{dx} = \left[x^2 \left(\frac{1}{2}\sec^2 \frac{1}{2}x \right) + \tan \frac{1}{2}x \times 2x \right] + \sec^2 \left(x - \frac{1}{2} \right)$$

$$= \frac{1}{2}x^2 \sec^2 \frac{1}{2}x + 2x\tan \frac{1}{2}x + \sec^2 \left(x - \frac{1}{2} \right)$$

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Exercise I, Question 1

Question:

Differentiate

(a) $\cot 4x$

(b) $\sec 5x$

(c) $\operatorname{cosec} 4x$

(d) $\sec^2 3x$

(e) $x \cot 3x$

(f) $\frac{\sec^2 x}{x}$

(g) $\operatorname{cosec}^3 2x$

(h) $\cot^2 (2x - 1)$

Solution:

(a) $y = \cot 4x$

Let $u = 4x$, then $y = \cot u$

$\frac{du}{dx} = 4$ and $\frac{dy}{du} = - \operatorname{cosec}^2 u$

$\therefore \frac{dy}{dx} = - \operatorname{cosec}^2 u \times 4 = - 4 \operatorname{cosec}^2 4x$ (From the chain rule)

(b) $y = \sec 5x$

Let $u = 5x$, then $y = \sec u$

$\frac{du}{dx} = 5$ and $\frac{dy}{du} = \sec u \tan u$

$\therefore \frac{dy}{dx} = 5 \sec u \tan u = 5 \sec 5x \tan 5x$

(c) $y = \operatorname{cosec} 4x$

Let $u = 4x$, then $y = \operatorname{cosec} u$

$\frac{du}{dx} = 4$ and $\frac{dy}{du} = -\operatorname{cosec} u \cot u$

$\therefore \frac{dy}{dx} = -4 \operatorname{cosec} u \cot u = -4 \operatorname{cosec} 4x \cot 4x$

(d) $y = \sec^2 3x$

Let $u = \sec 3x$, then $y = u^2$

$\frac{du}{dx} = 3 \sec 3x \tan 3x$ and $\frac{dy}{du} = 2u$

From the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 2u \times 3 \sec 3x \tan 3x$$
$$= 2 \sec 3x \times 3 \sec 3x \tan 3x$$
$$= 6 \sec^2 3x \tan 3x$$

(e) $y = x \cot 3x$

This is a product so use the product formula.

Let $u = x$ and $v = \cot 3x$

$\frac{du}{dx} = 1$ and $\frac{dv}{dx} = -3 \operatorname{cosec}^2 3x$

$\therefore \frac{dy}{dx} = x (-3 \operatorname{cosec}^2 3x) + \cot 3x \times 1 = \cot 3x - 3x \operatorname{cosec}^2 3x$

(f) $y = \frac{\sec^2 x}{x}$

This is a quotient so use the quotient rule.

Let $u = \sec^2 x$ and $v = x$

$\frac{du}{dx} = 2 \sec x (\sec x \tan x)$ and $\frac{dv}{dx} = 1$

$$\therefore \frac{dy}{dx} = \frac{x (2 \sec^2 x \tan x) - \sec^2 x \times 1}{x^2} = \frac{\sec^2 x (2x \tan x - 1)}{x^2}$$

(g) $y = \operatorname{cosec}^3 2x$

Let $u = \operatorname{cosec} 2x$, then $y = u^3$

$\frac{du}{dx} = -2 \operatorname{cosec} 2x \cot 2x$ and $\frac{dy}{du} = 3u^2$

From the chain rule

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$$\frac{dy}{dx} = 3u^2 \left(-2 \operatorname{cosec} 2x \cot 2x \right)$$
$$= -6 \operatorname{cosec}^2 2x \operatorname{cosec} 2x \cot 2x$$
$$= -6 \operatorname{cosec}^3 2x \cot 2x$$

(h) $y = \cot^2 \left(2x - 1 \right)$

Let $u = \cot \left(2x - 1 \right)$ then $y = u^2$

$\frac{du}{dx} = -2 \operatorname{cosec}^2 \left(2x - 1 \right)$ and $\frac{dy}{du} = 2u$

From the chain rule

$$\frac{dy}{dx} = 2u \left[-2 \operatorname{cosec}^2 \left(2x - 1 \right) \right] = -4 \cot \left(2x - 1 \right) \operatorname{cosec}^2 \left(2x - 1 \right)$$

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Exercise J, Question 1

Question:

Find the function $f'(x)$ where $f(x)$ is

(a) $\sin 3x$

(b) $\cos 4x$

(c) $\tan 5x$

(d) $\sec 7x$

(e) $\operatorname{cosec} 2x$

(f) $\cot 3x$

(g) $\sin \frac{2x}{5}$

(h) $\cos \frac{3x}{7}$

(i) $\tan \frac{2x}{5}$

(j) $\operatorname{cosec} \frac{x}{2}$

(k) $\cot \frac{1}{3}x$

(l) $\sec \frac{3x}{2}$

Solution:

(a) $f(x) = \sin 3x$
$f'(x) = 3\cos 3x$

(b) $f(x) = \cos 4x$
$f'(x) = -4\sin 4x$

(c) $f(x) = \tan 5x$

$f'(x) = 5\sec^2 5x$

(d) $f(x) = \sec 7x$

$f'(x) = 7\sec 7x \tan 7x$

(e) $f(x) = \operatorname{cosec} 2x$

$f'(x) = -2\operatorname{cosec} 2x \cot 2x$

(f) $f(x) = \cot 3x$

$f'(x) = -3\operatorname{cosec}^2 3x$

(g) $f(x) = \sin \frac{2x}{5}$

$f'(x) = \frac{2}{5}\cos \frac{2x}{5}$

(h) $f(x) = \cos \frac{3x}{7}$

$f'(x) = -\frac{3}{7}\sin \frac{3x}{7}$

(i) $f(x) = \tan \frac{2x}{5}$

$f'(x) = \frac{2}{5}\sec^2 \frac{2x}{5}$

(j) $f(x) = \operatorname{cosec} \frac{x}{2}$

$f'(x) = -\frac{1}{2}\operatorname{cosec} \frac{x}{2} \cot \frac{x}{2}$

(k) $f(x) = \cot \frac{1}{3}x$

$f'(x) = -\frac{1}{3}\operatorname{cosec}^2 \frac{1}{3}x$

(l) $f(x) = \sec \frac{3x}{2}$

$f'(x) = \frac{3}{2}\sec \frac{3x}{2} \tan \frac{3x}{2}$

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Exercise J, Question 2

Question:

Find the function $f'(x)$ where f(x) is

(a) $\sin^2 x$

(b) $\cos^3 x$

(c) $\tan^4 x$

(d) $(\sec x)^{\frac{1}{2}}$

(e) $\sqrt{\cot x}$

(f) $\operatorname{cosec}^2 x$

(g) $\sin^3 x$

(h) $\cos^4 x$

(i) $\tan^2 x$

(j) $\sec^3 x$

(k) $\cot^3 x$

(l) $\operatorname{cosec}^4 x$

Solution:

(a) $f(x) = \sin^2 x = (\sin x)^2$
$f'(x) = 2(\sin x)^1 \cos x = 2\sin x \cos x = \sin 2x$

(b) $f(x) = \cos^3 x = (\cos x)^3$
$f'(x) = 3(\cos x)^2(-\sin x) = -3\cos^2 x \sin x$

(c) $f(x) = \tan^4 x = (\tan x)^4$
$f'(x) = 4(\tan x)^3(\sec^2 x) = 4\tan^3 x \sec^2 x$

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(d) $f(x) = (\sec x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(\sec x)^{-\frac{1}{2}} \times \sec x \tan x = \frac{1}{2}(\sec x)^{\frac{1}{2}} \tan x$

(e) $f(x) = (\cot x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(\cot x)^{-\frac{1}{2}} \times (-\mathrm{cosec}^2 x) = -\frac{1}{2}(\cot x)^{-\frac{1}{2}} \mathrm{cosec}^2 x$

(f) $f(x) = \mathrm{cosec}^2 x = (\mathrm{cosec}\, x)^2$

$f'(x) = 2(\mathrm{cosec}\, x)^1(-\mathrm{cosec}\, x \cot x) = -2\,\mathrm{cosec}^2 x \cot x$

(g) $f(x) = (\sin x)^3$

$f'(x) = 3(\sin x)^2 \cos x = 3\sin^2 x \cos x$

(h) $f(x) = (\cos x)^4$

$f'(x) = 4(\cos x)^3(-\sin x) = -4\cos^3 x \sin x$

(i) $f(x) = (\tan x)^2$

$f'(x) = 2\tan x \times \sec^2 x = 2\tan x \sec^2 x$

(j) $f(x) = (\sec x)^3$

$f'(x) = 3(\sec x)^2 \sec x \tan x = 3\sec^3 x \tan x$

(k) $f(x) = (\cot x)^3$

$f'(x) = 3(\cot x)^2(-\mathrm{cosec}^2 x) = -3\,\mathrm{cosec}^2 x \cot^2 x$

(l) $f(x) = (\mathrm{cosec}\, x)^4$

$f'(x) = 4(\mathrm{cosec}\, x)^3(-\mathrm{cosec}\, x \cot x) = -4\,\mathrm{cosec}^4 x \cot x$

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Exercise J, Question 3

Question:

Find the function $f'(x)$ where f(x) is

(a) $x \cos x$

(b) $x^2 \sec 3x$

(c) $\frac{\tan 2x}{x}$

(d) $\sin^3 x \cos x$

(e) $\frac{x^2}{\tan x}$

(f) $\frac{1 + \sin x}{\cos x}$

(g) $e^{2x} \cos x$

(h) $e^x \sec 3x$

(i) $\frac{\sin 3x}{e^x}$

(j) $e^x \sin^2 x$

(k) $\frac{\ln x}{\tan x}$

(l) $\frac{e^{\sin x}}{\cos x}$

Solution:

(a) $f(x) = x \cos x$

$f'(x) = x(-\sin x) + \cos x(1)$ (Product rule)

$= -x \sin x + \cos x$

(b) $f(x) = x^2 \sec 3x$

$f'(x) = x^2(3\sec 3x \tan 3x) + \sec 3x(2x)$ (Product rule)

$= x\sec 3x(3x\tan 3x + 2)$

(c) $f(x) = \frac{\tan 2x}{x}$

$f'(x) = \frac{x(2\sec^2 2x) - \tan 2x(1)}{x^2}$ (Quotient rule)

$= \frac{2x\sec^2 2x - \tan 2x}{x^2}$

(d) $f(x) = \sin^3 x \cos x$

$f'(x) = \sin^3 x(-\sin x) + \cos x(3\sin^2 x\cos x)$ (Product rule)

$= 3\sin^2 x\cos^2 x - \sin^4 x$

(e) $f(x) = \frac{x^2}{\tan x}$

$f'(x) = \frac{\tan x(2x) - x^2(\sec^2 x)}{\tan^2 x}$ (Quotient rule)

$= \frac{2x\tan x - x^2\sec^2 x}{\tan^2 x}$

(f) $f(x) = \frac{1 + \sin x}{\cos x}$

$f'(x) = \frac{\cos x(\cos x) - (1 + \sin x)(-\sin x)}{\cos^2 x}$ (Quotient rule)

$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$

$= \frac{(\cos^2 x + \sin^2 x) + \sin x}{\cos^2 x}$ (Use $\cos^2 x + \sin^2 x \equiv 1$)

$= \frac{1 + \sin x}{\cos^2 x}$

(g) $f(x) = e^{2x}\cos x$

$f'(x) = e^{2x}(-\sin x) + \cos x(2e^{2x})$ (Product rule)

$= e^{2x}(2\cos x - \sin x)$

(h) $f(x) = e^x \sec 3x$

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$$f'(x) = e^x(3\sec 3x\tan 3x) + \sec 3x(e^x) \quad \text{(Product rule)}$$
$$= e^x\sec 3x(3\tan 3x + 1)$$

(i) $f(x) = \frac{\sin 3x}{e^x}$

$$f'(x) = \frac{e^x(3\cos 3x) - \sin 3x(e^x)}{(e^x)^2} \quad \text{(Quotient rule)}$$
$$= \frac{e^x(3\cos 3x - \sin 3x)}{(e^x)^2}$$
$$= \frac{3\cos 3x - \sin 3x}{e^x}$$

(j) $f(x) = e^x\sin^2 x$

$$f'(x) = e^x(2\sin x\cos x) + \sin^2 x(e^x) \quad \text{(Product rule)}$$
$$= e^x\sin x(2\cos x + \sin x)$$

(k) $f(x) = \frac{\ln x}{\tan x}$

$$f'(x) = \frac{\tan x\left(\frac{1}{x}\right) - \ln x(\sec^2 x)}{\tan^2 x} \quad \text{(Quotient rule)}$$

$$= \frac{\tan x - x\sec^2 x\ln x}{x\tan^2 x} \quad \text{(Multiply numerator and denominator by } x\text{)}$$

(l) $f(x) = \frac{e^{\sin x}}{\cos x}$

$$f'(x) = \frac{\cos x[e^{\sin x}\times\cos x] - e^{\sin x}(-\sin x)}{\cos^2 x} \quad \text{(Quotient rule)}$$
$$= \frac{e^{\sin x}(\cos^2 x + \sin x)}{\cos^2 x}$$

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Exercise K, Question 1

Question:

Differentiate with respect to x:

(a) $\ln x^2$

(b) $x^2 \sin 3x$

[E]

Solution:

(a) $y = \ln x^2 = 2 \ln x$ (This uses properties of logs)

$\therefore \frac{dy}{dx} = 2 \times \frac{1}{x} = \frac{2}{x}$

Alternative method

When $y = \ln f(x)$, $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$ (From the chain rule)

$\therefore y = \ln x^2 \Rightarrow \frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x}$

(b) $y = x^2 \sin 3x$

$\frac{dy}{dx} = x^2 (3 \cos 3x) + \sin 3x (2x)$ (Product rule)

$= 3x^2 \cos 3x + 2x \sin 3x$

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Exercise K, Question 2

Question:

Given that

$$f(x) \equiv 3 - \frac{x^2}{4} + \ln \frac{x}{2}, x > 0$$

find $f'(x)$.

[E]

Solution:

$$f(x) = 3 - \frac{x^2}{4} + \ln \frac{x}{2}$$

$$f'(x) = 0 - \frac{2x}{4} + \frac{\frac{1}{2}}{\frac{1}{2}x} = -\frac{x}{2} + \frac{1}{x}$$

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Exercise K, Question 3

Question:

Given that $2y = x - \sin x \cos x$, show that $\frac{dy}{dx} = \sin^2 x$.

[E]

Solution:

$$2y = x - \sin x \cos x$$

$$\therefore y = \frac{x}{2} - \frac{1}{2}\sin x \cos x$$

$$\frac{dy}{dx} = \frac{1}{2} - \left[\frac{1}{2}\sin x \left(-\sin x \right) + \cos x \left(\frac{1}{2}\cos x \right) \right] \quad \text{(Product rule)}$$

$$= \frac{1}{2} + \frac{1}{2}\left(\sin^2 x \right) - \frac{1}{2}\cos^2 x$$

$$= \frac{1}{2}\left(1 - \cos^2 x \right) + \frac{1}{2}\sin^2 x$$

$$= \frac{1}{2}\sin^2 x + \frac{1}{2}\sin^2 x \quad \text{(Using } \cos^2 x + \sin^2 x \equiv 1)$$

$$= \sin^2 x$$

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Exercise K, Question 4

Question:

Differentiate, with respect to x,

(a) $\frac{\sin x}{x}$, $x > 0$

(b) $\ln \frac{1}{x^2 + 9}$

[E]

Solution:

(a) $y = \frac{\sin x}{x}$

$\frac{dy}{dx} = \frac{x \cos x - \sin x \times 1}{x^2}$ (Using the quotient rule)

$= \frac{x \cos x - \sin x}{x^2}$

(b) $y = \ln \frac{1}{x^2 + 9} = \ln 1 - \ln (x^2 + 9)$ (Using laws of logarithms)

$\Rightarrow y = - \ln (x^2 + 9)$

$\therefore \frac{dy}{dx} = \frac{- 2x}{x^2 + 9}$

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Exercise K, Question 5

Question:

Use the derivatives of sin x and cos x to prove that the derivative of tan x is $\sec^2 x$.

[E]

Solution:

Let $y = \tan x = \frac{\sin x}{\cos x}$

Use the quotient rule with $u = \sin x$ and $v = \cos x$.

Then $\frac{du}{dx} = \cos x$ and $\frac{dv}{dx} = -\sin x$

As

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \quad (\text{Use } \cos^2 x + \sin^2 x \equiv 1)$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x \quad \left(\text{As } \sec x = \frac{1}{\cos x}\right)$$

So derivative of tan x is $\sec^2 x$.

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Exercise K, Question 6

Question:

$$f(x) = \frac{x}{x^2 + 2}, x \in \mathbb{R}$$

Find the set of values of x for which $f'(x) < 0$.

[E]

Solution:

$$f(x) = \frac{x}{x^2 + 2}$$

$$f'(x) = \frac{(x^2 + 2)(1) - x(2x)}{(x^2 + 2)^2} = \frac{x^2 + 2 - 2x^2}{(x^2 + 2)^2} = \frac{2 - x^2}{(x^2 + 2)^2}$$

When $f'(x) < 0$, $\frac{2 - x^2}{(x^2 + 2)^2} < 0$

$\therefore 2 - x^2 < 0$ [As $(x^2 + 2)^2 > 0$]

$\therefore x^2 > 2$

$\therefore x < -\sqrt{2}, x > \sqrt{2}$

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Exercise K, Question 7

Question:

The function f is defined for positive real values of x by

$$f(x) = 12\ln x - x^{\frac{3}{2}}$$

Write down the set of values of x for which f(x) is an increasing function of x

[E].

Solution:

$$f(x) = 12\ln x - x^{\frac{3}{2}}$$

$$f'(x) = 12 \times \frac{1}{x} - \frac{3}{2}x^{\frac{1}{2}}$$

When f(x) is an increasing function, $f'(x) > 0$.

$$\therefore \frac{12}{x} - \frac{3}{2}x^{\frac{1}{2}} > 0$$

$$\therefore \frac{12}{x} > \frac{3}{2}x^{\frac{1}{2}}$$

As $x > 0$, multiply both sides by x to give

$$12 > \frac{3}{2}x^{1\frac{1}{2}}$$

$$\therefore x^{\frac{3}{2}} < 8$$

$$\therefore x < 8^{\frac{2}{3}}$$

i.e. $x < 4$

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Exercise K, Question 8

Question:

Given that $y = \cos 2x + \sin x$, $0 < x < 2\pi$, and x is in radians, find, to 2 decimal places, the values of x for which $\frac{dy}{dx} = 0$.

[E]

Solution:

$y = \cos 2x + \sin x$

$\frac{dy}{dx} = -2 \sin 2x + \cos x$

Put $\frac{dy}{dx} = 0$

$\therefore \cos x - 2 \sin 2x = 0$

$\therefore \cos x - 4 \sin x \cos x = 0$ (Using double angle formula)

i.e. $\cos x (1 - 4 \sin x) = 0$.

$\therefore \cos x = 0$ or $\sin x = \frac{1}{4}$

$\therefore x = 1.57$ or 4.71 or 0.25 or 2.89

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Exercise K, Question 9

Question:

The maximum point on the curve with equation $y = x\sqrt{\sin x}$, $0 < x < \pi$, is the point A. Show that the x-coordinate of point A satisfies the equation $2\tan x + x = 0$.

[E]

Solution:

$y = x\sqrt{\sin x}$

$$\frac{dy}{dx} = x\left[\frac{1}{2}\left(\sin x\right)^{-\frac{1}{2}} \times \cos x\right] + \left(\sin x\right)^{\frac{1}{2}} \times 1$$

At the maximum point, $\frac{dy}{dx} = 0$.

$$\therefore \frac{x}{2}\left(\sin x\right)^{-\frac{1}{2}}\cos x + \left(\sin x\right)^{\frac{1}{2}} = 0$$

Multiply equation by $2\left(\sin x\right)^{\frac{1}{2}}$:

$x\cos x + 2\sin x = 0$

Divide equation by $\cos x$:

$x + 2\tan x = 0$

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Exercise K, Question 10

Question:

$f(x) = e^{0.5x} - x^2, x \in \mathbb{R}$

(a) Find $f'(x)$.

(b) By evaluating $f'(6)$ and $f'(7)$, show that the curve with equation $y = f(x)$ has a stationary point at $x = p$, where $6 < p < 7$.

[E]

Solution:

(a) $f(x) = e^{0.5x} - x^2$
$\therefore f'(x) = 0.5e^{0.5x} - 2x$

(b) $f'(6) = -1.96 < 0$
$f'(7) = 2.56 > 0$
As the sign changes and the function is continuous, $f'(x) = 0$ has a root p where $6 < p < 7$.
So $y = f(x)$ has a stationary point at $x = p$.

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Exercise K, Question 11

Question:

$f(x) \equiv e^{2x}\sin 2x,\ 0 \le x \le \pi$

(a) Use calculus to find the coordinates of the turning points on the graph of $y = f(x)$.

(b) Show that $f''(x) = 8e^{2x}\cos 2x$.

(c) Hence, or otherwise, determine which turning point is a maximum and which is a minimum.

[E]

Solution:

(a) $f(x) = e^{2x}\sin 2x$

$\therefore f'(x) = e^{2x}(2\cos 2x) + \sin 2x(2e^{2x})$

When $f'(x) = 0$, $2e^{2x}(\cos 2x + \sin 2x) = 0$

$\therefore \sin 2x = -\cos 2x$

Divide both sides by cos $2x$:

$\tan 2x = -1$

$\therefore 2x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

$\therefore x = \frac{3\pi}{8}$ or $\frac{7\pi}{8}$ (As $0 \le x \le \pi$)

As $y = f(x)$, when $x = \frac{3\pi}{8}$, $y = \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}$

and when $x = \frac{7\pi}{8}$, $y = -\frac{1}{\sqrt{2}}e^{\frac{7\pi}{4}}$

So $\left(\frac{3\pi}{8}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right)$ and $\left(\frac{7\pi}{8}, \frac{-1}{\sqrt{2}}e^{\frac{7\pi}{4}}\right)$ are stationary values.

(b) As $f'(x) = 2e^{2x}(\cos 2x + \sin 2x)$

$f''(x) = 2e^{2x}(-2\sin 2x + 2\cos 2x) + 4e^{2x}(\cos 2x + \sin 2x)$

$= 8e^{2x}\cos 2x$

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(c) $f''\left(\frac{3\pi}{8}\right) = 8e^{\frac{3\pi}{4}}\cos\frac{3\pi}{4} = -4\sqrt{2}e^{\frac{3\pi}{4}} < 0$

$\therefore$ maximum at $\left(\frac{3\pi}{8}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right)$

$f''\left(\frac{7\pi}{8}\right) = 8e^{\frac{7\pi}{4}}\cos\frac{7\pi}{4} = +4\sqrt{2}e^{\frac{7\pi}{4}} > 0$

$\therefore$ minimum at $\left(\frac{7\pi}{8}, \frac{-1}{\sqrt{2}}e^{\frac{7\pi}{4}}\right)$.

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Exercise K, Question 12

Question:

The curve **C** has equation $y = 2e^x + 3x^2 + 2$. The point A with coordinates (0, 4) lies on **C**. Find the equation of the tangent to **C** at A.

[E]

Solution:

$y = 2e^x + 3x^2 + 2$

$\frac{dy}{dx} = 2e^x + 6x$

At the point A (0, 4), $x = 0$, so the gradient of the tangent at A is $2e^0 + 6 \times 0 = 2$.

$\therefore$ the equation of the tangent at A is

$y - 4 = 2 (x - 0)$

i.e. $y = 2x + 4$

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Exercise K, Question 13

Question:

The curve **C** has equation $y = f(x)$, where

$f(x) = 3\ln x + \frac{1}{x}, x > 0$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.
The point Q on **C** has x-coordinate 1.

(b) Find an equation for the normal to **C** at Q.

[E]

Solution:

(a) $f(x) = 3\ln x + \frac{1}{x}$

$f'(x) = \frac{3}{x} - \frac{1}{x^2}$

When $f'(x) = 0$, $\frac{3}{x} - \frac{1}{x^2} = 0$

Multiply equation by x^2:
$3x - 1 = 0$
$3x = 1$
$x = \frac{1}{3}$

So the x-coordinate of the stationary point P is $\frac{1}{3}$.

(b) At the point Q, $x = 1$. $\therefore y = f(1) = 1$
The gradient of the curve at point Q is $f'(1) = 3 - 1 = 2$.
So the gradient of the normal to the curve at Q is $-\frac{1}{2}$.

$\therefore$ the equation of the normal is $y - 1 = -\frac{1}{2}(x - 1)$

i.e. $y = -\frac{1}{2}x + 1\frac{1}{2}$

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Exercise K, Question 14

Question:

Differentiate $e^{2x}\cos x$ with respect to x.
The curve **C** has equation $y = e^{2x}\cos x$.

(a) Show that the turning points on **C** occur when $\tan x = 2$.

(b) Find an equation of the tangent to **C** at the point where $x = 0$.

[E]

Solution:

Let $f(x) = e^{2x}\cos x$
Then $f'(x) = e^{2x}(-\sin x) + \cos x(2e^{2x})$

(a) The turning points occur when $f'(x) = 0$.
$\therefore e^{2x}(2\cos x - \sin x) = 0$
$\therefore \sin x = 2\cos x$
Divide both sides by cos x:
$\tan x = 2$

(b) When $x = 0$, $y = f(0) = e^{0}\cos 0 = 1$
The gradient of the curve at (0, 1) is $f'(0)$.
$f'(0) = 0 + 2 = 2$
This is the gradient of the tangent at (0, 1) also.
So the equation of the tangent at (0, 1) is
$y - 1 = 2(x - 0)$
$\therefore y = 2x + 1$

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Exercise K, Question 15

Question:

Given that $x = y^2 \ln y$, $y > 0$,

(a) find $\frac{dx}{dy}$

(b) use your answer to part (a) to find in terms of e, the value of $\frac{dy}{dx}$ at $y = e$.

[E]

Solution:

(a) $x = y^2 \ln y$

Use the product rule to give

$$\frac{dx}{dy} = y^2 \left(\frac{1}{y} \right) + \ln y \times 2y = y + 2y \ln y$$

(b) $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{y + 2y \ln y}$

when $y = e$,

$$\frac{dy}{dx} = \frac{1}{e + 2e \ln e} = \frac{1}{3e}$$

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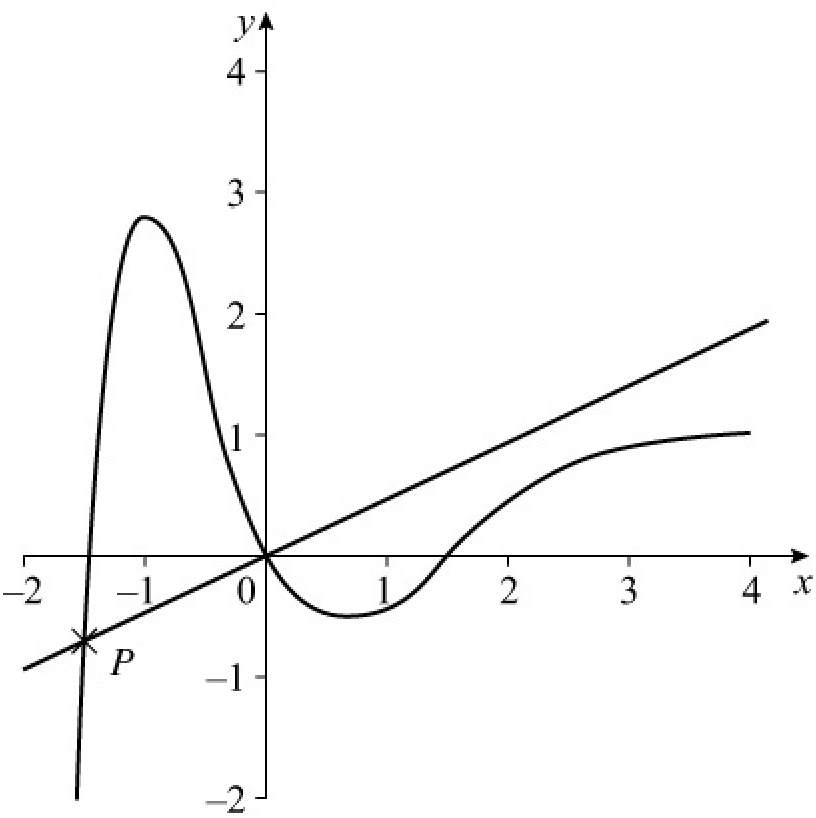
Exercise K, Question 16

Question:



The figure shows part of the curve **C** with equation $y = f(x)$, where $f(x) = (x^3 - 2x)e^{-x}$

(a) Find $f'(x)$.

The normal to **C** at the origin O intersects **C** at a point P, as shown in the figure.

(b) Show that the x-coordinate of P is the solution of the equation $2x^2 = e^x + 4$.

[E]

Solution:

(a) $f(x) = (x^3 - 2x)e^{-x}$

$\therefore f'(x) = (x^3 - 2x)(-e^{-x}) + e^{-x}(3x^2 - 2) = e^{-x}(-x^3 + 3x^2 + 2x - 2)$

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(b) The gradient of the curve at (0, 0) is $f'(0) = -2$

The normal at the origin has gradient $\frac{1}{2}$.

So the equation of the normal at the origin is $y = \frac{1}{2}x$

This normal meets the curve $y = (x^3 - 2x)e^{-x}$ at the point P.

∴ the x-coordinate of P satisfies

$\frac{1}{2}x = (x^3 - 2x)e^{-x}$

Multiply both sides by $2e^x$:

$xe^x = 2x^3 - 4x$

Divide both sides by x and rearrange to give

$2x^2 = e^x + 4$

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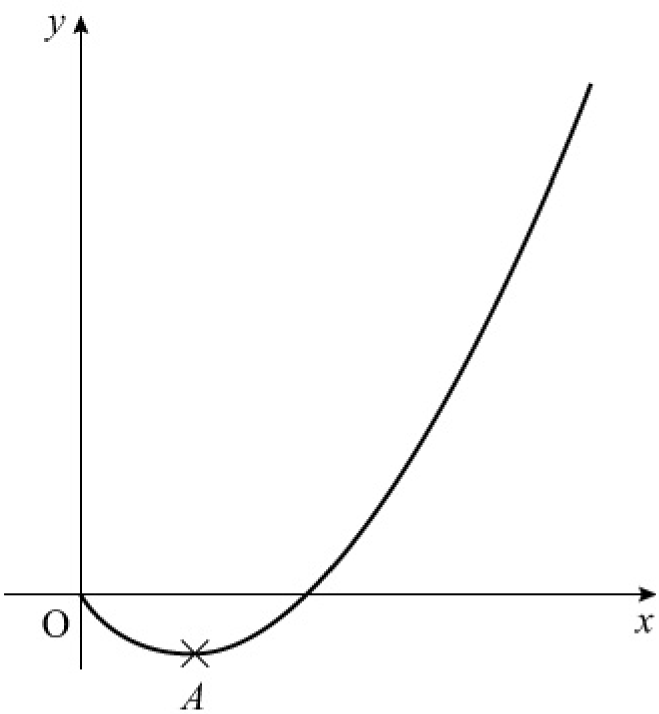
Exercise K, Question 17

Question:



The diagram shows part of the curve with equation $y = f(x)$ where
$f(x) = x(1 + x)\ln x \quad \{ x > 0 \{$
The point A is the minimum point of the curve.

(a) Find $f'(x)$.

(b) Hence show that the x-coordinate of A is the solution of the equation $x = g(x)$, where

$g(x) = e^{-\frac{1+x}{1+2x}}$

[E]

Solution:

(a) $f(x) = x(1 + x)\ln x = (x + x^2)\ln x$

$f'(x) = (x + x^2) \times \frac{1}{x} + \ln x(1 + 2x) = (1 + x) + (1 + 2x)\ln x$

(b) A is the minimum point on the curve $y = f(x)$
$\therefore f'(x) = 0$ at point A.

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$\therefore \ (1 + x) + (1 + 2x) \ln x = 0$

$\therefore \ (1 + 2x) \ln x = -(1 + x)$

$\therefore \ \ln x = -\frac{1 + x}{1 + 2x}$

$\therefore \ x = e^{-\frac{1 + x}{1 + 2x}}$

i.e. the x-coordinate of A is a solution of $x = g(x)$ where $g(x) = e^{-\frac{1 + x}{1 + 2x}}$.