

Paper Reference(s) 66663 Edexcel GCE Core Mathematics C4 Advanced Subsidiary Set A: Practice Paper 1

Time: 1 hour 30 minutes

Materials required for examination Mathematical Formulae **Items included with question papers**

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Nil

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. This paper has nine questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.



Question Number	Leave Blank		
1			
2			
3			
4			
5			
6			
7			
8			
9			
Total			

Turn over

1. The function f is given by

$$f(x) = \frac{3(x+1)}{(x+2)(x-1)}, x \in \mathbb{R}, x \neq -2, x \neq 1$$

(*a*) Express f(x) in partial fractions.

(b) Hence, or otherwise, prove that f'(x) < 0 for all values of x in the domain.

2. The curve *C* is described by the parametric equations

$$x = 3 \cos t$$
, $y = \cos 2t$, $0 \le t \le \pi$.

(*a*) Find a cartesian equation of the curve *C*.

(*b*) Draw a sketch of the curve *C*.

3. Use the substitution $x = \sin \theta$ to show that, for $|x| \le 1$,

$$\int \frac{1}{(1-x^2)^{\frac{3}{2}}} \, \mathrm{d}x = \frac{x}{(1-x^2)^{\frac{1}{2}}} + c, \text{ where } c \text{ is an arbitrary constant.}$$

(6)

4. A measure of the effective voltage, *M* volts, in an electrical circuit is given by

$$M^2 = \int_0^1 V^2 \, \mathrm{d}t$$

where V volts is the voltage at time t seconds. Pairs of values of V and t are given in the following table.

t	0	0.25	0.5	0.75	1
V	-48	207	37	-161	-29
V^2					

Use the trapezium rule with five values of V^2 to estimate the value of M.

(6)

(3)

(3)

(2)

(2)





Figure 1 shows part of the curve with equation $y = 1 + \frac{1}{2\sqrt{x}}$. The shaded region *R*, bounded by the curve, that *x*-axis and the lines x = 1 and x = 4, is rotated through 360° about the *x*-axis. Using integration, show that the volume of the solid generated is $\pi (5 + \frac{1}{2} \ln 2)$.

(8)

6. Liquid is poured into a container at a constant rate of 30 cm³ s⁻¹. At time *t* seconds liquid is leaking from the container at a rate of $\frac{2}{15}V$ cm³ s⁻¹, where V cm³ is the volume of liquid in the container at that time.

(a) Show that

5.

$$-15\frac{\mathrm{d}V}{\mathrm{d}t} = 2V - 450.$$
(3)

Given that V = 1000 when t = 0,

(*b*) find the solution of the differential equation, in the form V = f(t).

(7)

(c) Find the limiting value of V as $t \to \infty$.

(1)

7. The curve C has equation $y = \frac{x}{4 + x^2}$.

(*a*) Use calculus to find the coordinates of the turning points of *C*.

(5)

Using the result
$$\frac{d^2 y}{dx^2} = \frac{2x(x^2 - 12)}{(4 + x^2)^3}$$
, or otherwise,

(b) determine the nature of each of the turning points.

(*c*) Sketch the curve *C*.

(3)

(3)

8. (i) Given that $\cos(x+30)^\circ = 3\cos(x-30)^\circ$, prove that $\tan x^\circ = -\frac{\sqrt{3}}{2}$. (5)

(ii) (a) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan \theta$.

(3)

- (b) Verify that $\theta = 180^{\circ}$ is a solution of the equation $\sin 2\theta = 2 2 \cos 2\theta$.
- (1)
- (c) Using the result in part (a), or otherwise, find the other two solutions, 0 < θ < 360°, of the equation using sin 2θ = 2 2 cos 2θ.
 (4)

9. The equations of the lines l_1 and l_2 are given by

 l_1 : $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$

$$l_2$$
: $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$

where λ and μ are parameters.

- (a) Show that l_1 and l_2 intersect and find the coordinates of Q, their point of intersection.
- (b) Show that l_1 is perpendicular to l_2 .

(6)

(2)

The point *P* with *x*-coordinate 3 lies on the line l_1 and the point *R* with *x*-coordinate 4 lies on the line l_2 .

(c) Find, in its simplest form, the exact area of the triangle PQR.

(6)

END