Paper Reference (complete below)	Centre No.	Surname	Initial(s)
6 6 6 3 / 0 1	Candidate No.	Signature	

Paper Reference(s)

6663

Edexcel GCE

Core Mathematics C4
Advanced Subsidiary
Set A: Practice Paper 6

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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9	
Total	

Materials required for examination
Mathematical Formulae

Items included with question papers
Nil

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has nine questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

Turn over

1. (*a*) Express 1.5 sin 2*x* + 2 cos 2*x* in the form *R* sin (2*x* + α), where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving your values of *R* and α to 3 decimal places where appropriate. **(4)**

(*b*) Express 3 sin *x* cos *x* + 4 $\cos^2 x$ in the form *a* cos 2*x* + *b* sin 2*x* + *c*, where *a*, *b* and *c* are constants to be found. **(2)**

(*c*) Hence, using your answer to part (*a*), deduce the maximum value of 3 sin *x* cos *x* + 4 $\cos^2 x$. **(2)**

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2. **Figure 1**

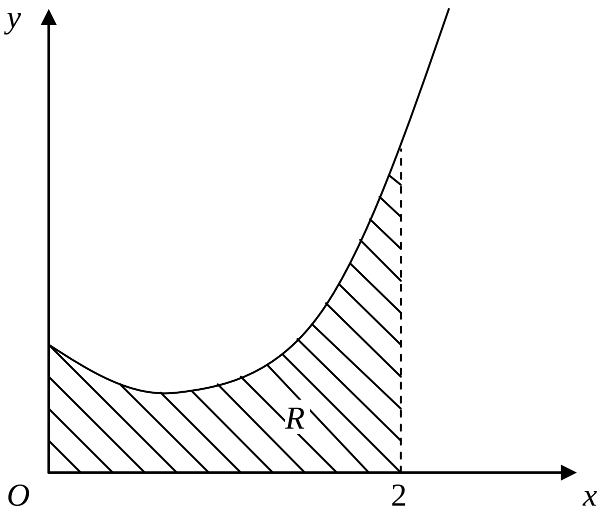

Figure 1 shows part of the curve with equation *y* = f(*x*), where

$$f(x) = \frac{x^2 + 1}{(1+x)(3-x)}, \quad 0 \le x < 3.$$

(*a*) Given that $f(x) = A + \frac{B}{1+x} + \frac{C}{3-x}$, find the values of the constants *A*, *B* and *C*.

(4)

The finite region *R*, shown in Fig. 1, is bounded by the curve with equation *y* = f(*x*), the *x*-axis, the *y*-axis and the line *x* = 2.

(*b*) Find the area of *R*, giving your answer in the form *p* + *q* ln *r*, where *p*, *q* and *r* are rational constants to be found.

(5)

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3. A student tests the accuracy of the trapezium rule by evaluating I, where

$$I = \int_{0.5}^{1.5} \left(\frac{3}{x} + x^4 \right) \mathrm{d}x .$$

(*a*) Complete the student's table, giving values to 2 decimal places where appropriate.

x	0.5	0.75	1	1.25	1.5
$\frac{3}{x} + x^4$	6.06	4.32			

(2)

(*b*) Use the trapezium rule, with all the values from your table, to calculate an estimate for the value of I.

(4)

(*c*) Use integration to calculate the exact value of I.

(4)

(*d*) Verify that the answer obtained by the trapezium rule is within 3% of the exact value.

(2)

4. **Figure 1**

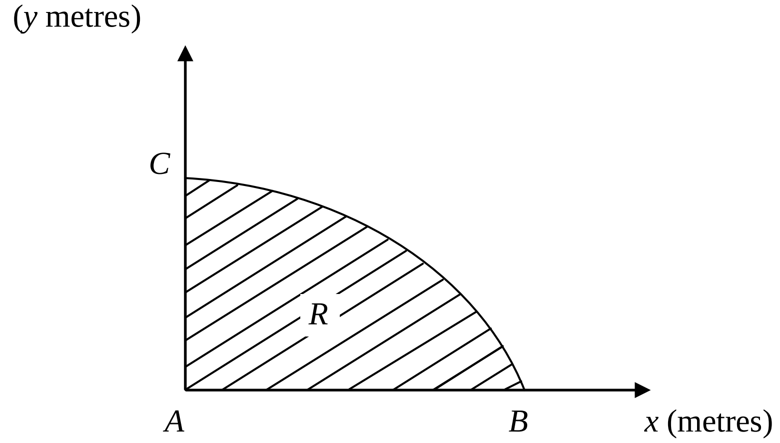

Figure 1 shows a cross-section R of a dam. The line AC is the vertical face of the dam, AB is the horizontal base and the curve BC is the profile. Taking x and y to be the horizontal and vertical axes, then A, B and C have coordinates (0, 0), $(3\pi^2, 0)$ and (0, 30) respectively. The area of the cross-section is to be calculated.

Initially the profile BC is approximated by a straight line.

(*a*) Find an estimate for the area of the cross-section R using this approximation.
(1)

The profile BC is actually described by the parametric equations.

$$x = 16t^2 - \pi^2, \quad y = 30 \sin 2t, \quad \frac{\pi}{4} \le t \le \frac{\pi}{2}.$$

(*b*) Find the exact area of the cross-section R.
(7)

(*c*) Calculate the percentage error in the estimate of the area of the cross-section R that you found in part (*a*).
(2)

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5. (*a*) Prove that, when $x = \frac{1}{15}$, the value of $(1+5x)^{-\frac{1}{2}}$ is exactly equal to sin 60°.
(3)

(*b*) Expand $(1+5x)^{-\frac{1}{2}}$, $|x| < 0.2$, in ascending powers of x up to and including the term in x^3, simplifying each term.
(4)

(*c*) Use your answer to part (*b*) to find an approximation for sin 60°.
(2)

(*d*) Find the difference between the exact value of sin 60° and the approximation in part (*c*).
(1)

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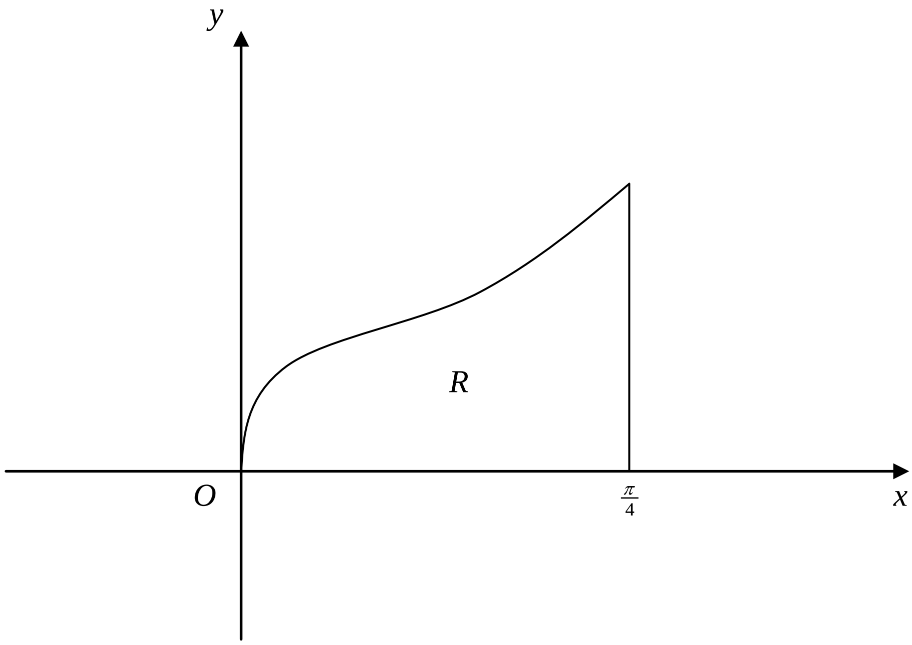
6. (*a*) Use integration by parts to show that

$$\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx = \tfrac{1}{4}\pi - \tfrac{1}{2}\ln 2.$$

(6)

Figure 1



The finite region R, bounded by the equation $y = x^{\frac{1}{2}}\sec x$, the line $x = \dfrac{\pi}{4}$ and the x-axis is shown in Fig. 1. The region R is rotated through 2π radians about the x-axis.

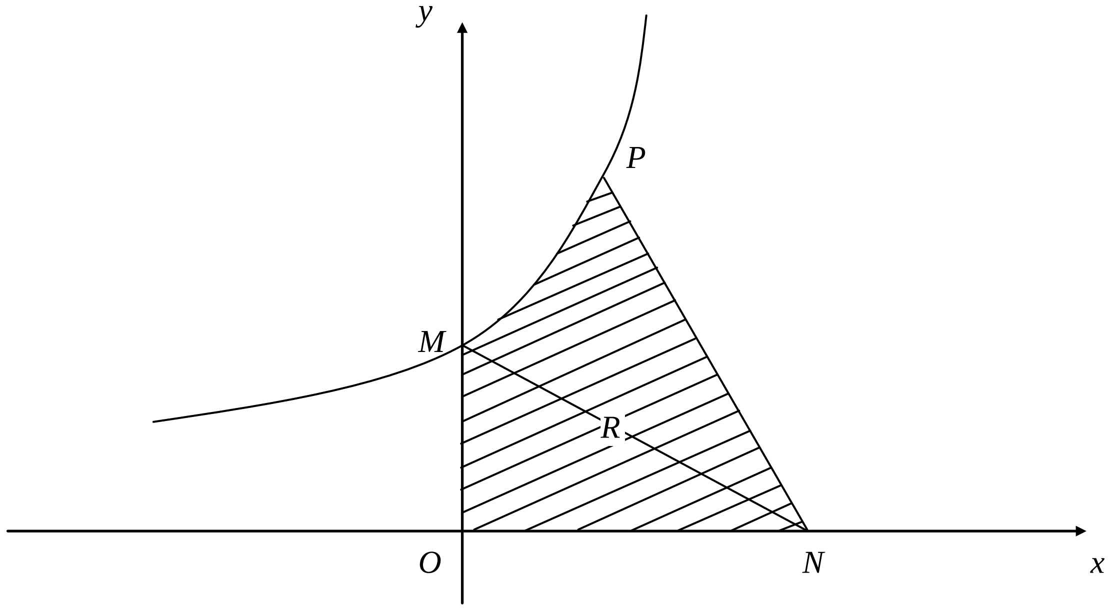
(*b*) Find the volume of the solid of revolution generated.

(2)

(*c*) Find the gradient of the curve with equation $y = x^{\frac{1}{2}}\sec x$ at the point where $x = \dfrac{\pi}{4}$.

(3)

7. **Figure 3**



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in Fig. 3.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in Fig. 3.

(c) Find the area of R, giving your answers in the form $p + q \ln 2$, where p and q are integers to be found.

(7)

8. Referred to an origin O, the points A, B and C have position vectors $(9\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, $(6\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$ and $(3\mathbf{i} + p\mathbf{j} + q\mathbf{k})$ respectively, where p and q are constants.

(*a*) Find, in vector form, an equation of the line l which passes through A and B.

(2)

Given that C lies on l,

(*b*) find the value of p and the value of q,

(2)

(*c*) calculate, in degrees, the acute angle between OC and AB.

(3)

The point D lies on AB and is such that OD is perpendicular to AB.

(*d*) Find the position vector of D.

(6)

END