

Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6665/01)

June 2009
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	<p>Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$</p> <p>$x_1 = \frac{2}{(2.5)^2} + 2$</p> <p>$x_1 = 2.32$</p> <p>$x_2 = 2.371581451\dots$</p> <p>$x_3 = 2.355593575\dots$</p> <p>$x_4 = 2.360436923\dots$</p>	<p>An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320</p> <p>Both $x_1 = 2.32(0)$ and $x_2 = \text{awrt } 2.372$</p> <p>Both $x_3 = \text{awrt } 2.356$ and $x_4 = \text{awrt } 2.360$ or 2.36</p> <p>M1</p> <p>A1</p> <p>A1 cso</p> <p style="text-align: right;">(3)</p>
(b)	<p>Let $f(x) = -x^3 + 2x^2 + 2 = 0$</p> <p>$f(2.3585) = 0.00583577\dots$</p> <p>$f(2.3595) = -0.00142286\dots$</p> <p>Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)</p>	<div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;"> <p>Choose suitable interval for x, e.g. $[2.3585, 2.3595]$ or tighter</p> </div> <p>any one value awrt 1 sf or truncated 1 sf</p> <div style="border: 1px solid black; padding: 2px; margin-bottom: 5px;"> <p>both values correct, sign change and conclusion</p> </div> <p>At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
		[6]

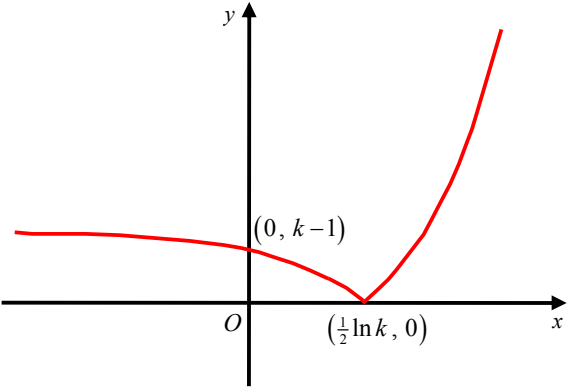
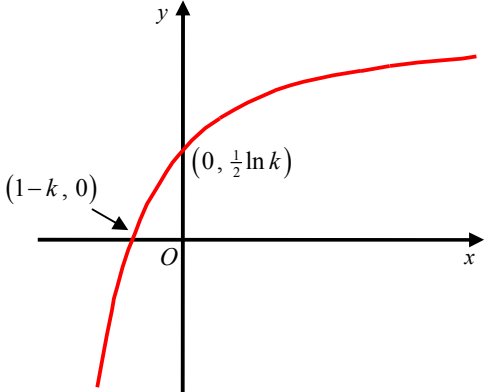
Question Number	Scheme	Marks
Q2 (a)	$\cos^2 \theta + \sin^2 \theta = 1 \quad (\div \cos^2 \theta)$ $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$ $1 + \tan^2 \theta = \sec^2 \theta$	M1
	$\tan^2 \theta = \sec^2 \theta - 1 \quad (\text{as required}) \quad \text{AG}$	Complete proof. No errors seen. A1 cso (2)
(b)	$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2, \quad (\text{eqn } *) \quad 0 \leq \theta < 360^\circ$	
	$2(\sec^2 \theta - 1) + 4 \sec \theta + \sec^2 \theta = 2$	Substituting $\tan^2 \theta = \sec^2 \theta - 1$ into eqn * to get a quadratic in $\sec \theta$ only M1
	$2 \sec^2 \theta - 2 + 4 \sec \theta + \sec^2 \theta = 2$	
	$3 \sec^2 \theta + 4 \sec \theta - 4 = 0$	Forming a three term "one sided" quadratic expression in $\sec \theta$. M1
	$(\sec \theta + 2)(3 \sec \theta - 2) = 0$	Attempt to factorise or solve a quadratic. M1
	$\sec \theta = -2 \quad \text{or} \quad \sec \theta = \frac{2}{3}$	
	$\frac{1}{\cos \theta} = -2 \quad \text{or} \quad \frac{1}{\cos \theta} = \frac{2}{3}$	
	$\underline{\cos \theta = -\frac{1}{2}}; \quad \text{or} \quad \underline{\cos \theta = \frac{3}{2}}$	$\underline{\cos \theta = -\frac{1}{2}}$ A1;
	$\alpha = 120^\circ \quad \text{or} \quad \alpha = \text{no solutions}$	
	$\theta_1 = \underline{120^\circ}$	$\underline{120^\circ}$ A1
$\theta_2 = 240^\circ$	$\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos \theta = \dots$ B1 $\sqrt{}$	
$\theta = \{120^\circ, 240^\circ\}$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Note the final A1 mark has been changed to a B1 mark. </div> (6)	

[8]

Question Number	Scheme	Marks
Q3	$P = 80e^{\frac{t}{5}}$	
	(a) $t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
	(b) $P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{1000}{80}\right)$ $t = 12.6286\dots$	M1 Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject.
		A1 awrt 12.6 or 13 years Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0 (2)
	(c) $\frac{dP}{dt} = 16e^{\frac{t}{5}}$	M1 A1 $ke^{\frac{t}{5}}$ and $k \neq 80$. $16e^{\frac{t}{5}}$ (2)
	(d) $50 = 16e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{50}{16}\right) \quad \{= 5.69717\dots\}$ $P = 80e^{\frac{1}{5}\left(5 \ln\left(\frac{50}{16}\right)\right)}$ or $P = 80e^{\frac{1}{5}(5.69717\dots)}$ $P = \frac{80(50)}{16} = \underline{250}$	M1 Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$. dM1 Substitutes their value of t back into the equation for P . A1 $\underline{250}$ or awrt 250 (3) [8]

Question Number	Scheme	Marks
Q4 (i)(a)	$y = x^2 \cos 3x$ <p>Apply product rule: $\left\{ \begin{array}{l} u = x^2 \\ \frac{du}{dx} = 2x \end{array} \quad \begin{array}{l} v = \cos 3x \\ \frac{dv}{dx} = -3 \sin 3x \end{array} \right\}$</p> $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$	<p>Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$</p> <p>M1</p> <p>Any one term correct</p> <p>A1</p> <p>Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.</p> <p>A1</p> <p>(3)</p>
(b)	$y = \frac{\ln(x^2 + 1)}{x^2 + 1}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \end{array} \quad \begin{array}{l} v = x^2 + 1 \\ \frac{dv}{dx} = 2x \end{array} \right\}$</p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1}\right)(x^2 + 1) - 2x \ln(x^2 + 1)}{(x^2 + 1)^2}$ $\left\{ \frac{dy}{dx} = \frac{2x - 2x \ln(x^2 + 1)}{(x^2 + 1)^2} \right\}$	$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ <p>M1</p> $\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ <p>A1</p> <p>Applying $\frac{vu' - uv'}{v^2}$</p> <p>M1</p> <p>Correct differentiation with correct bracketing but allow recovery.</p> <p>A1</p> <p>{Ignore subsequent working.}</p> <p>(4)</p>

Question Number	Scheme	Marks
(ii)	$y = \sqrt{4x+1}, \quad x > -\frac{1}{4}$ <p>At P, $y = \sqrt{4(2)+1} = \sqrt{9} = 3$</p> $\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$ $\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$ <p>At P, $\frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$</p> <p>Hence $m(\mathbf{T}) = \frac{2}{3}$</p> <p>Either $\mathbf{T}: y - 3 = \frac{2}{3}(x - 2)$;</p> <p>or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;</p> <p>Either $\mathbf{T}: 3y - 9 = 2(x - 2)$;</p> <p>$\mathbf{T}: 3y - 9 = 2x - 4$</p> <p>$\mathbf{T}: \underline{2x - 3y + 5 = 0}$</p> <p>or $\mathbf{T}: y = \frac{2}{3}x + \frac{5}{3}$</p> <p>$\mathbf{T}: 3y = 2x + 5$</p> <p>$\mathbf{T}: \underline{2x - 3y + 5 = 0}$</p>	<p>At P, $y = \sqrt{9}$ or $\underline{3}$</p> <p>$\pm k(4x+1)^{-\frac{1}{2}}$</p> <p>$2(4x+1)^{-\frac{1}{2}}$</p> <p>Substituting $x = 2$ into an equation involving $\frac{dy}{dx}$;</p> <p>$y - y_1 = m(x - 2)$ or $y - y_1 = m(x - \text{their stated } x)$ with ‘their TANGENT gradient’ and their y_1; or uses $y = mx + c$ with ‘their TANGENT gradient’, their x and their y_1.</p> <p>$\underline{2x - 3y + 5 = 0}$</p> <p>Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.</p> <p>(6)</p> <p>[13]</p>

Question Number	Scheme	Marks
Q5 (a)	 <p style="text-align: right;">Curve retains shape when $x > \frac{1}{2} \ln k$</p> <p style="text-align: right;">Curve reflects through the x-axis when $x < \frac{1}{2} \ln k$</p> <p style="text-align: right;">$(0, k-1)$ and $(\frac{1}{2} \ln k, 0)$ marked in the correct positions.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(b)	 <p style="text-align: right;">Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)</p> <p style="text-align: right;">$(1-k, 0)$ and $(0, \frac{1}{2} \ln k)$</p>	<p>B1</p> <p>B1</p> <p>(2)</p>
(c)	<p>Range of f: $f(x) > -k$ or $y > -k$ or $(-k, \infty)$</p>	<p>Either $f(x) > -k$ or $y > -k$ or $(-k, \infty)$ or $f > -k$ or <u>Range $> -k$.</u></p> <p>B1</p> <p>(1)</p>
(d)	<p>$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y + k) = 2x$ $\Rightarrow \frac{1}{2} \ln(y + k) = x$</p> <p>Hence $f^{-1}(x) = \frac{1}{2} \ln(x + k)$</p>	<p>Attempt to make x (or swapped y) the subject M1</p> <p>Makes e^{2x} the subject and takes \ln of both sides M1</p> <p>$\frac{1}{2} \ln(x + k)$ or $\ln \sqrt{x + k}$ A1 cao</p> <p>(3)</p>
(e)	<p>$f^{-1}(x)$: Domain: $x > -k$ or $(-k, \infty)$</p>	<p>Either $x > -k$ or $(-k, \infty)$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer B1 $\sqrt{\quad}$</p> <p>(1)</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>Q6 (a)</p> <p>(b)</p> <p>(c)</p>	<p>$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$</p> <p>$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives</p> <p>$\underline{\cos 2A = 1 - \sin^2 A - \sin^2 A = 1 - 2\sin^2 A}$ (as required)</p> <p>$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$</p> <p>$3\sin 2x = 4\left(\frac{1 - \cos 2x}{2}\right) - 2\cos 2x$</p> <p>$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$</p> <p>$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$</p> <p>$3\sin 2x + 4\cos 2x = 2$</p> <p>$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$</p> <p>$3\sin 2x + 4\cos 2x = R\cos 2x \cos \alpha + R\sin 2x \sin \alpha$</p> <p>Equate $\sin 2x$: $3 = R\sin \alpha$ Equate $\cos 2x$: $4 = R\cos \alpha$</p> <p>$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$</p> <p>$\tan \alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765\dots^\circ$</p> <p>Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$</p>	<p>Applies $A = B$ to $\cos(A + B)$ to give the <u>underlined</u> equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$</p> <p>M1</p> <p><u>Complete proof, with a link between LHS and RHS.</u> No errors seen.</p> <p>A1 AG (2)</p> <p>Eliminating y correctly.</p> <p>Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.</p> <p>M1</p> <p>Rearranges to give correct result</p> <p>A1 AG (3)</p> <p>$R = 5$</p> <p>$\tan \alpha = \pm \frac{3}{4}$ or $\tan \alpha = \pm \frac{4}{3}$ or $\sin \alpha = \pm \frac{3}{\text{their } R}$ or $\cos \alpha = \pm \frac{4}{\text{their } R}$</p> <p>awrt 36.87</p> <p>A1</p> <p>(3)</p>

Question Number	Scheme	Marks
(d)	$3\sin 2x + 4\cos 2x = 2$ $5\cos(2x - 36.87) = 2$ $\cos(2x - 36.87) = \frac{2}{5}$ $(2x - 36.87) = 66.42182\dots^\circ$ $(2x - 36.87) = 360 - 66.42182\dots^\circ$ <p>Hence, $x = 51.64591\dots^\circ, 165.22409\dots^\circ$</p>	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$ M1 awrt 66 A1 One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 A1 Both awrt 51.6 AND awrt 165.2 A1 (4) If there are any EXTRA solutions inside the range $0 \leq x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \leq x < 180^\circ$. [12]

Question Number	Scheme	Marks
<p>Q7</p> <p>$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$</p> <p>$x \in \mathbb{R}, x \neq -4, x \neq 2.$</p> <p>(a)</p> <p>$f(x) = \frac{(x-2)(x+4) - 2(x-2) + x-8}{(x-2)(x+4)}$</p> <p>$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$</p> <p>$= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$</p> <p>$= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$</p> <p>$= \frac{(x-3)}{(x-2)}$</p> <p>(b)</p> <p>$g(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq \ln 2.$</p> <p>Apply quotient rule: $\left\{ \begin{array}{l} u = e^x - 3 \quad v = e^x - 2 \\ \frac{du}{dx} = e^x \quad \frac{dv}{dx} = e^x \end{array} \right\}$</p> <p>$g'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$</p> <p>$= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$</p> <p>$= \frac{e^x}{(e^x - 2)^2}$</p>	<p>An attempt to combine to one fraction</p> <p>Correct result of combining all three fractions</p> <p>Simplifies to give the correct numerator. Ignore omission of denominator</p> <p>An attempt to factorise the numerator.</p> <p>Correct result</p> <p>Applying $\frac{vu' - uv'}{v^2}$</p> <p>Correct differentiation</p> <p>Correct result</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>dM1</p> <p>A1 cso AG</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>A1 AG</p> <p>cso</p> <p>(3)</p>

Question Number	Scheme	Marks
(c)	$g'(x) = 1 \Rightarrow \frac{e^x}{(e^x - 2)^2} = 1$ $e^x = (e^x - 2)^2$ $e^x = e^{2x} - 2e^x - 2e^x + 4$ $\underline{e^{2x} - 5e^x + 4 = 0}$ $(e^x - 4)(e^x - 1) = 0$ $e^x = 4 \text{ or } e^x = 1$ $x = \ln 4 \text{ or } x = 0$	<p>Puts their differentiated numerator equal to their denominator. M1</p> <p>$\underline{e^{2x} - 5e^x + 4}$ A1</p> <p>Attempt to factorise or solve quadratic in e^x M1</p> <p>both $x = 0, \ln 4$ A1</p> <p>(4)</p> <p>[12]</p>

Question Number	Scheme	Marks
Q8 (a)	$\sin 2x = \underline{2 \sin x \cos x}$	$\underline{2 \sin x \cos x}$
(b)	$\operatorname{cosec} x - 8 \cos x = 0, \quad 0 < x < \pi$ $\frac{1}{\sin x} - 8 \cos x = 0$ $\frac{1}{\sin x} = 8 \cos x$ $1 = 8 \sin x \cos x$ $1 = 4(2 \sin x \cos x)$ $1 = 4 \sin 2x$ $\underline{\sin 2x = \frac{1}{4}}$ <p>Radians $2x = \{0.25268\dots, 2.88891\dots\}$ Degrees $2x = \{14.4775\dots, 165.5225\dots\}$</p> <p>Radians $x = \{0.12634\dots, 1.44445\dots\}$ Degrees $x = \{7.23875\dots, 82.76124\dots\}$</p>	<p>B1 aef (1)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>Both <u>0.13</u> and <u>1.44</u></p> <p>A1 cao (5)</p> <p>Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.</p> <p>[6]</p>