

GCE

Edexcel GCE

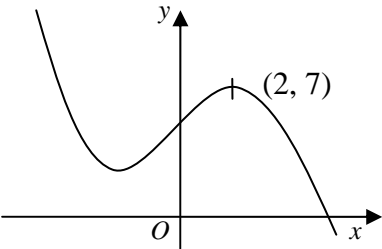
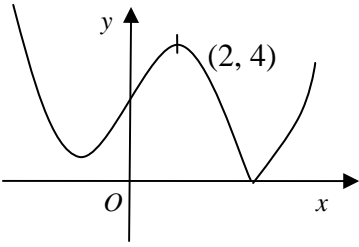
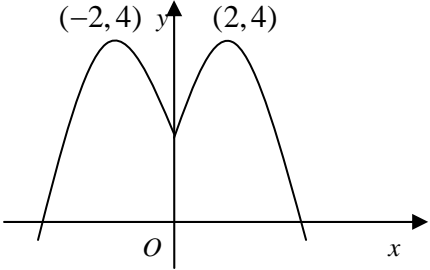
Core Mathematics C3 (6665)

January 2006

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Mark Scheme (Results)

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6665 Core Mathematics C3  
Mark Scheme

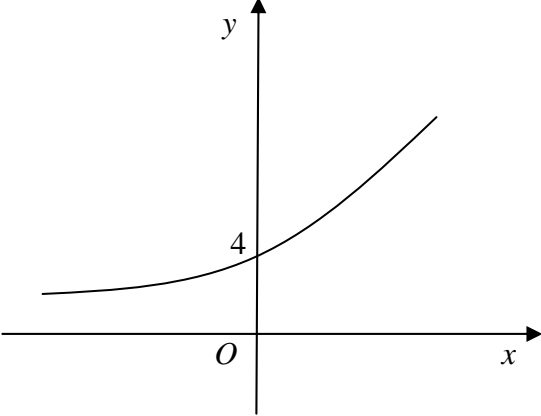
Question Number	Scheme	Marks
<p><b>1.</b></p>	<p>(a)</p> 	<p>Shape unchanged Point</p> <p>B1 B1</p> <p><b>(2)</b></p>
	<p>(b)</p> 	<p>Shape Point</p> <p>B1 B1</p> <p><b>(2)</b></p>
	<p>(c)</p> 	<p>Shape (2, 4) (-2, 4)</p> <p>B1 B1 B1</p> <p><b>(3)</b> <b>[7]</b></p>

Question Number	Scheme	Marks
2.	$x^2 - x - 2 = (x-2)(x+1)$ <p style="text-align: right;">At any stage</p> $\frac{2x^2 + 3x}{(2x+3)(x-2)} = \frac{x(2x+3)}{(2x+3)(x-2)} = \frac{x}{x-2}$ $\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2} = \frac{x(x+1) - 6}{(x-2)(x+1)}$ $= \frac{x^2 + x - 6}{(x-2)(x+1)}$ $= \frac{(x+3)(x-2)}{(x-2)(x+1)}$ $= \frac{x+3}{x+1}$	B1 B1 M1 A1 M1 A1 A1 (7) [7]
	<p>Alternative method</p> $x^2 - x - 2 = (x-2)(x+1)$ <p style="text-align: right;">At any stage</p> <p>(2x+3) appearing as a factor of the numerator at any stage</p> $\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)} = \frac{(2x^2 + 3x)(x+1) - 6(2x+3)}{(2x+3)(x-2)(x+1)}$ $= \frac{2x^3 + 5x^2 - 9x - 18}{(2x+3)(x-2)(x+1)}$ <p style="text-align: right;">can be implied</p> $= \frac{(x-2)(2x^2 + 9x + 9)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(2x+3)(x^2 + x - 6)}{(2x+3)(x-2)(x+1)} \quad \text{or} \quad \frac{(x+3)(2x^2 - x - 6)}{(2x+3)(x-2)(x+1)}$ <p style="text-align: center;">Any one linear factor <math>\times</math> quadratic</p> $= \frac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ <p style="text-align: right;">Complete factors</p> $= \frac{x+3}{x+1}$	B1 B1 M1 A1 M1 A1 (7)

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} = \frac{1}{x}$ <p style="text-align: right;">accept <math>\frac{3}{3x}</math></p> $\text{At } x = 3, \frac{dy}{dx} = \frac{1}{3} \Rightarrow m' = -3$ <p style="text-align: right;">Use of <math>mm' = -1</math></p> $y - \ln 1 = -3(x - 3)$ $y = -3x + 9$ <p style="text-align: right;">Accept <math>y = 9 - 3x</math></p> <p><math>\frac{dy}{dx} = \frac{1}{3x}</math> leading to <math>y = -9x + 27</math> is a maximum of M1 A0 M1 M1 A0 = 3/5</p>	M1 A1 M1 M1 A1 (5) [5]
4.	<p>(a) (i)</p> $\frac{d}{dx}(e^{3x+2}) = 3e^{3x+2} \quad (\text{or } 3e^2 e^{3x})$ <p style="text-align: right;">At any stage</p> $\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$ <p style="text-align: right;">Or equivalent</p> <p>(ii)</p> $\frac{d}{dx}(\cos(2x^3)) = -6x^2 \sin(2x^3)$ <p style="text-align: right;">At any stage</p> $\frac{dy}{dx} = \frac{-18x^3 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$ <p style="text-align: right;">M1 A1 (4)</p> <p>Alternatively using the product rule for second M1 A1</p> $y = (3x)^{-1} \cos(2x^3)$ $\frac{dy}{dx} = -3(3x)^{-2} \cos(2x^3) - 6x^2 (3x)^{-1} \sin(2x^3)$ <p style="text-align: right;">Accept equivalent unsimplified forms</p> <p>(b)</p> $1 = 8 \cos(2y + 6) \frac{dy}{dx} \quad \text{or} \quad \frac{dx}{dy} = 8 \cos(2y + 6)$ $\frac{dy}{dx} = \frac{1}{8 \cos(2y + 6)}$ $\frac{dy}{dx} = \frac{1}{8 \cos\left(\arcsin\left(\frac{x}{4}\right)\right)} \quad \left( = (\pm) \frac{1}{2\sqrt{(16-x^2)}} \right)$	B1 M1 A1+A1 (4) M1 A1 M1 A1 (4) M1 M1 A1 M1 A1 (5) [13]

Question Number	Scheme	Marks
5.	<p>(a) <math>2x^2 - 1 - \frac{4}{x} = 0</math> Dividing equation by <math>x</math> M1  <math>x^2 = \frac{1}{2} + \frac{4}{2x}</math> Obtaining <math>x^2 = \dots</math> M1  <math>x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}</math> * cso A1 (3)</p> <p>(b) <math>x_1 = 1.41, x_2 = 1.39, x_3 = 1.39</math> B1, B1, B1  If answers given to more than 2 dp, penalise first time then accept awrt above. (3)</p> <p>(c) Choosing (1.3915, 1.3925) or a tighter interval M1  <math>f(1.3915) \approx -3 \times 10^{-3}, f(1.3925) \approx 7 \times 10^{-3}</math> Both, awrt A1  Change of sign (and continuity) <math>\Rightarrow \alpha \in (1.3915, 1.3925)</math>  <math>\Rightarrow \alpha = 1.392</math> to 3 decimal places * cso A1 (3)</p>	<p>M1 M1 A1 (3)</p> <p>B1, B1, B1 (3)</p> <p>M1 A1 A1 (3) [9]</p>
6.	<p>(a) <math>R \cos \alpha = 12, R \sin \alpha = 4</math>  <math>R = \sqrt{(12^2 + 4^2)} = \sqrt{160}</math> Accept if just written down, awrt 12.6 M1 A1  <math>\tan \alpha = \frac{4}{12}, \Rightarrow \alpha \approx 18.43^\circ</math> awrt <math>18.4^\circ</math> M1, A1(4)</p> <p>(b) <math>\cos(x + \text{their } \alpha) = \frac{7}{\text{their } R}</math> (<math>\approx 0.5534</math>) M1  <math>x + \text{their } \alpha = 56.4^\circ</math> awrt <math>56^\circ</math> A1  <math>= \dots, 303.6^\circ</math> <math>360^\circ - \text{their principal value}</math> M1  <math>x = 38.0^\circ, 285.2^\circ</math> Ignore solutions out of range A1, A1 (5)  If answers given to more than 1 dp, penalise first time then accept awrt above.</p> <p>(c)(i) minimum value is <math>-\sqrt{160}</math> ft their <math>R</math> B1ft</p> <p>(ii) <math>\cos(x + \text{their } \alpha) = -1</math> M1  <math>x \approx 161.57^\circ</math> cao A1 (3)</p>	<p>M1 A1 M1, A1(4)</p> <p>M1 A1 M1 A1, A1 (5)</p> <p>B1ft</p> <p>M1 A1 (3) [12]</p>

Question Number	Scheme	Marks
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1
	$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x \quad *$	cso A1 (2)
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2} \quad *$	cso A1 (3)
	(b)	Using (a)(i) M1
	$\cos \theta (\cos \theta - \sin \theta) = \frac{1}{2}$	
	$\cos^2 \theta - \cos \theta \sin \theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$	Using (a)(ii) M1
	$\cos 2\theta = \sin 2\theta \quad *$	A1 (3)
(c)	Using (a)(ii) M1	
$\tan 2\theta = 1$	M1	
$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$	any one correct value of $2\theta$ A1	
$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$	Obtaining at least 2 solutions in range M1	
<p>The 4 correct solutions If decimals (0.393, 1.963, 3.534, 5.105) or degrees (22.5°, 112.5°, 202.5°, 292.5°) are given, but all 4 solutions are found, penalise one A mark only. Ignore solutions out of range.</p>	A1 (4) [12]	

Question Number	Scheme	Marks
8.	<p>(a) <math display="block">gf(x) = e^{2(2x+\ln 2)}</math> <math display="block">= e^{4x} e^{2\ln 2}</math> <math display="block">= e^{4x} e^{\ln 4}</math> <math display="block">= 4e^{4x}</math>           Give mark at this point, cso            (Hence <math>gf : x \mapsto 4e^{4x}, x \in \mathbb{R}</math>)</p> <p>(b) </p> <p>(c) Range is <math>\mathbb{R}_+</math>      Accept <math>gf(x) &gt; 0, y &gt; 0</math></p> <p>(d) <math display="block">\frac{d}{dx}[gf(x)] = 16e^{4x}</math> <math display="block">e^{4x} = \frac{3}{16}</math> <math display="block">4x = \ln \frac{3}{16}</math> <math display="block">x \approx -0.418</math></p>	<p>M1 M1 M1 A1      (4)</p> <p>Shape and point      B1      (1)</p> <p>B1      (1)</p> <p>M1 A1 M1 A1      (4) [10]</p>