

GCE

Edexcel GCE

Core Mathematics C3 (6665)

Summer 2005

advancing learning, changing lives

Mark Scheme (Results)

June 2005
6665 Core C3
Mark Scheme

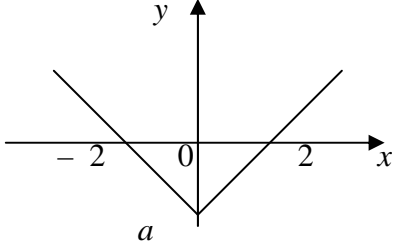
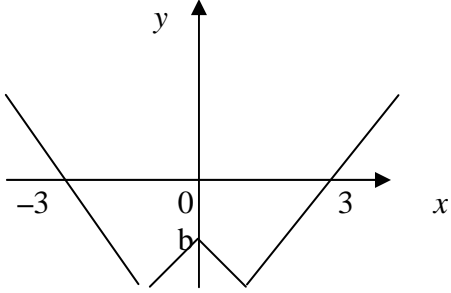
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2 \theta$: $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \equiv \frac{1}{\cos^2 \theta}$ Completion : $1 + \tan^2 \theta \equiv \sec^2 \theta$ (no errors seen)	M1 A1 (2)
(b)	Use of $1 + \tan^2 \theta = \sec^2 \theta$: $2(\sec^2 \theta - 1) + \sec \theta = 1$ $[2\sec^2 \theta + \sec \theta - 3 = 0]$ Factorising or solving: $(2\sec \theta + 3)(\sec \theta - 1) = 0$ $[\sec \theta = -\frac{3}{2} \text{ or } \sec \theta = 1]$ $\theta = 0$ $\cos \theta = -\frac{2}{3}$; $\theta_1 = 131.8^\circ$ $\theta_2 = 228.2^\circ$ $[A1ft \text{ for } \theta_2 = 360^\circ - \theta_1]$	M1 M1 B1 M1 A1 A1√ (6) [8]

Question Number	Scheme	Marks
2. (a)	<p>(i) $6 \sin x \cos x + 2 \sec 2x \tan 2x$ or $3 \sin 2x + 2 \sec 2x \tan 2x$ [M1 for $6 \sin x$]</p> <p>(ii) $3(x + \ln 2x)^2 \left(1 + \frac{1}{x}\right)$ [B1 for $3(x + \ln 2x)^2$]</p>	<p>M1A1A1 (3)</p> <p>B1M1A1 (3)</p>
(b)	<p>Differentiating numerator to obtain $10x - 10$ Differentiating denominator to obtain $2(x-1)$</p> <p>Using quotient rule formula correctly:</p> <p>To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2-10x+9)2(x-1)}{(x-1)^4}$</p> <p>Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2-10x+9)]}{(x-1)^4}$</p> <p>$= -\frac{8}{(x-1)^3}$ * (c.s.o.)</p> <p>Alternatives for (b) Either Using product rule formula correctly: Obtaining $10x - 10$ Obtaining $-2(x-1)^{-3}$ To obtain $\frac{dy}{dx} = (5x^2-10x+9)\{-2(x-1)^{-3}\} + (10x-10)(x-1)^{-2}$</p> <p>Simplifying to form $\frac{10(x-1)^2 - 2(5x^2-10x+9)}{(x-1)^3}$</p> <p>$= -\frac{8}{(x-1)^3}$ * (c.s.o.)</p> <p>Or Splitting fraction to give $5 + \frac{4}{(x-1)^2}$ Then differentiating to give answer</p>	<p>B1 B1</p> <p>M1 A1</p> <p>M1 A1 (6)</p> <p>[12]</p> <p>M1 B1 B1 A1 cao</p> <p>M1 A1 (6)</p> <p>M1 B1 B1 M1 A1 A1 (6)</p>

Question Number	Scheme	Marks
3(a)	$\frac{5x + 1}{(x + 2)(x - 1)} - \frac{3}{x + 2}$ $= \frac{5x + 1 - 3(x - 1)}{(x + 2)(x - 1)}$ <p>M1 for combining fractions even if the denominator is not lowest common</p> $= \frac{2x + 4}{(x + 2)(x - 1)} = \frac{2(x + 2)}{(x + 2)(x - 1)} = \frac{2}{x - 1} \quad *$ <p>M1 must have linear numerator</p>	<p>B1</p> <p>M1</p> <p>M1 A1 cso</p> <p>(4)</p>
(b)	$y = \frac{2}{x - 1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$ $f^{-1}(x) = \frac{2 + x}{x} \quad \text{o.e.}$	<p>M1A1</p> <p>A1</p> <p>(3)</p>
(c)	$fg(x) = \frac{2}{x^2 + 4} \quad (\text{attempt}) \quad \left[\frac{2}{"g" - 1} \right]$ <p>Setting $\frac{2}{x^2 + 4} = \frac{1}{4}$ and finding $x^2 = \dots; \quad x = \pm 2$</p>	<p>M1</p> <p>M1; A1</p> <p>(3)</p> <p>[10]</p>

Question Number	Scheme	Marks
4	<p>(a) $f'(x) = 3e^x - \frac{1}{2x}$</p> <p>(b) $3e^x - \frac{1}{2x} = 0$</p> $\Rightarrow 6\alpha e^\alpha = 1 \quad \Rightarrow \alpha = \frac{1}{6} e^{-\alpha} \quad (*)$ <p>(c) $x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots$</p> <p>[M1 at least x_1 correct, A1 all correct to 4 d.p.]</p> <p>(d) Using $f'(x) = 3e^x - \frac{1}{2x}$ with suitable interval</p> <p>e.g. $f'(0.14425) = -0.0007$ $f'(0.14435) = +0.002(1)$</p> <p>Accuracy (change of sign and correct values)</p>	<p>M1A1A1 (3)</p> <p>M1</p> <p>A1 cso (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A \quad (+ \text{ use of } \cos^2 A + \sin^2 A \equiv 1)$ $= (1 - \sin^2 A) - \sin^2 A = 1 - 2 \sin^2 A \quad (*)$	M1 A1 (2)
(b)	$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv 4 \sin \theta \cos \theta - 3(1 - 2 \sin^2 \theta) - 3 \sin \theta + 3$ $\equiv 4 \sin \theta \cos \theta + 6 \sin^2 \theta - 3 \sin \theta$ $\equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3) \quad (*)$	B1; M1 M1 A1 (4)
(c)	$4 \cos \theta + 6 \sin \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ <p>Complete method for R (may be implied by correct answer)</p> $[R^2 = 4^2 + 6^2, R \sin \alpha = 4, R \cos \alpha = 6]$ $R = \sqrt{52} \text{ or } 7.21$ <p>Complete method for α; $\alpha = 0.588$ (allow 33.7°)</p>	M1 A1 M1 A1 (4)
(d)	$\sin \theta (4 \cos \theta + 6 \sin \theta - 3) = 0$ $\theta = 0$ $\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160.. \quad (24.6^\circ)$ $\theta + 0.588 = (0.4291), 2.7125 \text{ [or } \theta + 33.7^\circ = (24.6^\circ), 155.4^\circ]$ $\theta = 2.12 \text{ cao}$	M1 B1 M1 dM1 A1 (5)
		[15]

Question Number	Scheme	Marks
6. (a)	 <p data-bbox="895 309 1147 342">Translation ← by 1</p> <p data-bbox="895 383 1123 416">Intercepts correct</p>	<p data-bbox="1302 309 1350 342">M1</p> <p data-bbox="1302 383 1449 416">A1 (2)</p>
(b)	 <p data-bbox="895 752 1283 860">$x \geq 0$, correct “shape” provided graph is not original graph</p> <p data-bbox="895 898 1150 931">Reflection in y-axis</p> <p data-bbox="895 972 1123 1005">Intercepts correct</p>	<p data-bbox="1302 748 1342 781">B1</p> <p data-bbox="1302 893 1362 927">B1√</p> <p data-bbox="1302 972 1461 1005">B1 (3)</p>
(c)	<p data-bbox="261 1084 485 1117">$a = -2, b = -1$</p>	<p data-bbox="1302 1084 1461 1117">B1B1 (2)</p>
(d)	<p data-bbox="245 1196 748 1229">Intersection of $y = 5x$ with $y = -x - 1$</p> <p data-bbox="517 1240 831 1296">Solving to give $x = -\frac{1}{6}$</p> <p data-bbox="320 1420 416 1453">[Notes:</p> <p data-bbox="325 1453 1171 1599">(i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -\frac{3}{4}$; required to eliminate $x = -\frac{3}{4}$ for final mark.</p> <p data-bbox="325 1599 1139 1711">(ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]</p>	<p data-bbox="1302 1196 1390 1229">M1A1</p> <p data-bbox="1302 1263 1461 1296">M1A1 (4)</p> <p data-bbox="1410 1632 1469 1666">[11]</p>

7. (a)	<p>Setting $p = 300$ at $t = 0 \Rightarrow 300 = \frac{2800a}{1 + a}$</p> <p>$(300 = 2500a); \quad a = 0.12$ (c.s.o) *</p>	M1 dM1A1 (3)
(b)	<p>$1850 = \frac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}} ; \quad e^{0.2t} = 16.2\dots$</p> <p>Correctly taking logs to $0.2 t = \ln k$ $t = 14$ (13.9..)</p>	M1A1 M1 A1 (4)
(c)	<p>Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)</p>	B1 (1)
(d)	<p>Using $t \rightarrow \infty, e^{-0.2t} \rightarrow 0,$</p> $p \rightarrow \frac{336}{0.12} = 2800$	M1 A1 (2)
		[10]