

1. The function f is given by

$$f : x \mapsto \frac{x}{x^2 - 1} - \frac{1}{x + 1}, \quad x > 1.$$

(a) Show that $f(x) = \frac{1}{(x-1)(x+1)}$. (3)

(b) Find the range of f . (2)

The function g is given by

$$g : x \mapsto \frac{2}{x}, \quad x > 0.$$

(c) Solve $gf(x) = 70$. (4)

2. Express $\frac{y+3}{(y+1)(y+2)} - \frac{y+1}{(y+2)(y+3)}$ as a single fraction in its simplest form. (5)

3. The function f is even and has domain \mathbb{R} . For $x \geq 0$, $f(x) = x^2 - 4ax$, where a is a positive constant.

(a) In the space below, sketch the curve with equation $y = f(x)$, showing the coordinates of all the points at which the curve meets the axes. (3)

(b) Find, in terms of a , the value of $f(2a)$ and the value of $f(-2a)$. (2)

Given that $a = 3$,

(c) use algebra to find the values of x for which $f(x) = 45$. (4)

4.
$$f(x) = x^3 + x^2 - 4x - 1.$$

The equation $f(x) = 0$ has only one positive root, α .

(a) Show that $f(x) = 0$ can be rearranged as

$$x = \sqrt{\left(\frac{4x+1}{x+1}\right)}, x \neq -1. \quad (2)$$

The iterative formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ is used to find an approximation to α .

(b) Taking $x_1 = 1$, find, to 2 decimal places, the values of x_2 , x_3 and x_4 . (3)

(c) By choosing values of x in a suitable interval, prove that $\alpha = 1.70$, *correct* to 2 decimal places. (3)

(d) Write down a value of x_1 for which the iteration formula $x_{n+1} = \sqrt{\left(\frac{4x_n+1}{x_n+1}\right)}$ does *not* produce a valid value for x_2 .

Justify your answer.

(2)

5. The functions f and g are defined by

$$f: x \mapsto |x - a| + a, \quad x \in \mathbb{R},$$

$$g: x \mapsto 4x + a, \quad x \in \mathbb{R}.$$

where a is a positive constant.

(a) On the same diagram, sketch the graphs of f and g , showing clearly the coordinates of any points at which your graphs meet the axes. (5)

(b) Use algebra to find, in terms of a , the coordinates of the point at which the graphs of f and g intersect. (3)

(c) Find an expression for $fg(x)$. (2)

(d) Solve, for x in terms of a , the equation

$$fg(x) = 3a. \quad (3)$$

6.

Figure 1

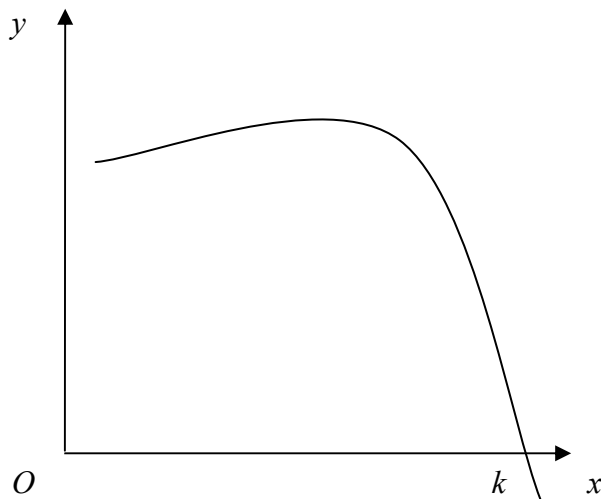


Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = 10 + \ln(3x) - \frac{1}{2}e^x, \quad 0.1 \leq x \leq 3.3.$$

Given that $f(k) = 0$,

(a) show, by calculation, that $3.1 < k < 3.2$. (2)

(b) Find $f'(x)$. (3)

The tangent to the graph at $x = 1$ intersects the y -axis at the point P .

(c) (i) Find an equation of this tangent.

(ii) Find the exact y -coordinate of P , giving your answer in the form $a + \ln b$. (5)

7. (a) Express $\sin x + \sqrt{3} \cos x$ in the form $R \sin (x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. (4)
- (b) Show that the equation $\sec x + \sqrt{3} \operatorname{cosec} x = 4$ can be written in the form
- $$\sin x + \sqrt{3} \cos x = 2 \sin 2x. \quad (3)$$
- (c) Deduce from parts (a) and (b) that $\sec x + \sqrt{3} \operatorname{cosec} x = 4$ can be written in the form
- $$\sin 2x - \sin (x + 60^\circ) = 0. \quad (1)$$

END