Mark Scheme (Results)

Summer 2013

GCE Core Mathematics C4 6666/01
Original Paper

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2013
Publications Code

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ or AG: The answer is printed on the paper
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ddM1 denotes a method mark which is dependent upon the award of the previous 2 method marks.
- dM1* denotes a method mark which is dependent upon the award of the M1* mark.

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Misreads

A misread must be consistent for the whole question to be interpreted as such.
These are not common. In clear cases, please deduct the first 2 A (or B) marks which would have been lost by following the scheme. (Note that 2 marks is the maximum misread penalty, but that misreads which alter the nature or difficulty of the question cannot be treated so generously and it will usually be necessary here to follow the scheme as written).

Sometimes following the scheme as written is more generous to the candidate than applying the misread rule, so in this case use the scheme as written.
If in doubt, send the response to Review.

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
1. (a)	$\sqrt{(9+8x)} = (9+8x)^{\frac{1}{2}} = \underline{(9)^{\frac{1}{2}}}\left(1+\frac{8x}{9}\right)^{\frac{1}{2}} = \underline{3}\left(1+\frac{8x}{9}\right)^{\frac{1}{2}}$	$\underline{(9)^{\frac{1}{2}}}$ or $\underline{3}$ outside brackets	<u>B1</u>
		Expands $(1+**x)^{\frac{1}{2}}$ to give a simplified or an un-simplified $1+(\frac{1}{2})(**x)$;	M1;
	$= 3\left[\underline{1+(\frac{1}{2})(**x);+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(**x)^2+...}\right]$ **with** $**\neq 1$	A correct simplified or an un-simplified $[\ \underline{.........}\ ]$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= 3\left[\underline{1+\left(\frac{1}{2}\right)\left(\frac{8x}{9}\right);+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{8x}{9}\right)^2+...}\right]$	Award SC M1 if you see $\frac{1}{2}(**x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(**x)^2$ or $1+...+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(**x)^2$	
	$= 3\left[1+\frac{4}{9}x;-\frac{8}{81}x^2+...\right]$	$3\left[1+\frac{4}{9}x;...\right]$ or SC $K\left[1+\frac{4}{9}x-\frac{8}{81}x^2+...\right]$	A1 oe
	$= 3+\frac{4}{3}x;-\frac{8}{27}x^2+...$	$-\frac{8}{27}x^2$	A1
			[5]
(b)	$\sqrt{11} = \sqrt{(9+8x)} \Rightarrow \underline{x=\frac{1}{4}}$	$\underline{x=\frac{1}{4}}$	B1 oe
	$\sqrt{11} \approx 3+\frac{4}{3}\left(\frac{1}{4}\right)-\frac{8}{27}\left(\frac{1}{4}\right)^2\ \left\{=3+\frac{1}{3}-\frac{1}{54}\right\}$	Substitutes their x into their binomial expansion	M1
	$= 3\frac{17}{54} = \frac{179}{54}$	$3\frac{17}{54}$ or $\frac{179}{54}$.	A1
			[3] **8**
	Notes on Question 1		
(b)	**B1:** Writes down or uses $x=\frac{1}{4}$ oe. **M1:** Substitutes their x, where $\lvert x\rvert<\frac{9}{8}$ into at least one of the x or x^2 term of their binomial expansion. **A1:** Either $3\frac{17}{54}$ or $\frac{179}{54}$.		

Question Number	Scheme	Marks
2. (a)	x: 0, 1, 2, 3, 4; y: 0, $e^{-\frac{1}{2}}$, $\underline{2e^{-1}}$, $3e^{-\frac{3}{2}}$, $4e^{-2}$	
	$\underline{2e^{-1}}$ or awrt 0.74	B1 **[1]**
(b)	$\text{Area}(R) \approx \frac{1}{2}\times 1\,;\times\underline{\left\{0+2\left(e^{-\frac{1}{2}}+2e^{-1}+3e^{-\frac{3}{2}}\right)+4e^{-2}\right\}}$	
	Outside brackets $\frac{1}{2}\times 1$ or 0.5;	B1
	<u>For structure of trapezium rule</u> $\underline{\{\text{............}\}}$	<u>M1</u>
	Correct expression <u>inside brackets</u>	A1
	$= \frac{1}{2}\times 4.564701... = 2.282351... = \underline{2.28}$ (2dp) — $\underline{2.28}$	A1 **cao** **[4]**
(c)(i)	$\int x e^{-\frac{1}{2}x}\,dx \Rightarrow \left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^{-\frac{1}{2}x} & \Rightarrow v = -2e^{-\frac{1}{2}x}\end{array}\right\}$	
	$\int x e^{-\frac{1}{2}x}\,dx = -2xe^{-\frac{1}{2}x} - \int -2e^{-\frac{1}{2}x}\,dx$	
	Use of 'integration by parts' formula in the correct direction.	<u>M1*</u>
	Correct expression.	A1 aef
	$= -2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x} + c$	
	$\pm\lambda x e^{-\frac{1}{2}x} \pm \mu e^{-\frac{1}{2}x}\ (+c)$	M1
	Correct answer with/without $+c$	A1
(ii)	$\int_0^4 x e^{-\frac{1}{2}x}\,dx = \left[-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right]_0^4$	
	$= \left(-2(4)e^{-\frac{1}{2}(4)} - 4e^{-\frac{1}{2}(4)}\right) - \left(-2(0)e^{-\frac{1}{2}(0)} - 4e^{-\frac{1}{2}(0)}\right)$	
	Substitutes limits of 4 and 0 and subtracts the correct way round.	d<u>M1*</u>
	$= \left(-8e^{-2} - 4e^{-2}\right) - (0-4)$	
	$= 4 - 12e^{-2}$ — $\underline{a=4}$, $\underline{b=-12}$ or $4-12e^{-2}$	A1 **[6]**
		11
	Notes on Question 2	
(b)	**M1: SC:** Allow either an extra term or one missing term in $\left(e^{-\frac{1}{2}} + 2e^{-1} + 3e^{-\frac{3}{2}}\right)$.	
(c)(ii)	**dM1: Complete method** of applying limits of 4 and 0 and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.	

Question Number	Scheme		Marks
3. (a)	$x = 2t + 5,\ \ y = 3 + \frac{4}{t}$		
	$\frac{dx}{dt} = 2,\ \ \frac{dy}{dt} = -4t^{-2}$		
	So, $\frac{dy}{dx} = \frac{-4t^{-2}}{2}\ \left\{ = -2t^{-2} = -\frac{2}{t^2} \right\}$	Candidate's $\frac{dy}{dt}$ divided by candidate's $\frac{dx}{dt}$	M1
		Correct $\frac{dy}{dx}$	A1
	At $(9, 5)$, $t = 2$ When $t = 2$, $\frac{dy}{dx} = \frac{-4(2)^{-2}}{2}\ \left\{ = -2(2)^{-2} = -\frac{2}{2^2} \right\}$	Substitutes their **found** t into their $\frac{dy}{dx}$	M1
	So, $\frac{dy}{dx} = -\frac{1}{2}$	$\frac{dy}{dx} = -\frac{1}{2}$	A1 cso
			[4]
(b)	$t = \frac{x-5}{2} \Rightarrow y = 3 + \frac{4}{\left(\frac{x-5}{2}\right)}$	An attempt to eliminate t.	M1
		Achieves a correct equation in x and y only.	A1oe
	$\Rightarrow y = 3 + \frac{8}{x-5}$		
	$\Rightarrow y = \frac{3(x-5)+8}{x-5}$		
	$\Rightarrow y = \frac{3x-7}{x-5}\qquad x \neq 5$	$\underline{a=3}$, $\underline{b=-7}$, $\underline{c=1}$ and $\underline{d=-5}$ or $\frac{3x-7}{x-5}$	A1 oe
			[3] **7**

	Notes on Question 3	
(a)	**Note:** Part (a) and part (b) can be marked together. **<u>Alternative Method for part (a)</u>**	
	$y = 3 + \frac{8}{x-5} = 3 + 8(x-5)^{-1} \Rightarrow \frac{dy}{dx} = -8(x-5)^{-2}$	M1 for $\pm\lambda(x-5)^{-2}$ where $\lambda \neq 0$ A1 for $-8(x-5)^{-2}$
	At $(9, 5)$, $\frac{dy}{dx} = -8(9-5)^{-2}$	M1 for substituting $x = 9$ into their $\frac{dy}{dx}$
	So, $\frac{dy}{dx} = -\frac{1}{2}$	A1 for $\frac{dy}{dx} = -\frac{1}{2}$ by correct solution only
(b)	Award M1A1 for either $x = \frac{8}{y-3} + 5$ or $\frac{4}{y-3} = \frac{x-5}{2}$ or equivalent.	

Question Number	Scheme		Marks
4.	$l_1 : \mathbf{r} = \begin{pmatrix} -9 \\ 8 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$		
(a)	$A(1, 0, -1)$	correct coordinates	B1 **[1]**
(b)	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\mathbf{d}_1 = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$ and θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{OA} \bullet \mathbf{d}_1}{\lvert\overrightarrow{OA}\rvert . \lvert\mathbf{d}_1\rvert} = \dfrac{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}}{\sqrt{(1)^2+(0)^2+(-1)^2} . \sqrt{(5)^2+(-4)^2+(-3)^2}}$	Applies dot product formula between $\overrightarrow{OA}$ and $\mathbf{d}_1$.	M1
	$\cos\theta = \dfrac{5+0+3}{\sqrt{(1)^2+(0)^2+(-1)^2} . \sqrt{(5)^2+(-4)^2+(-3)^2}} \left\{ = \dfrac{8}{(\sqrt{2})(5\sqrt{2})} \right.$	Correct ft expression or equation.	A1 ft
	$\cos\theta = \underline{\dfrac{8}{10}}$ or $\underline{\dfrac{4}{5}}$ or $\underline{0.8}$	$\underline{\dfrac{8}{10}}$ or $\underline{\dfrac{4}{5}}$ or $\underline{0.8}$ isw	A1 **cao** **[3]**
(c)	$\overrightarrow{OB} = 3\overrightarrow{OA} = 3\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$		
	$l_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$	In the form of their $\overrightarrow{OB} + \lambda\mathbf{d}$ with any one of either $\mathbf{d}_1$ or their ft $\overrightarrow{OB}$ correct.	M1
		Correct equation and $\mathbf{r} =$	A1ft oe **[2]**
(d)	$OB = \sqrt{(3)^2 + (0)^2 + (-3)^2}$ $= \sqrt{18} = 3\sqrt{2}$	$3\sqrt{2}$	B1 ft **[1]**
(e)	So, $\dfrac{OX}{3\sqrt{2}} = \sin\theta$	$\dfrac{OX}{\text{their } OB} = \sin\theta$	M1
	$\left\{ \cos\theta = \dfrac{4}{5} \Rightarrow \right\} \sin\theta = \dfrac{3}{5}$	Converts $\cos\theta$ into an expression for $\sin\theta$	M1 oe
	$OX \left\{ = 3\sqrt{2}\left(\dfrac{3}{5}\right) = \dfrac{9}{5}\sqrt{2} \right\} = 2.5455844...$	$OX =$ awrt 2.55	A1
			[3] **10**

	Notes on Question 4
(b) (e)	**Note:** Obtaining $\cos\theta = -\frac{4}{5}$ is M1A1A0. **Note:** 2^{nd} M1 mark can be awarded instead for candidate using $\sin(\text{awrt } 37)$
(e)	**<u>Alternative Method 1 for part (e)</u>** $\mathbf{d}_2 = \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}, \quad \overrightarrow{OX} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = \begin{pmatrix} 3+5\lambda \\ -4\lambda \\ -3-3\lambda \end{pmatrix}$ $\overrightarrow{OX} \bullet \mathbf{d}_2 = 0 \Rightarrow \begin{pmatrix} 3+5\lambda \\ -4\lambda \\ -3-3\lambda \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = 15 + 25\lambda + 16\lambda + 9 + 9\lambda = 0$ **M1:** Applies $\overrightarrow{OX} \bullet \mathbf{d}_2 = 0$ and solves the resulting equation to find a value for λ. leading to $50\lambda + 24 = 0 \Rightarrow \lambda = -\frac{12}{25}$ Position vector $\overrightarrow{OX} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} - \frac{12}{25}\begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} \\ \frac{48}{25} \\ -\frac{39}{25} \end{pmatrix}$ **dM1:** Substitutes their value of λ into $\begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ -3 \end{pmatrix}$. **Note:** This mark is dependent upon the previous M1 mark if a candidate uses this alternative method. $OX = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{48}{25}\right)^2 + \left(-\frac{39}{25}\right)^2} = 2.5455844...$ **A1:** For $OX =$ awrt 2.55
(e)	**<u>Alternative Method 2 for part (e)</u>** $\frac{BX}{3\sqrt{2}} = \cos\theta \left\{ \Rightarrow BX = 3\sqrt{2}\left(\frac{4}{5}\right) = \frac{12\sqrt{2}}{5} \right\}$ **M1:** $\frac{BX}{\text{their } OB} = \cos\theta$ So, $OX = \sqrt{\left(3\sqrt{2}\right)^2 - \left(2.4\sqrt{2}\right)^2}$ $OX = 2.5455844...$ **M1:** Subtracts using Pythagoras to find OX. **A1:** For $OX =$ awrt 2.55

Question Number	Scheme		Marks
5.	$\sin(\pi y) - y - x^2 y = -5$		
		Differentiates implicitly to include either $\pm k\cos(\pi y)\frac{dy}{dx}$ or $-\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
(a)	$\underline{\pi\cos(\pi y)\frac{dy}{dx}} - \underline{\frac{dy}{dx}} - \left(\underline{\underline{2xy + x^2\frac{dy}{dx}}}\right) = \underline{0}$	$(\sin(\pi y)) \to \left(\pi\cos(\pi y)\frac{dy}{dx}\right)$, $(-y) \to \left(-\frac{dy}{dx}\right)$ and $(-5 \to 0)$	<u>A1</u>
		$\pm 2xy \pm x^2\frac{dy}{dx}$	M1
	$\frac{dy}{dx}\left(\pi\cos(\pi y) - 1 - x^2\right) = 2xy$	Grouping terms and factorising out $\frac{dy}{dx}$.	dM1
	$\frac{dy}{dx} = \frac{2xy}{\left(\pi\cos(\pi y) - 1 - x^2\right)}$	$\frac{2xy}{\left(\pi\cos(\pi y) - 1 - x^2\right)}$	A1 oe
			[5]
(b)	At $(2, 1)$, $\frac{dy}{dx} = \frac{2(2)(1)}{\left(\pi\cos(\pi(1)) - 1 - (2)^2\right)}$; $\left(= \frac{4}{-\pi-5}\right)$	Substituting $x = 2$ & $y = 1$ into an equation involving $\frac{dy}{dx}$;	M1;
	T: $y - 1 = \frac{4}{-\pi-5}(x-2)$	$y - 1 = m_T(x-2)$ with 'their TANGENT gradient';	M1
	Cuts x-axis $\Rightarrow y = 0 \Rightarrow -1 = \frac{4}{-\pi-5}(x-2)$	Setting $y = 0$ in their tangent equation.	M1
	So, $x = \frac{\pi+5}{4} + 2$ $\left\{= \frac{\pi+13}{4}\right\}$	$\frac{\pi+5}{4} + 2$	A1 oe cso
			[4] **9**
	Notes on Question 5		
(b)	**Note:** 2nd M1 can be implied for $-1 = \frac{4}{-\pi-5}(x-2)$ or $\frac{-1}{x-2} = \frac{-4}{\pi+5}$ if no equation of tangent is given. **Note:** Award 2nd M0 where m in $y - 1 = m(x-2)$ is either a changed tangent gradient or a normal gradient.		

Question Number	Scheme		Marks
6. (i)(a)	$\dfrac{7x}{(x+3)(2x-1)} = \dfrac{A}{(x+3)} + \dfrac{B}{(2x-1)}$		
	$7x \equiv A(2x-1) + B(x+3)$	Forms the correct identity.	B1
	When $x = -3$, $A = 3$. When $x = \dfrac{1}{2}$, $B = 1$.	Substitutes either $x = -3$ or $x = \frac{1}{2}$ into their identity and correctly finds one of either A or B.	M1
	Hence, $\left\{\dfrac{7x}{(x+3)(2x-1)}\right\} = \dfrac{3}{(x+3)} + \dfrac{1}{(2x-1)}$	Correct partial fraction.	A1
			[3]
(b)	$\displaystyle\int \frac{7x}{(x+3)(2x-1)}\,dx = \int \frac{3}{(x+3)} + \frac{1}{(2x-1)}\,dx$		
		Either $\pm a\ln(x+3)$ or $\pm b\ln(2x-1)$	M1
	$= 3\ln(x+3) + \dfrac{1}{2}\ln(2x-1) + c$	At least one ln term correct	A1 ft
		Correct integration with $+c$	A1
			[3]
(ii)	$\displaystyle\int \frac{1}{x + x^{\frac{1}{3}}}\,dx$, $u^3 = x$		
	$3u^2\dfrac{du}{dx} = 1$	$3u^2\dfrac{du}{dx} = 1$ or $\dfrac{dx}{du} = 3u^2$ or $\dfrac{du}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}}$	B1 oe
	$\displaystyle = \int \frac{1}{u^3 + u} \, .\, 3u^2\,du$	Attempt to substitute $u^3 = x$ and $3u^2\dfrac{du}{dx} = 1$ or $\dfrac{du}{dx} = \dfrac{1}{3}x^{-\frac{2}{3}}$ to give an expression to be integrated which is in terms of u only.	M1
	$\displaystyle = \int \frac{3u}{u^2 + 1}\,du$	$\displaystyle\int \frac{3u}{u^2+1}\,du$	A1
	$= \dfrac{3}{2}\ln\left(u^2 + 1\right) + c$	$\pm\lambda\ln\left(u^2+1\right)$	M1
	$= \dfrac{3}{2}\ln\left(x^{\frac{2}{3}} + 1\right) + c$	Correct answer in x with or without $+c$.	A1
			[5] **11**
	Notes on Question 6		
(ii)	**Note:** 1st M1 can be implied by $\displaystyle\int \frac{1}{u^3 + u} \, .\, 3u^2$ if the du is missing.		

Question Number	Scheme		Marks
7. (a)	$x = \tan\theta,\quad y = 1 + 2\cos 2\theta\,,\quad ,\quad 0 \leqslant \theta < \frac{\pi}{2}$		
	$V = \underline{\pi}\int \underline{(1 + 2\cos 2\theta)^2}.\sec^2\theta\,\{\mathrm{d}\theta\}$	attempt at $V = \underline{\pi}\int \underline{y^2}\mathrm{d}x$	M1
		Correct expression ignoring limits and π.	B1
	$V = (\pi)\int (1 + 2(2\cos^2\theta - 1))^2 \sec^2\theta\,\{\mathrm{d}\theta\}$	Using either $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos 2\theta = 1 - 2\sin^2\theta$ or $\cos 2\theta = \cos^2\theta - \sin^2\theta$	M1
	$V = (\pi)\int (4\cos^2\theta - 1)^2 \sec^2\theta\,\{\mathrm{d}\theta\}$		
	$V = (\pi)\int (16\cos^4\theta - 8\cos^2\theta + 1)\sec^2\theta\,\{\mathrm{d}\theta\}$		
	$V = \pi\int (16\cos^2\theta - 8 + \sec^2\theta)\{\mathrm{d}\theta\}$	Manipulates to give the final answer where $k = \pi$	A1 *
	change limits: when $x = 1 \Rightarrow 1 = \tan\theta \Rightarrow \theta = \frac{\pi}{4}$ and when $x = \sqrt{3} \Rightarrow \sqrt{3} = \tan\theta \Rightarrow \theta = \frac{\pi}{3}$	Evidence of changing both limits.	B1
			[5]
(b)	$(\pi)\int 16\left(\frac{1 + \cos 2\theta}{2}\right) - 8 + \sec^2\theta\,\mathrm{d}\theta$	Using the identity $\cos 2\theta = 2\cos^2\theta - 1$ to substitute for $\cos^2\theta$.	M1
	$= (\pi)\int 8 + 8\cos 2\theta - 8 + \sec^2\theta\,\mathrm{d}\theta$		
	$= (\pi)\int 8\cos 2\theta + \sec^2\theta\,\mathrm{d}\theta$		
	$= (\pi)\left(\frac{8\sin 2\theta}{2} + \tan\theta\right)$	Either $\pm 4\sin 2\theta$ or $\tan\theta$	M1
		$\frac{8\sin 2\theta}{2} + \tan\theta$	A1
	So, $V = (\pi)\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(8\cos 2\theta + \sec^2\theta\right)\mathrm{d}\theta = (\pi)\left[\frac{8\sin 2\theta}{2} + \tan\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$		
	$= (\pi)\left[\left(\frac{4\sqrt{3}}{2} + \sqrt{3}\right) - (4 + 1)\right]$	Substitutes limits of $\frac{\pi}{3}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \left(3\sqrt{3} - 5\right)\pi$	$\left(3\sqrt{3} - 5\right)\pi$	A1
			[5] **10**
	Notes on Question 7		
(a)	**Note:** The use of $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\{\mathrm{d}\theta\}$ (i.e. an expression for area and not volume) is the 1st M0, 1st B0.		
	Note: For the 1st B1, the correct expression of $\int \underline{(1 + 2\cos 2\theta)^2}.\sec^2\theta$ must be stated on one line.		
	Note: Award 2nd M0 for applying $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$ to give an expression in terms of $\cos 2\theta$.		
	Note: The π in the volume formula is only required for the 1st M1 mark and the A1 mark.		

Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt} = -32\pi\sqrt{h}$		
	$V = \pi(40)^2 h \; \{= 1600\pi h\}$	$V = \pi(40)^2 h$	B1
	$\frac{dV}{dh} = 1600\pi$	$\frac{dV}{dh} = 1600\pi$	B1ft
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$		
	$\frac{dh}{dt} = \frac{1}{1600\pi} \times -32\pi\sqrt{h}$	$\frac{dh}{dt} = \left(\pm 32\pi\sqrt{h}\right) \div \left(\text{their } \frac{dV}{dh}\right)$	M1
	So, $\frac{dh}{dt} = -0.02\sqrt{h}$	Correct proof.	A1 * cso
			[4]
(b)	$\int \frac{dh}{\sqrt{h}} = \int -0.02\, dt$	Attempt to separate variables. Integral signs not necessary.	B1
	$\Rightarrow \int h^{-\frac{1}{2}}\, dh = \int -0.02\, dt$		
	$\Rightarrow \frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = -0.02t\ (+c)$	Separates variables and integrates to give $\pm \alpha h^{\frac{1}{2}} = \pm \beta t\ (+c)$	M1
		Correct integration with/without $+c$	A1
	$t = 0,\ h = 100 \Rightarrow 2\sqrt{100} = -0.02(0) + c \Rightarrow c = 20$ $h = 50 \Rightarrow 2\sqrt{50} = -0.02t + 20$	Uses boundary conditions for t and h to find c. Then uses h with found c to form an equation in order to find t.	M1
	So, $0.02t = 20 - 2\sqrt{50}$ $\Rightarrow t = 1000 - 500\sqrt{2} = 292.8932188...$ $\Rightarrow t = 293$ (minutes) (nearest minute)	awrt 293	A1 cso
			[5] **9**
	Notes on Question 8		
(a)	**Note:** Use of $V = \pi r^2 h$ is 1st B0 until $r = 40$ is substituted.		
(b)	**Note:** Award final A0 for dividing by 60 after achieving $t = 292.8932188...$ **Note:** The final A1 mark is for correct solution only. If the candidate makes a sign error then award final A0.		

	Notes on Question 8 continued	
(a)	**Alternative Method for part (a)**	
	$\frac{d}{dt}\left(\pi 40^2 h\right) = -32\pi\sqrt{h}$	**B1B1:** $\frac{d}{dt}\left(\pi 40^2 h\right) = -32\pi\sqrt{h}$
	$\Rightarrow \frac{dh}{dt} = \frac{-32\pi\sqrt{h}}{\pi 40^2}$	**M1:** Simplifies to give an expression for $\frac{dh}{dt}$.
	So, $\frac{dh}{dt} = -0.02\sqrt{h}$ *	**A1:** Correct proof.
(b)	**Alternative Method for part (b)**	
	$\int_{100}^{50} \frac{dh}{\sqrt{h}} = \int_0^T -0.02\ dt$	**B1:** Attempt to separate variables. Integral signs and limits not necessary.
	$\Rightarrow \int_{100}^{50} h^{-\frac{1}{2}}\ dh = \int_0^T -0.02\ dt$	
	$\Rightarrow \left[\frac{h^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_{100}^{50} = \left[-0.02t\right]_0^T$	**M1:** $\pm \alpha h^{\frac{1}{2}} = \pm \beta t\ (+c)$ **A1:** Correct integration with/without limits
	$2\sqrt{50} - 2\sqrt{100} = -0.02T$	**M1:** Attempts to use limits in order to find T.
	So, $0.02T = 20 - 2\sqrt{50}$	
	$\Rightarrow T = 1000 - 500\sqrt{2} = 292.8932188...$	
	$\Rightarrow T = 293$ (minutes) (nearest minute)	**A1:** A correct solution (with a correct application of limits) leading to awrt 293.

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com
Summer 2013

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning