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Mark Scheme (Results)

January 2012

GCE Core Mathematics C4 (6666) Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principals for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*), leading to x = …

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

January 2012
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\cancel{\times}\right\}\quad \underline{2+6y\frac{dy}{dx}}+\left(\underline{\underline{6xy+3x^2\frac{dy}{dx}}}\right)=\underline{8x}$	M1 A1 B1
	$\left\{\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $\mathrm{m}(\mathbf{T})=\frac{dy}{dx}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $\mathrm{m}(\mathbf{N})=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	N: $y-1=\frac{9}{4}(x+1)$	M1
	N: $9x-4y+13=0$	A1
		[3]
		8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $3x^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx}=\right)$). **A1**: $(2x+3y^2)\to\left(\underline{2+6y\frac{dy}{dx}}\right)$ and $(4x^2\to\underline{8x})$. **Note:** If an extra "sixth" term appears then award A0. **B1**: $6xy+3x^2\frac{dy}{dx}$. **dM1**: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly. If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0. Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T})=-4$ Note that this mark is dependent on the previous method mark being awarded. **A1**: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44 If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1:** Applies $\mathrm{m}(\mathbf{N})=-\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$. **M1:** Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(\mathrm{m}_N)x+"c"$, Where $m_N=-\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N=\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N=-\text{their } \mathrm{m}(\mathbf{T})$. **A1:** $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc. Must be "$=0$". So do not allow $9x+13=4y$ etc. **Note**: $m_N=-\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	

Alternative method for part (a): Differentiating with respect to y

$$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}y}} \cancel{=}\right\} \quad \underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y} + \underline{\underline{\left(6xy\frac{\mathrm{d}x}{\mathrm{d}y} + 3x^2\right)}} = \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$$

M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} =\right)$).

A1: $\left(2x+3y^2\right) \to \left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y}\right)$ and $\left(4x^2 \to \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6x\,y + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x = -1$ and $y = 1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x = 1$ and $y = -1$, then award M0.

Candidates who substitute $x = 1$ and $y = -1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme	Marks
2. (a)	$\int x\sin 3x\ \mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\ \{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\ \{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x\ \mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\ \{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)\ \{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x\ \ \{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x\ \{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without + c. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x\ \{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without + c, can be un-simplified. You can ignore subsequent working here. **Special Case**: If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.	

Question Number	Scheme		Marks
3. (a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$	<u>B1</u>
	$=\left\{\frac{1}{4}\right\}\left[1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2+\ldots\right]$	see notes	M1 A1ft
	$=\left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x}{2}\right)^2+\ldots}\right]$		
	$=\frac{1}{4}\left[1+5x;+\frac{75}{4}x^2+\ldots\right]$	See notes below!	
	$=\frac{1}{4}+\frac{5}{4}x;+\frac{75}{16}x^2+\ldots$		A1; A1
			[5]
(b)	$\left\{\frac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\left\{\frac{75}{16}x^2+\ldots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\frac{2(5x)}{4}+\frac{kx}{4}=\frac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\frac{150x^2}{16}+\frac{5kx^2}{4}$		M1
	So, $A = \frac{75}{8}+\frac{5(-3)}{4} = \frac{75}{8}-\frac{15}{4} = \underline{\frac{45}{8}}$	$\underline{\frac{45}{8}}$ or $\underline{5\frac{5}{8}}$ or $\underline{5.625}$	A1
			[2] **9**

(a)

<u>**B1**</u>: $\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$ outside brackets or $\underline{\frac{1}{4}}$ as candidate's constant term in their binomial expansion.

M1: Expands to give a simplified or an un-simplified,

$1+(-2)(**x)$ or $(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2$ or $1+\ldots\ldots+\frac{(-2)(-3)}{2!}(**x)^2$, **where** $**\neq 1$**.**

A1: A correct simplified or an un-simplified $1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2$ expansion with candidate's follow through $(**x)$. Note that $(**x)$ must be consistent.

You would award B1M1A0 for $=\frac{1}{4}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}(-5x)^2+\ldots}\right]$ because ** is not consistent.

Invisible brackets $\left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x^2}{2}\right)+\ldots}\right]$ is M1A0 unless recovered.

A1: For $\frac{1}{4}+\frac{5}{4}x$ **(simplified fractions)** or Also allow $0.25+1.25x$ or $\frac{1}{4}+1\frac{1}{4}x$**.**

Allow Special Case A1 for either SC: $\frac{1}{4}[1+5x;\ldots]$ or **SC**: $K\left[1+5x+\frac{75}{4}x^2+\ldots\right]$.

A1: Accept only $\frac{75}{16}x^2$ or $4\frac{11}{16}x^2$ or $4.6875x^2$

<u>**Alternative method:**</u> Candidates can apply an alternative form of the binomial expansion. (See next page).

3. (b)	**M1:** Candidate writes down $(2+kx)\big($their part (a) answer, at least up to the term in x.$\big)$ $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\ \ldots\right)$ or $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\ldots\right)$ are fine. This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x. **A1:** $k=-3$
(c)	**M1:** Multiplies out their $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\ldots\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find A using a numerical value of k. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0. *Alternative method for part (a)* $(2-5x)^{-2}=(2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2$ **B1:** $\frac{1}{4}$ or $(2)^{-2}$, **M1:** Any two of three (un-simplified) terms correct. **A1:** All three (un-simplified) terms correct. **A1:** $\frac{1}{4}+\frac{5}{4}x$ **A1:** $\frac{75}{16}x^2$ **Note:** The terms in C need to be evaluated, so ${}^{-2}C_0(2)^{-2}+{}^{-2}C_1(2)^{-3}(-5x);+{}^{-2}C_2(2)^{-4}(-5x)^2$ without further working is B0M0A0. *Alternative method for parts (b) and (c)* $(2+kx)=(2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+\ldots\right)$ $(2+kx)=(4-20x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+\ldots\right)$ $(2+kx)=2+(7x-10x)+\left(4Ax^2-35x^2+\frac{25}{2}x^2\right)$ Equate x terms: $\underline{k=-3}$ Equate x^2 terms: $0=4A-35+\frac{25}{2}\Rightarrow 4A=\frac{45}{2}\Rightarrow \underline{A=\frac{45}{8}}$
(b)	**M1:** For $(2+kx)=(4\pm\lambda x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+\ldots\right)$, where $\lambda\neq 0$ **A1:** $k=-3$
(c)	**M1:** Multiplies out to obtain three x^2 terms/coefficients, equates to 0 and attempts to find A. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

Question Number	Scheme		Marks
4.	Volume $= \pi \int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 \mathrm{d}x$	Use of $V = \pi \int y^2 \,\mathrm{d}x$.	B1
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right) - \left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \frac{1}{3}\pi\ln 4$	$\frac{1}{3}\pi\ln 4$ or $\frac{2}{3}\pi\ln 2$	A1 oe isw
			[5] **5**

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and $\mathrm{d}x$. This can be implied by later working,

but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note**: $\pi\left(\int y\right)^2$ would be B0.

Working in x

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16 - \ln 4)$ would be A0.

Working in u: where $u = 3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln(pu)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

A1: As above!

Question Number	Scheme	Marks
5.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1 $\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}, \quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] **8**

(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. *Alternative differentiation in part (a)* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$	

5. (b)	**M1:** Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage. **M1:** Candidate substitutes a found value of *t*, to attempt to find either one of *x* or *y*. The first two method marks can be implied by ONE correct set of coordinates for $(x,\ y)$ or $(y,\ x)$ interchanged. A correct point coming from NO WORKING can be awarded M1M1. **A1:** At least TWO sets of coordinates. **A1:** At least THREE sets of coordinates. **A1:** ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. **Note:** Candidate can use the diagram's symmetry to write down some of their coordinates. **Note:** When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. **Note:** The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0\ \&\ \cos t = 0$ has the potential to achieve all five marks. **Note:** It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a). *(b) An alternative method for finding the coordinates of the two maximum points.* Some candidates may use $y = 3\cos 2t$ to write down that the *y*-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the *x*-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the *x*-coordinates for the maximum point. **M1M1:** Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$. **Note:** these marks can only be awarded together for a candidate using this method. **A1:** For both $(2, 3)$ or $(-2, 3)$. **A0A0**: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

Question Number	Scheme		Marks
6. (a)	0.73508		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}$; $\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 <u>M1</u>
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp)	awrt 1.1504	A1 **[3]**
(c)	$\{u = 1+\cos x\} \Rightarrow \underline{\frac{du}{dx} = -\sin x}$		<u>B1</u>
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)}\,dx =\right\} \int \frac{2(2\sin x\cos x)}{(1+\cos x)}\,dx$	$\sin 2x = 2\sin x\cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\,du \quad \left\{= 4\int \frac{(1-u)}{u}\,du\right\}$		M1
	$= 4\int\left(\frac{1}{u}-1\right)du = 4(\ln u - u)+c$		dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x)+c = 4\ln(1+\cos x) - 4\cos x + k$	**AG**	A1 **cso** **[5]**
(d)	$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$	Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$		
	$= 4 - 4\ln 2 \ \{= 1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \left\|(4-4\ln 2) - 1.1504...\right\|$ $= 0.0770112776... = 0.077$ (2sf)	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
			12

(a) **B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.

(b) **B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196

M1: <u>For structure of trapezium rule</u>$\left[\ \underline{............}\ \right]$; (0 can be implied).

A1: anything that rounds to 1.1504

<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8} + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552).

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}\ (0+0) + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589).

<u>Alternative method for part (b): Adding individual trapezia</u>

$$\text{Area} \approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$$

B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets.

M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.

A1: anything that rounds to 1.1504

6. (c)

B1: $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$ or $\mathrm{d}u = -\sin x\,\mathrm{d}x$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{-\sin x}$ oe.

B1: For seeing, applying or implying $\sin 2x = 2\sin x \cos x$.

M1: After applying substitution candidate achieves $\pm k\int \frac{(u-1)}{u}(\mathrm{d}u)$ or $\pm k\int \frac{(1-u)}{u}(\mathrm{d}u)$.

Allow M1 for "invisible" brackets here, eg: $\pm\int \frac{(\lambda u-1)}{u}(\mathrm{d}u)$ or $\pm\int \frac{(-\lambda+u)}{u}(\mathrm{d}u)$, where λ is a positive constant.

dM1: An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)\mathrm{d}u \to \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded.

<u>**Alternative method:**</u> Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below).

A1: Correctly combines their $+c$ and "-4" together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$

As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \to 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \to 4\ln(1+\cos x) - 4\cos x + k$

Note: that this mark is also for a correct solution only.

Note: those candidates who attempt to find the value of k will usually achieve A0.

(d)

M1: Substitutes limits of $x = \frac{\pi}{2}$ and $x = 0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note** that: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0.

A1: $4(1-\ln 2)$ or $4 - 4\ln 2$ or awrt 1.2, however found.

This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3

A1: For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0.

<u>*Alternative method for dM1 in part (c)*</u>

$$\int \frac{(1-u)}{u}\,\mathrm{d}u = \left((1-u)\ln u - \int -\ln u\, du\right) = \left((1-u)\ln u + u\ln u - \int \frac{u}{u}\,\mathrm{d}u\right) = \left((1-u)\ln u + u\ln u - u\right)$$

or $$\int \frac{(u-1)}{u}\,\mathrm{d}u = \left((u-1)\ln u - \int \ln u\,\mathrm{d}u\right) = \left((u-1)\ln u - \left(u\ln u - \int \frac{u}{u}\,\mathrm{d}u\right)\right) = \left((u-1)\ln u - u\ln u + u\right)$$

So **dM1** is for $\int \frac{(1-u)}{u}\,\mathrm{d}u$ going to $\left((1-u)\ln u + u\ln u - u\right)$ or $\left((u-1)\ln u - u\ln u + u\right)$ oe.

<u>*Alternative method for part (d)*</u>

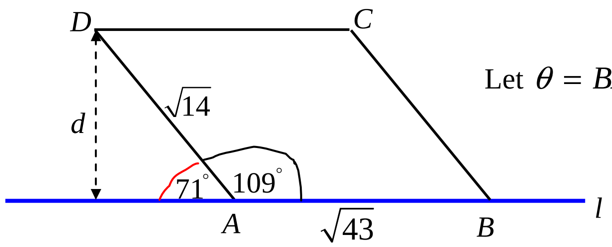
M1A1 for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)\mathrm{d}u = \right\} 4\left[\ln u - u\right]_2^1 = 4\left[(\ln 1 - 1) - (\ln 2 - 2)\right] = 4(1-\ln 2)$

<u>*Alternative method for part (d): Using an extra constant λ from their integration.*</u>

$$\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - \left[4\ln(1+\cos 0) - 4\cos 0 + \lambda\right]$$

λ is usually -4, but can be a value of k that the candidate has found in part (d).

Note: The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

Question Number	Scheme		Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\left\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\right\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1
			[2]
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	See notes	M1 A1ft
			[2]
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD} \text{ or } \overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9+6-5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	$\boxed{\text{Correct followed through expression or } \textbf{equation}.}$	A1 $\sqrt{}$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109 \text{ (nearest °)}$	awrt 109	A1 **cso AG**
			[4]
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$		
	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1
			[2]
(e)	Area $ABCD = \left(\tfrac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1
			[3]
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1
			[2]
			15

7. (a)	**M1:** Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Can be implied by two out of three components correct in $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ or $-3\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ **A1:** $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$
(b)	**M1:** An expression of the form $(3\text{ component vector})\pm\lambda(3\text{ component vector})$ **A1ft:** $\mathbf{r}=\overrightarrow{OA}+\lambda(\text{their }\pm\overrightarrow{AB})$ or $\mathbf{r}=\overrightarrow{OB}+\lambda(\text{their }\pm\overrightarrow{AB})$. **Note:** Candidate must begin writing their line as $\mathbf{r}=$ or $l=...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=...$ So, Line = ... would be A0.
(c)	**M1:** An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$. Can be implied by two out of three components correct in $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ or $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, respectively. **M1:** Applies dot product formula between their $(\overrightarrow{AB}\text{ or }\overrightarrow{BA})$ and their $(\overrightarrow{AD}\text{ or }\overrightarrow{DA})$. **A1ft:** Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly. **A1:** Obtains an angle of awrt 109 ***by correct solution only***. Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between: (i) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (ii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly. Award A0, cso for those candidates who take the dot product between: (iii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review.
(d)	**M1:** Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$. This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$. **A1:** For $2\mathbf{i}+4\mathbf{j}+9\mathbf{k}$.
(e)	**M1:** $\frac{1}{2}(\text{their }AB)(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$. Awrt 11.6 will usually imply this mark. **dM1:** Multiplies this by 2 for the parallelogram. Can be implied. **Note:** $\frac{1}{2}((\text{their }AB+\text{their }AB))(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$ **A1:** awrt 23.2
(f)	**M1:** $\dfrac{d}{\text{their }AD}=\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$ or $(\text{their }AB)\,d=(\text{their Area }ABCD)$ Award M0 for $(\text{their }AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their }AB)(\text{their }CB)$. Award M0 for $\dfrac{d}{\text{their }\sqrt{43}}=\sin 71$ or $(\text{their }\sqrt{14})\,d=23.19894905...$ **A1:** awrt 3.54 **Note: Some candidates will use their answer to part (f) in order to answer part (e).**

7.

Alternative method for part (c): Applying the cosine rule:

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$\overrightarrow{DB} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix} -6 \\ -1 \\ -6 \end{pmatrix}$

M1: as above.

So $\left|\overrightarrow{AB}\right| = \sqrt{43}$, $\left|\overrightarrow{AD}\right| = \sqrt{14}$ and $\left|\overrightarrow{DB}\right| = \sqrt{73}$

$$\cos\theta = \frac{\left(\sqrt{43}\right)^2 + \left(\sqrt{14}\right)^2 - \left(\sqrt{73}\right)^2}{2\sqrt{43}.\sqrt{14}}$$

M1: Cosine rule structure of $\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$ assigned each of $\left|\overrightarrow{AB}\right|, \left|\overrightarrow{AD}\right|$ and $\left|\overrightarrow{DB}\right|$ in any order as their *a*, *b* and *c*.

A1: Correct application of cosine rule.

$\left\{\cos\theta = \frac{-16}{2\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544...\right\} = 109$ (nearest °) A1: awrt 109 (no errors seen). **AG**

Alternative method for part (d):

$\overrightarrow{OE} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$

$\overrightarrow{DE} = \begin{pmatrix} 2+3\lambda \\ -1+3\lambda \\ 5+5\lambda \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3+3\lambda \\ -2+3\lambda \\ 1+5\lambda \end{pmatrix}$

$\overrightarrow{DE} \bullet \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix} 3+3\lambda \\ -2+3\lambda \\ 1+5\lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} = 0$

M1: Takes the dot product between $\overrightarrow{DE}$ and $\overrightarrow{AB}$ and progresses to find a value of λ

$9 + 9\lambda - 6 + 9\lambda + 5 + 3\lambda = 0 \Rightarrow \lambda = -\frac{8}{43}$

$\overrightarrow{DE} = \begin{pmatrix} 2+3\lambda \\ -1+3\lambda \\ 5+5\lambda \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{103}{43} \\ -\frac{110}{43} \\ \frac{3}{43} \end{pmatrix}$

dM1: Uses their value of λ to find $\overrightarrow{DE}$

Length DE = 3.537806563...

A1: awrt 3.54

Question Number	Scheme		Marks
8. (a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\displaystyle\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int \frac{1}{15}\,\mathrm{d}t$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\ \ \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \dfrac{5e^{\frac{1}{3}t}}{(4+e^{\frac{1}{3}t})} \quad \left\{\dfrac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \dfrac{5}{(1+4e^{-\frac{1}{3}t})}$ or $P = \dfrac{25}{(5+20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **12**

(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question. **A1:** Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$. **A1:** $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\dfrac{1}{5P} + \dfrac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working. Candidate can use 'cover-up' rule to write down $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$, as so gain all three marks. Candidate cannot gain the marks for part (a) in part (b).

8. (b)	**B1:** Separates variables as shown. dP and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1*: Both** $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants. **Or** $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5\pm P))$, where λ, μ, m and n are constants. **A1ft:** Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$ with or without $+c$ **dM1*:** Use of $t=0$ and $P=1$ in an integrated equation containing c **dM1*:** Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY. **dM1*:** Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation. **dM1*:** A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)** **A1:** $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ $\{$where $a=5$, $b=1$, $c=4\}$. Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ **Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.** **Note:** $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft. ***<u>dM1* for making P the subject</u>*** **Note there are three type of manipulations here which are considered acceptable to make *P* the subject.** **(1)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(1+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1+\mathrm{e}^{-\frac{1}{3}t})}$ **(2)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1+\mathrm{e}^{\frac{1}{3}t})}$ **(3)** M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P - \frac{5}{2}\right)^2 - \frac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$ **Note**: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*. **Note:** $(P) - (5-P) = \mathrm{e}^{\frac{1}{3}t} \Rightarrow 2P - 5 = \frac{1}{3}t$ leading to $P = \ldots$ or equivalent is awarded this dM0*
(c)	**B1:** $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. **For** $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b + c\mathrm{e}^{-\frac{1}{3}t})}$. **Award B0 for:** As $t \to \infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000, **unless** the candidate also proves that $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function. **If unsure here, then send to review!**

8.	*Alternative method for part (b)*
	B1M1*A1: as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \quad (+c)$
	Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$
	Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$
	Award 2nd M1 for $t=0,\ P=1 \Rightarrow \frac{1}{5-1} = Ae^0 \quad \left\{\Rightarrow \quad A = \frac{1}{4}\right\}$
	$\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$
	then award the final M1A1 in the same way.

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