

Mark Scheme (Results)

January 2011

GCE

GCE Core Mathematics C3 (6665) Paper 1

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General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M marks:** method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A marks:** Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B marks** are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol \checkmark will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- \square The second mark is dependent on gaining the first mark

January 2011
Core Mathematics C3 6665
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) $7\cos x - 24\sin x = R\cos(x + \alpha)$</p> <p>$7\cos x - 24\sin x = R\cos x \cos \alpha - R\sin x \sin \alpha$</p> <p>Equate $\cos x$: $7 = R\cos \alpha$</p> <p>Equate $\sin x$: $24 = R\sin \alpha$</p> <p>$R = \sqrt{7^2 + 24^2} = 25$ $R = 25$</p> <p>$\tan \alpha = \frac{24}{7} \Rightarrow \alpha = 1.287002218...^{\circ}$ $\tan \alpha = \frac{24}{7}$ or $\tan \alpha = \frac{7}{24}$</p> <p style="text-align: right;">awrt 1.287</p> <p>Hence, $7\cos x - 24\sin x = 25\cos(x + 1.287)$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
(b)	<p>Minimum value = <u>-25</u> -25 or -R</p>	<p>B1ft</p> <p>(1)</p>
(c)	<p>$7\cos x - 24\sin x = 10$</p> <p>$25\cos(x + 1.287) = 10$</p> <p>$\cos(x + 1.287) = \frac{10}{25}$ $\cos(x \pm \text{their } \alpha) = \frac{10}{(\text{their } R)}$</p> <p>PV = 1.159279481...^c or 66.42182152...^o For applying $\cos^{-1}\left(\frac{10}{(\text{their } R)}\right)$</p> <p>So, $x + 1.287 = \{1.159279...^{\circ}, 5.123906...^{\circ}, 7.442465...^{\circ}\}$ either $2\pi +$ or $-$ their PV^c or $360^{\circ} +$ or $-$ their PV^o</p> <p>gives, $x = \{3.836906..., 6.155465...\}$ awrt 3.84 OR 6.16</p> <p style="text-align: right;">awrt 3.84 AND 6.16</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(5)</p> <p>[9]</p>

Question Number	Scheme	Marks
2.		
(a)	$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$ $= \frac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$ $= \frac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$ $= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ $= \frac{4x+1}{2x-1}$ <p>An attempt to form a single fraction</p> <p>Simplifies to give a correct quadratic numerator over a correct quadratic denominator</p> <p>An attempt to factorise a 3 term quadratic numerator</p>	M1 A1 aef M1 A1 (4)
(b)	$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$ $f(x) = \frac{(4x+1)}{(2x-1)} - 2$ $= \frac{(4x+1) - 2(2x-1)}{(2x-1)}$ $= \frac{4x+1-4x+2}{(2x-1)}$ $= \frac{3}{(2x-1)}$ <p>An attempt to form a single fraction</p> <p>Correct result</p>	M1 A1 * (2)
(c)	$f(x) = \frac{3}{(2x-1)} = 3(2x-1)^{-1}$ $f'(x) = 3(-1)(2x-1)^{-2}(2)$ $f'(2) = \frac{-6}{9} = -\frac{2}{3}$ <p>$\pm k(2x-1)^{-2}$</p> <p>Either $\frac{-6}{9}$ or $-\frac{2}{3}$</p>	M1 A1 aef A1 (3) [9]

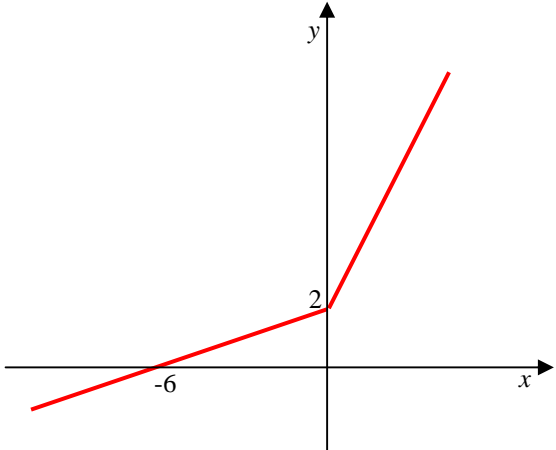
Question Number	Scheme	Marks
3.	$2 \cos 2\theta = 1 - 2 \sin \theta$ $2(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$ $2 - 4 \sin^2 \theta = 1 - 2 \sin \theta$ $4 \sin^2 \theta - 2 \sin \theta - 1 = 0$ $\sin \theta = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$ $\theta = \{54, 126, 198, 342\}$	<p>Substitutes either $1 - 2 \sin^2 \theta$ or $2 \cos^2 \theta - 1$ or $\cos^2 \theta - \sin^2 \theta$ for $\cos 2\theta$. M1</p> <p>Forms a “quadratic in sine” = 0 M1(*)</p> <p>Applies the quadratic formula See notes for alternative methods. M1</p> <p>Any one correct answer A1 180-their pv dM1(*) All four solutions correct. A1</p> <p>[6]</p>

Question Number	Scheme	Marks
4.		
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *) $\{t = 0, \theta = 90 \Rightarrow\} \quad 90 = 20 + Ae^{-k(0)} \quad \text{Substitutes } t = 0 \text{ and } \theta = 90 \text{ into eqn } *$ $90 = 20 + A \Rightarrow \underline{A = 70} \quad \underline{A = 70}$	M1 A1 (2)
(b)	$\theta = 20 + 70e^{-kt}$ $\{t = 5, \theta = 55 \Rightarrow\} \quad 55 = 20 + 70e^{-k(5)} \quad \text{Substitutes } t = 5 \text{ and } \theta = 55 \text{ into eqn } *$ $\frac{35}{70} = e^{-5k} \quad \text{and rearranges eqn } * \text{ to make } e^{\pm 5k} \text{ the subject.}$ $\ln\left(\frac{35}{70}\right) = -5k \quad \text{Takes 'lns' and proceeds to make '}\pm 5k\text{' the subject.}$ $-5k = \ln\left(\frac{1}{2}\right)$ $-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2} \quad \text{Convincing proof that } k = \frac{1}{5}\ln 2$	M1 dM1 A1 * (3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$ $\frac{d\theta}{dt} = -\frac{1}{5}\ln 2 \cdot (70)e^{-\frac{1}{5}t\ln 2} \quad \pm \alpha e^{-kt} \text{ where } k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$ When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$ $\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$ Rate of decrease of $\theta = 2.426^\circ\text{C/min}$ (3 dp.) awrt ± 2.426	M1 A1 oe A1 (3) [8]

Question Number	Scheme	Marks
5.		
(a)	<p>Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8 - x)\ln x = 0$</p> <p>Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$</p> <p>Coordinates are $A(1, 0)$ and $B(8, 0)$.</p>	<p>Either one of $\{x\}=1$ OR $x=\{8\}$ B1</p> <p>Both $A(1, \{0\})$ and $B(8, \{0\})$ B1</p> <p>(2)</p>
(b)	<p>Apply product rule: $\left\{ \begin{array}{l} u = (8 - x) \quad v = \ln x \\ \frac{du}{dx} = -1 \quad \frac{dv}{dx} = \frac{1}{x} \end{array} \right\}$</p> <p>$f'(x) = -\ln x + \frac{8-x}{x}$</p>	<p>$vu' + uv'$ M1</p> <p>Any one term correct A1</p> <p>Both terms correct A1</p> <p>(3)</p>
(c)	<p>$f'(3.5) = 0.032951317...$</p> <p>$f'(3.6) = -0.058711623...$</p> <p>Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.</p>	<p>Attempts to evaluate both $f'(3.5)$ and $f'(3.6)$ M1</p> <p>both values correct to at least 1 sf, sign change and conclusion A1</p> <p>(2)</p>
(d)	<p>At Q, $f'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$</p> <p>$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$</p> <p>$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$</p> <p>$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)</p>	<p>Setting $f'(x) = 0$. M1</p> <p>Splitting up the numerator and proceeding to $x=$ M1</p> <p>For correct proof. No errors seen in working. A1</p> <p>(3)</p>

Question Number	Scheme	Marks
(e)	<p>Iterative formula: $x_{n+1} = \frac{8}{\ln x_n + 1}$</p> <p>$x_1 = \frac{8}{\ln(3.55) + 1}$</p> <p>$x_1 = 3.528974374...$</p> <p>$x_2 = 3.538246011...$</p> <p>$x_3 = 3.534144722...$</p> <p>$x_1 = 3.529, x_2 = 3.538, x_3 = 3.534, \text{ to 3 dp.}$</p>	<p>An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$ Both $x_1 = \text{awrt } 3.529$ and $x_2 = \text{awrt } 3.538$</p> <p>M1</p> <p>A1</p> <p>x_1, x_2, x_3 all stated correctly to 3 dp</p> <p>A1</p> <p>(3) [13]</p>

Question Number	Scheme	Marks
6.		
(a)	$y = \frac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$ <p>Attempt to make x (or swapped y) the subject</p> $xy - 5y = 3 - 2x$ $\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$ <p>Collect x terms together and factorise.</p> $\Rightarrow x = \frac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \frac{3+5x}{x+2}$	M1 A1 oe (3)
(b)	Range of g is $-9 \leq g(x) \leq 4$ or $-9 \leq y \leq 4$ <p>Correct Range</p>	B1 (1)
(c)	<p>Deduces that $g(2)$ is 0. Seen or implied.</p> $g(2) = g(0) = -6$, from sketch.	M1 -6 A1 (2)
(d)	$fg(8) = f(4)$ <p>Correct order g followed by f</p> $= \frac{3-4(2)}{4-5} = \frac{-5}{-1} = 5$	M1 5 A1 (2)

Question Number	Scheme	Marks
(e)(ii)	 <p>Correct shape</p> <p>Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.</p>	<p>B1</p> <p>B1</p> <p>(4)</p>
(f)	<p>Domain of g^{-1} is $-9 \leq x \leq 4$</p> <p>Either correct answer or a follow through from part (b) answer</p>	<p>B1 $\sqrt{\quad}$</p> <p>(1)</p> <p>[13]</p>

Question Number	Scheme	Marks
7	<p>(a) $y = \frac{3 + \sin 2x}{2 + \cos 2x}$</p> <p>Apply quotient rule:</p> $\left\{ \begin{array}{l} u = 3 + \sin 2x \quad v = 2 + \cos 2x \\ \frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = -2 \sin 2x \end{array} \right\}$ $\frac{dy}{dx} = \frac{2 \cos 2x(2 + \cos 2x) - -2 \sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 2 \cos^2 2x + 6 \sin 2x + 2 \sin^2 2x}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2}{(2 + \cos 2x)^2} \quad (\text{as required})$	<p>Applying $\frac{u'v - uv'}{v^2}$ M1</p> <p>Any one term correct on the numerator A1</p> <p>Fully correct (unsimplified). A1</p> <p>For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. A1*</p> <p>No errors seen in working. A1*</p> <p>(4)</p>
(b)	<p>When $x = \frac{\pi}{2}$, $y = \frac{3 + \sin \pi}{2 + \cos \pi} = \frac{3}{1} = 3$</p> <p>At $(\frac{\pi}{2}, 3)$, $m(\mathbf{T}) = \frac{6 \sin \pi + 4 \cos \pi + 2}{(2 + \cos \pi)^2} = \frac{-4 + 2}{1^2} = -2$</p> <p>Either \mathbf{T}: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$; \mathbf{T}: $y = -2x + (\pi + 3)$</p>	<p>$y = 3$ B1</p> <p>$m(\mathbf{T}) = -2$ B1</p> <p>$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; M1 or uses $y = mx + c$ with 'their TANGENT gradient';</p> <p>$y = -2x + \pi + 3$ A1</p> <p>(4) [8]</p>

Question Number	Scheme	Marks
8.	<p>(a) $y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$</p> <p>$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$</p> <p>$\frac{dy}{dx} = \frac{\left\{ \frac{\sin x}{\cos^2 x} \right\}}{\left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right)} = \underline{\underline{\sec x \tan x}}$</p> <p>Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\frac{dy}{dx} = \pm((\cos x)^{-2}(\sin x))$ $-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$</p> <p>Convincing proof. Must see both <u>underlined steps.</u></p>	<p>M1</p> <p>A1</p> <p>A1 AG</p> <p>(3)</p>
(b)	<p>$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$</p> <p>$\frac{dx}{dy} = 2 \sec 2y \tan 2y$</p> <p>$K \sec 2y \tan 2y$ $2 \sec 2y \tan 2y$</p>	<p>M1</p> <p>A1</p> <p>(2)</p>
(c)	<p>$\frac{dy}{dx} = \frac{1}{2 \sec 2y \tan 2y}$</p> <p>$\frac{dy}{dx} = \frac{1}{2x \tan 2y}$</p> <p>$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$</p> <p>So $\tan^2 2y = x^2 - 1$</p> <p>$\frac{dy}{dx} = \frac{1}{2x\sqrt{(x^2-1)}}$</p> <p>Applies $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$</p> <p>Substitutes x for $\sec 2y$.</p> <p>Attempts to use the identity $1 + \tan^2 A = \sec^2 A$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>[9]</p>

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