

1. Use the derivatives of $\sin x$ and $\cos x$ to prove that the derivative of $\tan x$ is $\sec^2 x$. (4)
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2. The function f is given by $f: x \mapsto 2 + \frac{3}{x+2}$, $x \in \mathbb{R}$, $x \neq -2$.

(a) Express $2 + \frac{3}{x+2}$ as a single fraction. (1)

(b) Find an expression for $f^{-1}(x)$. (3)

(c) Write down the domain of f^{-1} . (1)

3. (a) Express as a fraction in its simplest form

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21}.$$

(3)

- (b) Hence solve

$$\frac{2}{x-3} + \frac{13}{x^2 + 4x - 21} = 1.$$

(3)

4. (a) Simplify $\frac{x^2 + 4x + 3}{x^2 + x}$. (2)

(b) Find the value of x for which $\log_2(x^2 + 4x + 3) - \log_2(x^2 + x) = 4$. (4)

5. (i) Prove, by counter-example, that the statement

$$\text{“sec}(A + B) \equiv \sec A + \sec B, \text{ for all } A \text{ and } B\text{”}$$

is false

(2)

- (ii) Prove that

$$\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta, \quad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}.$$

(5)

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6. (a) Prove that

$$\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta, \quad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}.$$

(3)

- (b) Solve, giving exact answers in terms of π ,

$$2(1 - \cos 2\theta) = \tan \theta, \quad 0 < \theta < \pi.$$

(6)

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7. Given that $y = \log_a x$, $x > 0$, where a is a positive constant,

- (a) (i) express x in terms of a and y ,

(1)

- (ii) deduce that $\ln x = y \ln a$.

(1)

- (b) Show that $\frac{dy}{dx} = \frac{1}{x \ln a}$.

(2)

The curve C has equation $y = \log_{10} x$, $x > 0$. The point A on C has x -coordinate 10. Using the result in part (b),

- (c) find an equation for the tangent to C at A .

(4)

The tangent to C at A crosses the x -axis at the point B .

- (d) Find the exact x -coordinate of B .

(2)

8. The curve with equation $y = \ln 3x$ crosses the x -axis at the point $P(p, 0)$.

(a) Sketch the graph of $y = \ln 3x$, showing the exact value of p .

(2)

The normal to the curve at the point Q , with x -coordinate q , passes through the origin.

(b) Show that $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$.

(4)

(c) Show that the equation in part (b) can be rearranged in the form $x = \frac{1}{3}e^{-x^2}$.

(2)

(d) Use the iteration formula $x_{n+1} = \frac{1}{3}e^{-x_n^2}$, with $x_0 = \frac{1}{3}$, to find x_1, x_2, x_3 and x_4 . Hence write down, to 3 decimal places, an approximation for q .

(3)

9.

Figure 3

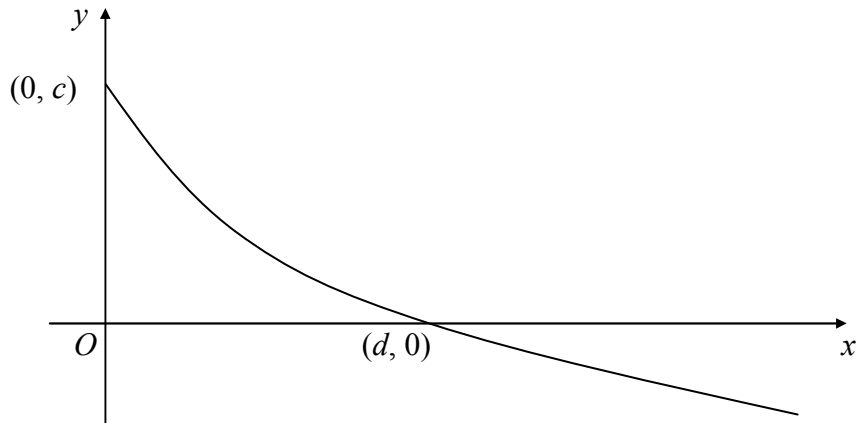


Figure 3 shows a sketch of the curve with equation $y = f(x)$, $x \geq 0$. The curve meets the coordinate axes at the points $(0, c)$ and $(d, 0)$.

In separate diagrams sketch the curve with equation

(a) $y = f^{-1}(x)$, (2)

(b) $y = 3f(2x)$. (3)

Indicate clearly on each sketch the coordinates, in terms of c or d , of any point where the curve meets the coordinate axes.

Given that f is defined by

$$f : x \mapsto 3(2^{-x}) - 1, \quad x \in \mathbb{R}, \quad x \geq 0,$$

(c) state (3)

(i) the value of c ,

(ii) the range of f .

(d) Find the value of d , giving your answer to 3 decimal places. (3)

The function g is defined by

$$g : x \mapsto \log_2 x, \quad x \in \mathbb{R}, \quad x \geq 1.$$

(e) Find $fg(x)$, giving your answer in its simplest form. (3)

END