



1. The function  $f$  is given by

$$f(x) = \frac{3(x+1)}{(x+2)(x-1)}, x \in \mathbb{R}, x \neq -2, x \neq 1.$$

(a) Express  $f(x)$  in partial fractions.

(3)

(b) Hence, or otherwise, prove that  $f'(x) < 0$  for all values of  $x$  in the domain.

(3)

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2. The curve  $C$  is described by the parametric equations

$$x = 3 \cos t, \quad y = \cos 2t, \quad 0 \leq t \leq \pi.$$

(a) Find a cartesian equation of the curve  $C$ .

(2)

(b) Draw a sketch of the curve  $C$ .

(2)

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3. Use the substitution  $x = \sin \theta$  to show that, for  $|x| \leq 1$ ,

$$\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx = \frac{x}{(1-x^2)^{\frac{1}{2}}} + c, \text{ where } c \text{ is an arbitrary constant.}$$

(6)

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4. A measure of the effective voltage,  $M$  volts, in an electrical circuit is given by

$$M^2 = \int_0^1 V^2 dt$$

where  $V$  volts is the voltage at time  $t$  seconds. Pairs of values of  $V$  and  $t$  are given in the following table.

|       |     |      |     |      |     |
|-------|-----|------|-----|------|-----|
| $t$   | 0   | 0.25 | 0.5 | 0.75 | 1   |
| $V$   | -48 | 207  | 37  | -161 | -29 |
| $V^2$ |     |      |     |      |     |

Use the trapezium rule with five values of  $V^2$  to estimate the value of  $M$ .

(6)

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5.

Figure 1

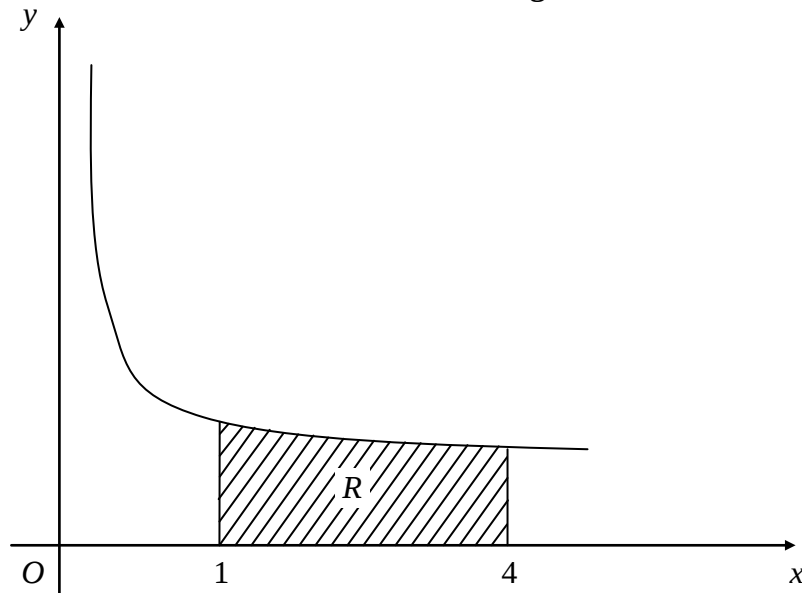


Figure 1 shows part of the curve with equation  $y = 1 + \frac{1}{2\sqrt{x}}$ . The shaded region  $R$ , bounded by the curve, that  $x$ -axis and the lines  $x = 1$  and  $x = 4$ , is rotated through  $360^\circ$  about the  $x$ -axis. Using integration, show that the volume of the solid generated is  $\pi(5 + \frac{1}{2} \ln 2)$ .

(8)

6. Liquid is poured into a container at a constant rate of  $30 \text{ cm}^3 \text{ s}^{-1}$ . At time  $t$  seconds liquid is leaking from the container at a rate of  $\frac{2}{15} V \text{ cm}^3 \text{ s}^{-1}$ , where  $V \text{ cm}^3$  is the volume of liquid in the container at that time.

(a) Show that

$$-15 \frac{dV}{dt} = 2V - 450. \quad (3)$$

Given that  $V = 1000$  when  $t = 0$ ,

(b) find the solution of the differential equation, in the form  $V = f(t)$ . (7)

(c) Find the limiting value of  $V$  as  $t \rightarrow \infty$ . (1)

7. The curve  $C$  has equation  $y = \frac{x}{4 + x^2}$ .

(a) Use calculus to find the coordinates of the turning points of  $C$ . (5)

Using the result  $\frac{d^2y}{dx^2} = \frac{2x(x^2 - 12)}{(4 + x^2)^3}$ , or otherwise,

(b) determine the nature of each of the turning points. (3)

(c) Sketch the curve  $C$ . (3)

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8. (i) Given that  $\cos(x + 30)^\circ = 3 \cos(x - 30)^\circ$ , prove that  $\tan x^\circ = -\frac{\sqrt{3}}{2}$ . (5)

(ii) (a) Prove that  $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$ . (3)

(b) Verify that  $\theta = 180^\circ$  is a solution of the equation  $\sin 2\theta = 2 - 2 \cos 2\theta$ . (1)

(c) Using the result in part (a), or otherwise, find the other two solutions,  $0 < \theta < 360^\circ$ , of the equation using  $\sin 2\theta = 2 - 2 \cos 2\theta$ . (4)

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9. The equations of the lines  $l_1$  and  $l_2$  are given by

$$l_1: \quad \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

$$l_2: \quad \mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$

where  $\lambda$  and  $\mu$  are parameters.

- (a) Show that  $l_1$  and  $l_2$  intersect and find the coordinates of  $Q$ , their point of intersection.

(6)

- (b) Show that  $l_1$  is perpendicular to  $l_2$ .

(2)

The point  $P$  with  $x$ -coordinate 3 lies on the line  $l_1$  and the point  $R$  with  $x$ -coordinate 4 lies on the line  $l_2$ .

- (c) Find, in its simplest form, the exact area of the triangle  $PQR$ .

(6)

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END