

1. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{5x - 1}.$$

(6)

- (b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x .

(4)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2. $f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

(3)

[illegible]

3.

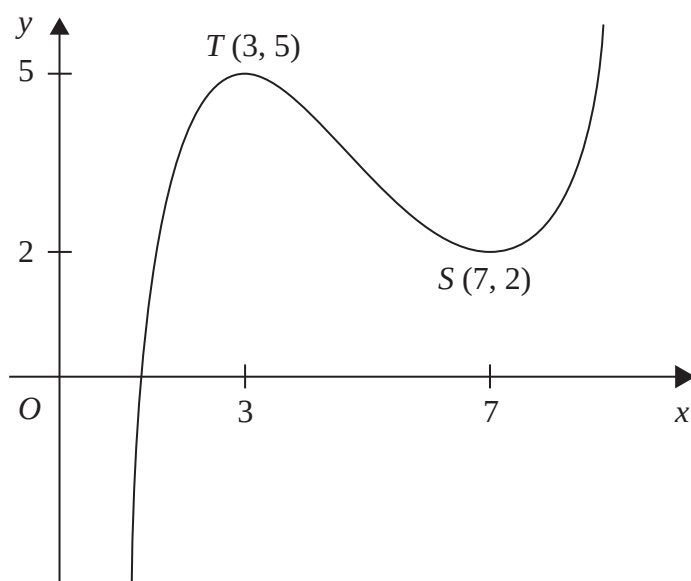


Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$, (3)

(b) $y = |f(x)|$. (3)

Indicate on each diagram the coordinates of any turning points on your sketch.



4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

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5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}$$

- (a) Write down the range of g .

(1)

- (b) Show that the composite function fg is defined by

$$\text{fg}: x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

- (c) Write down the range of fg .

(1)

- (d) Solve the equation $\frac{d}{dx}[fg(x)] = x(xe^{x^2} + 2)$.

(6)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π .

(5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)



7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

- (a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

- (b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 .

(3)

- (c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

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(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)^\circ + 4 \sin(15t)^\circ,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(d) Find the value of t when this minimum temperature occurs.

[illegible]

Question 8 continued

Lined area for writing the answer to Question 8 continued.

Q8

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(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

