

GCE

Edexcel GCE

Core Mathematics C4 (6666)

January 2006

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Mark Scheme (Results)

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6666 Core Mathematics C4
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Question Number	Scheme	Marks												
1.	<p>Differentiates</p> <p>to obtain : $6x + 8y \frac{dy}{dx} - 2,$ $\dots\dots\dots + (6x \frac{dy}{dx} + 6y) = 0$</p> $\left[\frac{dy}{dx} = \frac{2 - 6x - 6y}{6x + 8y} \right]$ <p>Substitutes $x = 1, y = -2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx} = -\frac{8}{10}$</p> <p>Uses line equation with numerical 'gradient' $y - (-2) = (\text{their gradient})(x - 1)$ or finds c and uses $y = (\text{their gradient})x + "c"$</p> <p>To give $5y + 4x + 6 = 0$ (or equivalent = 0)</p>	<p>M1</p> <p>A1,</p> <p>+(B1)</p> <p>M1, A1</p> <p>M1</p> <p>A1√ [7]</p>												
2. (a)	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 10%;">x</td> <td style="width: 10%;">0</td> <td style="width: 10%;">$\frac{\pi}{16}$</td> <td style="width: 10%;">$\frac{\pi}{8}$</td> <td style="width: 10%;">$\frac{3\pi}{16}$</td> <td style="width: 10%;">$\frac{\pi}{4}$</td> </tr> <tr> <td>y</td> <td>1</td> <td>1.01959</td> <td>1.08239</td> <td>1.20269</td> <td>1.41421</td> </tr> </table> <p>M1 for one correct, A1 for all correct</p>	x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	y	1	1.01959	1.08239	1.20269	1.41421	<p>M1 A1</p> <p>(2)</p>
x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$									
y	1	1.01959	1.08239	1.20269	1.41421									
(b)	<p>Integral = $\frac{1}{2} \times \frac{\pi}{16} \times \{1 + 1.4142 + 2(1.01959 + \dots + 1.20269)\}$</p> $\left(= \frac{\pi}{32} \times 9.02355 \right) = 0.8859$	<p>M1 A1√</p> <p>A1 cao (3)</p>												
(c)	<p>Percentage error = $\frac{\text{approx} - 0.88137}{0.88137} \times 100 = 0.51\%$ (allow 0.5% to 0.54% for A1)</p> <p>M1 gained for $(\pm) \frac{\text{approx} - \ln(1 + \sqrt{2})}{\ln(1 + \sqrt{2})}$</p>	<p>M1 A1 (2)</p> <p>[7]</p>												

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3.	<p>Uses substitution to obtain $x = f(u) \left[\frac{u^2 + 1}{2} \right]$,</p> <p>and to obtain $u \frac{du}{dx} = \text{const. or equiv.}$</p> <p>Reaches $\int \frac{3(u^2 + 1)}{2u} u du$ or equivalent</p> <p>Simplifies integrand to $\int \left(3u^2 + \frac{3}{2} \right) du$ or equiv.</p> <p>Integrates to $\frac{1}{2}u^3 + \frac{3}{2}u$</p> <p>A1√ dependent on all previous Ms</p> <p>Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)</p> <p>To give 16 cso</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1 A1√</p> <p>M1</p> <p>A1</p>
[8]		
	<p>“By Parts”</p> <p>Attempt at “right direction” by parts M1</p> $\left[3x \left(2x - 1 \right)^{\frac{1}{2}} \right] - \left\{ \int 3 \left(2x - 1 \right)^{\frac{1}{2}} dx \right\}$ <p>..... - $\left(2x - 1 \right)^{\frac{3}{2}}$ M1A1√</p> <p>Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1</p>	<hr style="width: 100%; margin-top: 20px;"/>

4.	<p>Attempts $V = \pi \int x^2 e^{2x} dx$</p> $= \pi \left[\frac{x^2 e^{2x}}{2} - \int x e^{2x} dx \right]$ <p>(M1 needs parts in the correct direction)</p> $= \pi \left[\frac{x^2 e^{2x}}{2} - \left(\frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right) \right]$ <p>(M1 needs second application of parts)</p> <p>M1A1√ refers to candidates $\int x e^{2x} dx$, but dependent on prev. M1</p> $= \pi \left[\frac{x^2 e^{2x}}{2} - \left(\frac{x e^{2x}}{2} - \frac{e^{2x}}{4} \right) \right]$ <p>Substitutes limits 3 and 1 and subtracts to give... [dep. on second and third Ms]</p> $= \pi \left[\frac{13}{4} e^6 - \frac{1}{4} e^2 \right]$ <p>or any correct exact equivalent.</p> <p>[Omission of π loses first and last marks only]</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1√</p> <p>A1 cao</p> <p>dM1</p> <p>A1</p> <p>[8]</p>
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Question Number	Scheme	Marks
5. (a)	<p>Considers $3x^2 + 16 = A(2+x)^2 + B(1-3x)(2+x) + C(1-3x)$</p> <p>and substitutes $x = -2$, or $x = 1/3$,</p> <p>or compares coefficients and solves simultaneous equations</p> <p>To obtain $A = 3$, and $C = 4$</p> <p>Compares coefficients or uses simultaneous equation to show $B = 0$.</p>	<p>M1</p> <p>A1, A1</p> <p>B1</p> <p>(4)</p>
5. (b)	<p>Writes $3(1-3x)^{-1} + 4(2+x)^{-2}$</p> <p>$= 3(1+3x+9x^2+27x^3+\dots) +$</p> $\frac{4}{4}\left(1 + \frac{(-2)}{1}\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3 + \dots\right)$ <p>$= 4 + 8x, + 27\frac{3}{4}x^2 + 80\frac{1}{2}x^3 + \dots$</p> <p>Or uses $(3x^2 + 16)(1-3x)^{-1}(2+x)^{-2}$</p> <p>$(3x^2 + 16)(1+3x+9x^2+27x^3+) \times$</p> $\frac{1}{4}\left(1 + \frac{(-2)}{1}\left(\frac{x}{2}\right) + \frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3\right)$ <p>$= 4 + 8x, + 27\frac{3}{4}x^2 + 80\frac{1}{2}x^3 + \dots$</p>	<p>M1</p> <p>(M1, A1)</p> <p>(M1 A1)</p> <p>A1, A1</p> <p>(7)</p> <p>M1</p> <p>(M1A1)×</p> <p>(M1A1)</p> <p>A1, A1</p> <p>(7)</p> <p>[11]</p>

<p>6. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$\lambda = -4 \rightarrow a = 18, \quad \mu = 1 \rightarrow b = 9$</p> $\begin{pmatrix} 8 + \lambda \\ 12 + \lambda \\ 14 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$ <p>$\therefore 8 + \lambda + 12 + \lambda - 14 + \lambda = 0$</p> <p>Solves to obtain $\lambda \quad (\lambda = -2)$</p> <p>Then substitutes value for λ to give P at the point (6, 10, 16) (any form)</p> <p>$OP = \sqrt{36 + 100 + 256}$</p> <p>$(= \sqrt{392}) = 14\sqrt{2}$</p>	<p>M1 A1, A1 (3)</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>M1, A1 (5)</p> <p>M1</p> <p>A1 cao (2)</p> <p>[10]</p>
<p>7. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>$\frac{dV}{dr} = 4\pi r^2$</p> <p>Uses $\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$ in any form, $= \frac{1000}{4\pi r^2 (2t+1)^2}$</p> <p>$V = \int 1000(2t+1)^{-2} dt$ and integrate to $p (2t+1)^{-1}, \quad = -500(2t+1)^{-1} (+c)$</p> <p>Using $V=0$ when $t=0$ to find c, ($c = 500$, or equivalent)</p> <p>$\therefore V = 500(1 - \frac{1}{2t+1})$ (any form)</p> <p>(i) Substitute $t = 5$ to give V, then use $r = \sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give $r, = 4.77$</p> <p>(ii) Substitutes $t = 5$ and $r =$ 'their value' into 'their' part (b)</p> <p>$\frac{dr}{dt} = 0.0289 \quad (\approx 2.90 \times 10^{-2}) \text{ (cm/s) } * \quad \text{AG}$</p>	<p>B1 (1)</p> <p>M1, A1 (2)</p> <p>M1, A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1,</p> <p>M1, A1 (3)</p> <p>M1</p> <p>A1 (2)</p> <p>[12]</p>

8. (a)	<p>Solves $y = 0 \Rightarrow \cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ (need both for A1)</p> <p>Or substitutes both values of t and shows that $y = 0$</p>	M1 A1 (2)
(b)	$\frac{dx}{dt} = 1 - 2\cos t$ $\text{Area} = \int y dx = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)(1 - 2\cos t) dt = \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 dt \quad * \quad \text{AG}$	M1 A1 B1 (3)
(c)	$\text{Area} = \int 1 - 4\cos t + 4\cos^2 t dt \quad \text{3 terms}$ $= \int 1 - 4\cos t + 2(\cos 2t + 1) dt \quad (\text{use of correct double angle formula})$ $= \int 3 - 4\cos t + 2\cos 2t dt$ $= [3t - 4\sin t + \sin 2t]$ <p>Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts.</p> $= 4\pi + 3\sqrt{3}$	M1 M1 M1 A1 M1 A1A1 (7) [12]