Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6666/01)

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June 2008
6666 Core Mathematics C4
Mark Scheme

Question	Scheme	Marks
1. (a)	x: 0, 0.4, 0.8, 1.2, 1.6, 2 y: e^0, $e^{0.08}$, $e^{0.32}$, $e^{0.72}$, $e^{1.28}$, e^2 or y: 1, 1.08329..., 1.37713..., 2.05443..., 3.59664..., 7.38906...	
	Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals)	B1
		[1]
(b) **Way 1**	$\text{Area} \approx \frac{1}{2} \times 0.4 \,; \times \underline{\left[e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2 \right]}$	
	Outside brackets $\frac{1}{2} \times 0.4$ or 0.2	B1;
	<u>For structure of trapezium rule</u> $\underline{[\;.............\;]}$;	<u>M1</u> $\sqrt{}$
	$= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf) $\underline{4.922}$	A1 **cao**
		[3]
Aliter (b) **Way 2**	$\text{Area} \approx 0.4 \times \left[\frac{e^0 + e^{0.08}}{2} + \frac{e^{0.08} + e^{0.32}}{2} + \frac{e^{0.32} + e^{0.72}}{2} + \frac{e^{0.72} + e^{1.28}}{2} + \frac{e^{1.28} + e^2}{2} \right]$ 0.4 and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	<u>M1</u> $\sqrt{}$
	$\text{Area} \approx \frac{1}{2} \times 0.4 \,; \times \underline{\left[e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2 \right]}$	
	$= 0.2 \times 24.61203164... = 4.922406... = \underline{4.922}$ (4sf) $\underline{4.922}$	A1 **cao**
		[3]
		4 marks

Note an expression like $\text{Area} \approx \frac{1}{2} \times 0.4 + e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the ( ) brackets for

The M1 mark for structure is for the material found in the curly brackets ie
$\underline{\left[\text{first } y \text{ ordinate} + 2\left(\text{intermediate ft } y \text{ ordinate}\right) + \text{final } y \text{ ordinate} \right]}$

Question Number	Scheme		Marks
2. (a)	$\left\{\begin{array}{l} u = x \;\Rightarrow\; \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \;\Rightarrow\; v = e^x \end{array}\right\}$		
	$\int x e^x \, dx = x e^x - \int e^x . 1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.)	M1
		Correct expression. (Ignore dx)	A1
	$= x e^x - \int e^x \, dx$		
	$= x e^x - e^x \; (+c)$	Correct integration with/without $+ c$	A1
			[3]
(b)	$\left\{\begin{array}{l} u = x^2 \;\Rightarrow\; \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \;\Rightarrow\; v = e^x \end{array}\right\}$		
	$\int x^2 e^x \, dx = x^2 e^x - \int e^x . 2x \, dx$	Use of 'integration by parts' formula in the **correct direction**.	M1
		Correct expression. (Ignore dx)	A1
	$= x^2 e^x - 2\int x e^x \, dx$		
	$= x^2 e^x - 2\left(x e^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 **ISW**
			[3]
	$\left\{\begin{array}{l} = x^2 e^x - 2x e^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{array}\right\}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that u and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

$+ c$ is not required in part (a).

Question Number	Scheme		Marks
3. (a)	From question, $\frac{\mathrm{d}A}{\mathrm{d}t} = 0.032$	$\frac{\mathrm{d}A}{\mathrm{d}t} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}x} = \right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}t} \div \frac{\mathrm{d}A}{\mathrm{d}x} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	$0.032 \div$ Candidate's $\frac{\mathrm{d}A}{\mathrm{d}x}$;	M1;
	When $x = 2\,\mathrm{cm}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{0.016}{2\pi}$		
	Hence, $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2 (5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2 (5x)}$ or $\underline{5\pi x^3}$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}x} = 15\pi x^2$	$\frac{\mathrm{d}V}{\mathrm{d}x} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = 15\pi x^2 . \left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	Candidate's $\frac{\mathrm{d}V}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$;	M1 $\sqrt{}$
	When $x = 2\,\mathrm{cm}$, $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.24(2) = \underline{0.48}$ ($\mathrm{cm}^3\,\mathrm{s}^{-1}$)	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Question Number	Scheme		Marks
4. (a)	$3x^2 - y^2 + xy = 4 \quad (\text{ eqn } *)$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{=}\right\} \quad \underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $\underline{\underline{(\ )}}$ of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	<u>A1</u>
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in x or terms in y together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 **cso**
			[6]
(b)	At P & Q, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the y terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	<u>A1</u>
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	<u>A1</u>
			[3]
			9 marks

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
5. (a)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	<u>B1</u>
		Expands $(1+{**}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({**}x)$;	M1;
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})({**}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2+...}\right]$ with $** \neq 1$	A correct simplified or an un-simplified $[\underline{.........}]$ expansion with candidate's followed through $({**}x)$	A1 $\sqrt{}$
	$= \frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+...}\right]$	Award SC M1 if you see $(-\frac{1}{2})({**}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2$	
	$= \frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+...\right]$	$\frac{1}{2}\left[1+\frac{3}{8}x;...\right]$	A1 isw
		SC: $K\left[1+\frac{3}{8}x+\frac{27}{128}x^2+...\right]$	
		$\frac{1}{2}\left[........;\frac{27}{128}x^2\right]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+...\right\}$	*Ignore subsequent working*	
			[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+...\right)$	Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+.....$ $+4+\frac{3}{2}x+\frac{27}{32}x^2+.....$	Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$= 4+2x;+\frac{33}{32}x^2+...$	Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
			[4]
			9 marks

Question Number	Scheme		Marks
6. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\mathbf{i}: -9+2\lambda=3+3\mu$ (1); $\mathbf{j}: \lambda=1-\mu$ (2); $\mathbf{k}: 10-\lambda=17+5\mu$ (3)	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) – 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda=1--2=3$	Both $\underline{\lambda=3}$ & $\underline{\mu=-2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively**. This mark can be implied by any two correct components of $(-3, 3, 7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3, 3, 7)$	A1
	Either check **k:** $\lambda=3$: LHS $=10-\lambda=10-3=7$ $\mu=-2$: RHS $=17+5\mu=17-10=7$	**Either** check that $\lambda=3$, $\mu=-2$ in a third equation **or** check that $\lambda=3$, $\mu=-2$ give the same coordinates on the other line. Conclusion not needed.	B1
	(As LHS = RHS then the lines intersect.)		**[6]**
(b)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times3)+(1\times-1)+(-1\times5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1
			[2]

Question Number	Scheme		Marks
6. (c)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$ $\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}.$ Hence the point A lies on l_1.)	Substitutes candidate's $\lambda=7$ into the line l_1 and finds $5\mathbf{i}+7\mathbf{j}+3\mathbf{k}$. The conclusion on this occasion is not needed.	B1 **[1]**
(d)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX}=\pm\left(\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1$\sqrt{\ }$ ±
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$		
	$\overrightarrow{OB}=\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	$\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}\text{their }\overrightarrow{AX}\end{pmatrix}$	dM1$\sqrt{\ }$
	Hence, $\overrightarrow{OB}=\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB}=\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$ or $\underline{(-11,-1,11)}$	A1 **[3]**
			12 marks

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$	$\underline{\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}}$, aef	<u>A1</u> **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of A or B is incorrect then M0A0A0.)		**[3]**

Question Number	Scheme		Marks
7. (b)	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$	Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
		their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their *A* and *B* and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y=0,\ x=\frac{\pi}{3} \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

Question Number	Scheme		Marks
8. (a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	$\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	$\underline{t = \frac{\pi}{3}}$ or $\underline{\text{awrt } 1.05}$ (radians) only stated in the range $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
			[2]
(b)	$x = 8\cos t$, $y = 4\sin 2t$		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = -8\sin t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$	A1
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression.	M1*
	$\left\{= \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence m(**N**) $= -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses m(**N**) $= -\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y - 2\sqrt{3} = -\sqrt{3}(x-4)$	Uses $y - 2\sqrt{3} = (\text{their } m_N)(x-4)$ or finds c using $x = 4$ and $y = 2\sqrt{3}$ and uses $y = (\text{their m}_N)x + "c"$.	dM1*
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ **AG**	$\underline{y = -\sqrt{3}x + 6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3} = -\sqrt{3}(4) + c \Rightarrow c = 2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ so **N**: $\left[\underline{y = -\sqrt{3}x + 6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
8. (c)	$A = \int_0^4 y\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\ \mathrm{d}t$	attempt at $A = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	M1
		correct expression (ignore limits and $\mathrm{d}t$)	A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\ \mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\ \mathrm{d}t$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\ \mathrm{d}t$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\ \mathrm{d}t$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$} {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$	M1
		Correct integration ignoring limits.	A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = 1 \text{ and } u = \frac{\sqrt{3}}{2}\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef **isw**
			[4]
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks