

- 1.** The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y -coordinate of P is 8.

- (a) Find, in terms of $\ln 2$, the x -coordinate of P .

(2)

- (b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)



2.

$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

- (a) find the value of R and the value of α to 3 decimal places.

(4)

- (b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

- (c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

- (ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

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3.

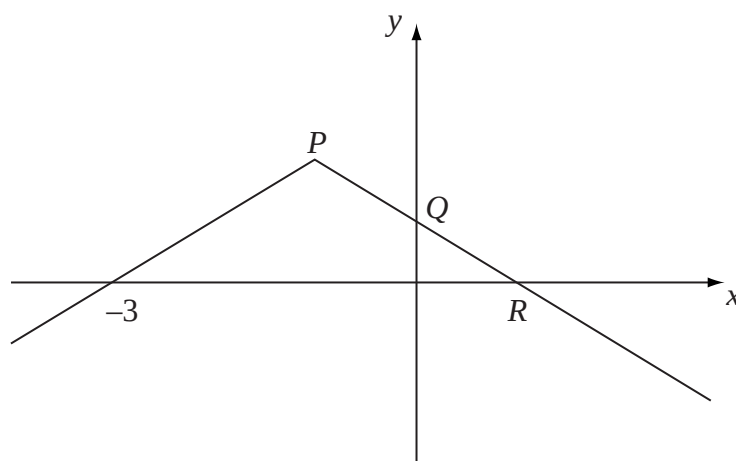


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point P .

The graph cuts the y -axis at the point Q and the x -axis at the points $(-3, 0)$ and R .

Sketch, on separate diagrams, the graphs of

$$(a) \quad y = |f(x)|, \tag{2}$$

$$(b) \quad y = f(-x).$$

Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P , Q and R ,

(d) solve $f(x) = \frac{1}{2}x$. (5)



Question 4 continued

This image shows a full page of blank, lined paper. It features approximately 28 horizontal grey lines spaced evenly apart, typical of standard notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings present.

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6. (a) Differentiate with respect to x ,

$$(i) \quad e^{3x}(\sin x + 2 \cos x), \quad (3)$$

$$(ii) \quad x^3 \ln (5x+2).$$

(3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)



7.

$$f(x) = 3x^3 - 2x - 6$$

- (a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$

(2)

- (b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

- (c) Starting with $x_0=1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places.

(3)

- (d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

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(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

