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Solutionbank C3
Edexcel AS and A Level Modular Mathematics

2 Review Exercise
Exercise A, Question 1

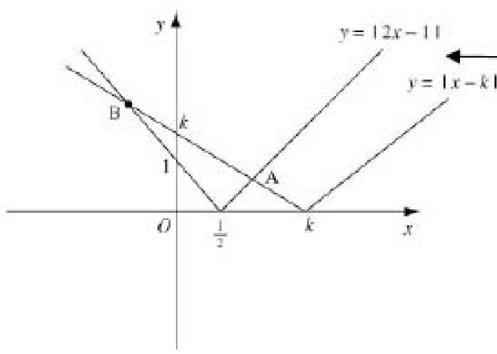
Question:

a On the same set of axes sketch the graphs of $y = |2x-1|$ and $y = |x-k|, k > 1$.

b Find, in terms of k, the values of x for which $|2x-1| = |x-k|$.

Solution:

a



For $y = |2x-1|$ draw $y = 2x-1$ and reflect part below x-axis in the x-axis. For $y = |x-k|$, intersects x-axis at $(k, 0)$ where $k > 1$ and gradient is 1, whereas the other line has gradient 2.

b For point A

$$2x - 1 = -(x-k)$$
$$\Rightarrow 3x = 1 + k$$
$$x = \frac{1+k}{3}$$

The line $|x-k|$ has been reflected so equation is $y = -(x-k)$.

For point B

$$-(2x-1) = -(x-k)$$
$$\Rightarrow -2x + 1 = -x + k$$
$$x = 1 - k$$

Both lines have been reflected.

As $k > 0$, this value is negative, which agrees with diagram.

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2 Review Exercise
Exercise A, Question 2

Question:

a Sketch the graph of $y = |3x+2| - 4$, showing the coordinates of the points of intersection of the graph with the axes.

b Find the values of x for which
$|3x+2| = 4 + x$.

Solution:

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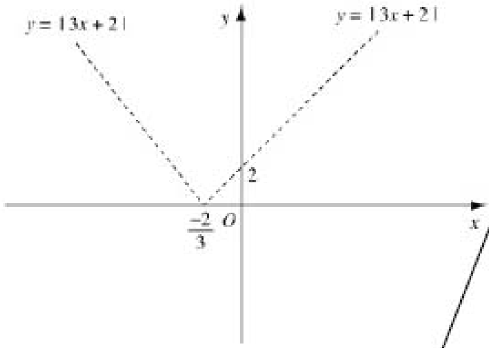
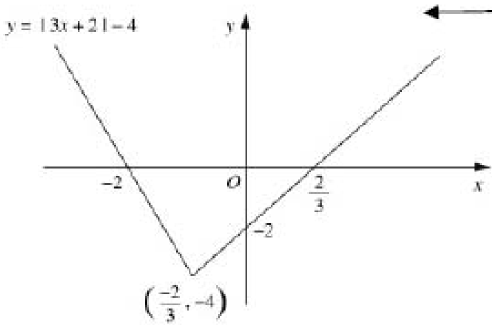
a



First draw $y = |3x+2|$ and then translate by 4 units in negative y-direction.



Meets x-axis where

i

$$y = 3x + 2 - 4 = 0$$
$$\Rightarrow 3x = 2$$
$$x = \frac{2}{3}$$

ii

$$y = -3x - 2 - 4 = 0$$
$$\Rightarrow 3x = -6$$
$$x = -2$$

b

$$|3x+2| = 4 + x$$
$$|3x+2| - 4 = x$$

The values of x are the x-coordinates of the intersections of $y = x$ and $y = |3x+2| - 4$.

So

i

$$3x + 2 - 4 = x$$
$$\Rightarrow 2x = 2$$
$$x = 1$$

ii

$$-3x - 2 - 4 = x$$
$$\Rightarrow 4x = -6$$
$$x = -1\frac{1}{2}$$

Alternatively, you could solve $(3x+2)^2 = (4+x)^2$.

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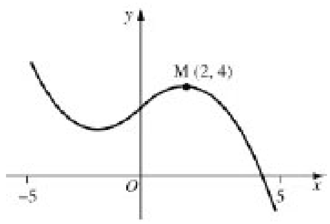
2 Review Exercise
Exercise A, Question 3

Question:



The figure shows the graph of $y = \mathrm{f}(x), -5 \le x \le 5$.

The point M (2, 4) is the maximum turning point of the graph.
Sketch, on separate diagrams, the graphs of

a $y = \mathrm{f}(x) + 3$

b $y = |\mathrm{f}(x)|$

c $y = \mathrm{f}(|x|)$.

Show on each graph the coordinates of any maximum turning points.

Solution:

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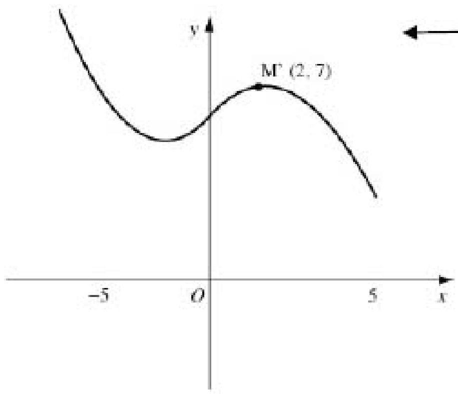
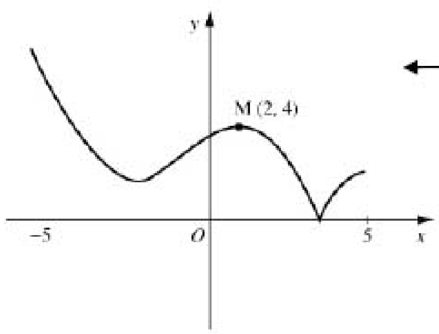
a



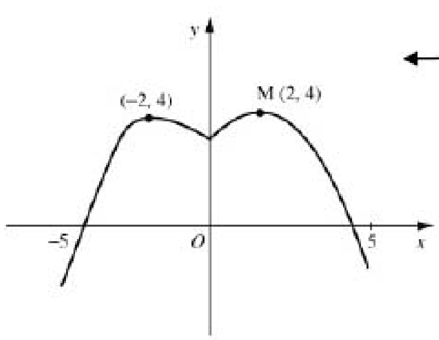
Translation of +3 in the y direction.

b



For $y \geq 0$, curve is $y = f(x)$.
For $y < 0$, reflect in x-axis.

c



For $x < 0$
$f|x| = f(-x)$
so draw $y = f(x)$
for $x \geq 0$, and then reflect this in
$x = 0$.

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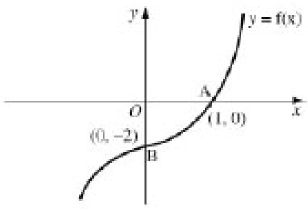
2 Review Exercise
Exercise A, Question 4

Question:



The diagram shows a sketch of the graph of the increasing function f.
The curve crosses the x-axis at the point A(1, 0) and the y-axis at the point B(0, −2). On separate diagrams, sketch the graph of:

a $y = f^{-1}(x)$
b $y = f(|x|)$
c $y = f(2x) + 1$
d $y = 3f(x - 1)$.

In each case, show the images of the points A and B.

Solution:

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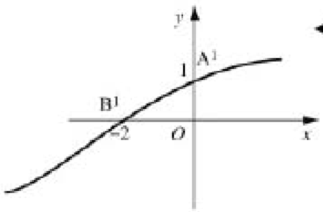
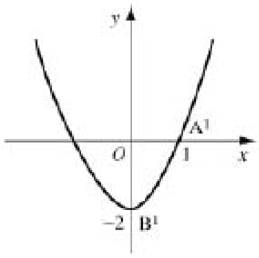
a



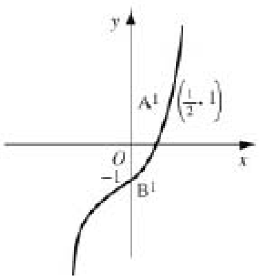
Reflect $y = f(x)$ in $y = x$.
$(1, 0) \rightarrow (0, 1)$
$(0, -2) \rightarrow (-2, 0)$

b



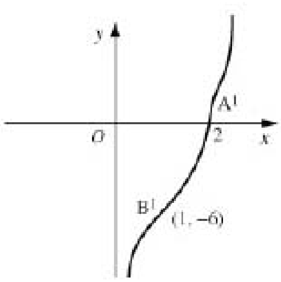
Draw $y = f(x), x \geq 0$
Reflect this in y-axis.

c



Sketch $y = f(x)$ in x-axis with scale factor $\frac{1}{2}$, and translate in the y direction by 1 unit.

d



Translate by +1 in the x direction and stretch in the y direction with scale factor 3.

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2 Review Exercise
Exercise A, Question 5

Question:

For the positive constant k, where $k > 1$ the functions f and g are defined by

$f : x \rightarrow \ln(x+k), x > -k,$

$g : x \rightarrow |2x-k|, \quad x \in \mathbb{R}$

a Sketch, on the same set of axes, the graphs of f and g. Give the coordinates of points where the graphs meet the axes.

b Write down the range of f.

c Find, in terms of k, $fg\left(\frac{k}{4}\right)$.

The curve C has equation $y = f(x)$. The tangent to C at the point with x- coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

d Find the value of k. ***E***

Solution:

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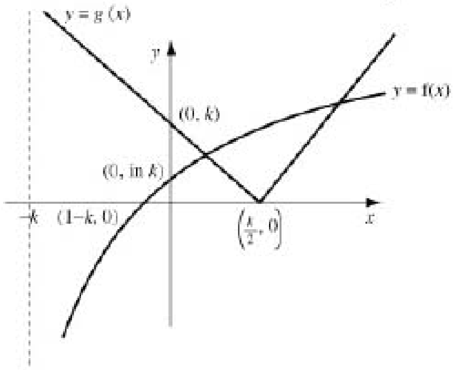
a



Graph is for $k > 1$ $y = f(x)$ has asymptote $x = -k$.
It meets the x-axis where $\ln(x+k) = 0$

$$\Rightarrow x + k = 1$$
$$\Rightarrow x = 1 - k$$

It meets the y-axis where $x = 0$, i.e. $y = \ln k$.

$y = g(x)$ is v-shaped passing through $(\frac{k}{2}, 0)$ and $(0, k)$.

b

Range of f is $f(x) \in \mathbb{R}$

f is an increasing function.

c

$$fg\left(\frac{k}{4}\right) = f\left(\left|-\frac{k}{2}\right|\right)$$
$$= f\left(\frac{k}{2}\right)$$
$$= \ln\left(\frac{3k}{2}\right)$$

d

$$y = \ln(x+k)$$
$$\frac{dy}{dx} = \frac{1}{x+k}$$

So when $x = 3$ $\frac{1}{3+k} = \frac{2}{9}$

Gradient of $9y = 2x + 1$ is $\frac{2}{9}$.

$$\Rightarrow 7 = 6 + 2k$$
$$\Rightarrow 2k = 3$$
$$k = 1\frac{1}{2}$$

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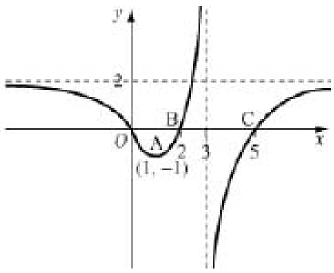
2 Review Exercise
Exercise A, Question 6

Question:



The diagram shows a sketch of the graph of $y = f(x)$.

The curve has a minimum at the point $A(1, -1)$, passes through x-axis at the origin, and the points B(2, 0) and C(5, 0); the asymptotes have equations $x = 3$ and $y = 2$.

a Sketch, on separate axes, the graph of

i $y = |f(x)|$

ii $y = -f(x+1)$

iii $y = f(-2x)$

in each case, showing the images of the points A, B and C.

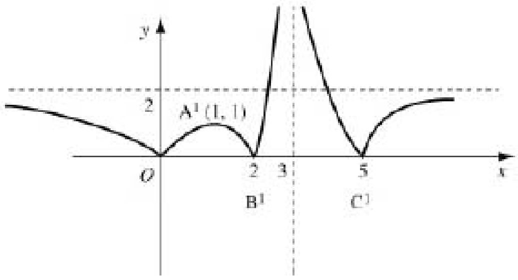
b State the number of solutions to the equation

i $3|f(x)| = 2$

ii $2|f(x)| = 3$.

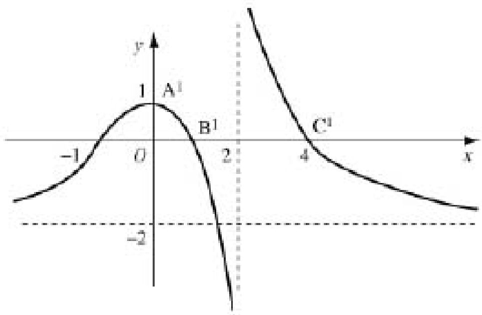
Solution:

a i



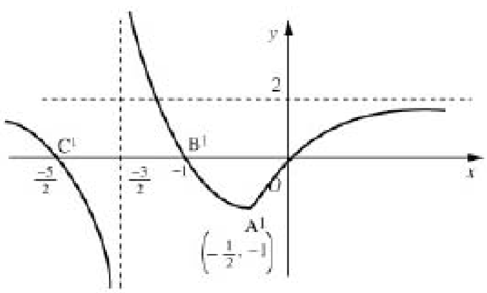
All parts of curve $y = f(x)$ below x-axis are reflected in x-axis.
$A \to (1, 1)$
B and C do not move.

ii



Translate by -1 in the x direction and reflect in the x −axis.

iii



Stretch in the x direction with scale factor $-\frac{1}{2}$ (or stretch in the x direction with scale factor $\frac{1}{2}$ and reflect in the y-axis).

b i

$3|f(x)| = 2 \Rightarrow |f(x)| = \frac{2}{3}$

number of solutions is 6

ii $2|f(x)| = 3 \Rightarrow |f(x)| = \frac{3}{2}$

number of solutions is 4

ie $2|f(x)| = 3 \Rightarrow |f(x) = \frac{3}{2}$

Consider graph a i.

i How many times does the line $y = \frac{2}{3}$ cross the curve?

Line is below A^1.

ii Draw the line $y = \frac{3}{2}$.

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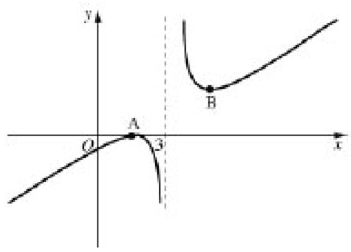
2 Review Exercise
Exercise A, Question 7

Question:



The diagram shows part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{(x-1)^2}{(x-3)}.$$

The points A and B are the stationary points of C.
The line $x = 3$ is a vertical asymptote to C.

a Using calculus, find the coordinates of A and B.

b Sketch the curve C^*, with equation $y = f(-x) + 2$, showing the coordinates of the images of A and B.

c State the equation of the vertical asymptote to C^*.

Solution:

a

$$\frac{dy}{dx} = \frac{(x-3)2(x-1)-(x-1)^2(1)}{(x-3)^2}$$

Use the quotient rule.

$$= \frac{(x-1)[2x-6-(x-1)]}{(x-3)^2}$$

$$= \frac{(x-1)(x-5)}{(x-3)^2}$$

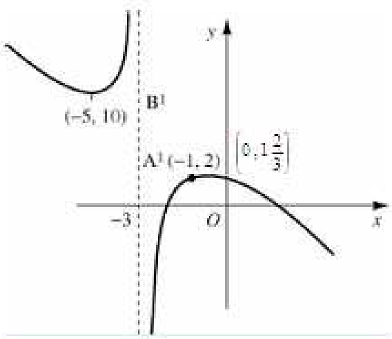
$$\frac{dy}{dx} = 0 \Rightarrow x=1 \text{ or } x=5$$

$f(x)=0 \Rightarrow x=1$, so, as a check, you know that the coordinates of A are (1, 0).

Coordinates of A: (1, 0) B: (5, 8)

$$y = \frac{(5-1)^2}{(5-3)} = \frac{16}{2} = 8$$

b



Reflect in the y-axis and translate by 2 units in the y direction.

c $x = -3$

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2 Review Exercise
Exercise A, Question 8

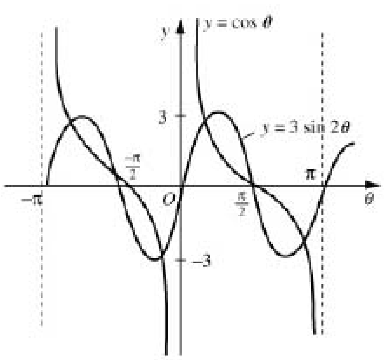
Question:

a On the same set of axes, in the interval $-\pi < \theta < \pi$, sketch the graphs of
i $y = \cot\theta$,
ii $y = 3\sin 2\theta$

b Solve, in the interval $-\pi < \theta < \pi$, the equation
$\cot\theta = 3\sin 2\theta$
giving your answers, in radians, to 3 significant figures where appropriate.

Solution:

a



b

$$\cot\theta = 3\sin 2\theta$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = 6\sin\theta\cos\theta$$

Do not cancel $\cos\theta$.

$$\Rightarrow \cos\theta\,(1-6\sin^2\theta) = 0$$

$$\Rightarrow \cos\theta = 0 \text{ or } \sin\theta = \pm\sqrt{\frac{1}{6}}$$

Do not forget $\pm$ for $\sin\theta$.

$$\cos\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2} = 1.57$$

$$\sin\theta = \pm\sqrt{\frac{1}{6}}$$

$$\theta = 0.421, 2.72, -0.421, -2.72$$

Give values in all 4 quadrants.
Remember to use the radian mode on your calculator.
$\alpha, \pi-\alpha, -\alpha, -\pi+\alpha$

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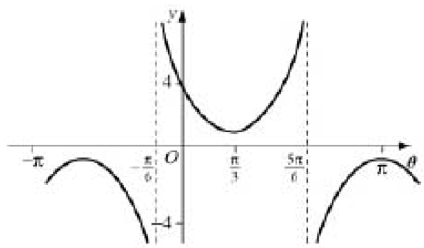
2 Review Exercise
Exercise A, Question 9

Question:



The diagram shows, in the interval $-\pi \le \theta \le \pi$, the graph of $y = k\sec(\theta - \alpha)$.

The curve crosses the y-axis at the point (0, 4) and the θ-coordinate of its minimum point is $\pi/3$.

a State, as a multiple of π, the value of α.

b Find the value of k.

c Find the exact values of θ at the points where the graph crosses the line $y = -2\sqrt{2}$.

d Show that the gradient at the point on the curve with θ-coordinate $\frac{7\pi}{12}$ is $2\sqrt{2}$.

Solution:

a $\frac{\pi}{3}$

$y = k\sec\theta$ has minimum on y-axis.
This curve has been translated by $\frac{\pi}{3}$ in the x direction.

b As (0, 4) lies on curve

$$4 = k\sec\left(-\frac{\pi}{3}\right)$$
$$\Rightarrow 4 = 2k$$
$$\Rightarrow k = 2$$

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$
$$\Rightarrow \sec\left(\frac{-\pi}{3}\right) = 2$$

c Solve

$$2\sec\left(\theta-\frac{\pi}{3}\right) = -2\sqrt{2}$$
$$\Rightarrow \sec\left(\theta-\frac{\pi}{3}\right) = -\sqrt{2}$$
$$\Rightarrow \cos\left(\theta-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta-\frac{\pi}{3} = -\frac{5\pi}{4}, -\frac{3\pi}{4}$$
$$\theta = \frac{\pi}{3}-\frac{5\pi}{4}, \frac{\pi}{3}-\frac{3\pi}{4}$$
$$= -\frac{11\pi}{12}, -\frac{5\pi}{12}$$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ gives values in the 2nd and 3rd quadrants.
$-\frac{4\pi}{3} \le \theta-\frac{\pi}{3} \le \frac{2\pi}{3}$
[$y = -2\sqrt{2}$ meets the graph where θ is negative.]

d

$$\frac{dy}{d\theta} = 2\sec\left(\theta-\frac{\pi}{3}\right)\tan\left(\theta-\frac{\pi}{3}\right)$$

$$\text{At } \theta = \frac{7\pi}{12}, \frac{dy}{dx} = 2\sec\frac{\pi}{4}\tan\frac{\pi}{4} = 2\sqrt{2}(1) = 2\sqrt{2}$$

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2 Review Exercise
Exercise A, Question 10

Question:

a Given that $\sin^2\theta+\cos^2\theta=1$, show that $1+\tan^2\theta\equiv\sec^2\theta$.

b Solve, for $0\le\theta<360^\circ$, the equation $2\tan^2\theta+\sec\theta=1$, giving your answers to 1 decimal place. *E*

Solution:

a

$$\sin^2\theta+\cos^2\theta \equiv 1$$
$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$$
$$\Rightarrow \tan^2\theta+1 \equiv \sec^2\theta$$

b

$$2\tan^2\theta+\sec\theta = 1$$
$$\Rightarrow 2(\sec^2\theta-1)+\sec\theta = 1$$
$$\Rightarrow 2\sec^2\theta+\sec\theta-3 = 0$$
$$\Rightarrow (2\sec\theta+3)(\sec\theta-1) = 0$$
$$\Rightarrow \sec\theta=-\frac{3}{2} \quad \text{or } \sec\theta = +1$$
$$\Rightarrow \cos\theta=-\frac{2}{3} \quad \text{or } \cos\theta = +1$$
$$\cos\theta = -\frac{2}{3}\Rightarrow\theta=131.8^\circ, 228.2^\circ$$
$$\cos\theta = +1\Rightarrow\theta=0^\circ$$

Use result in **a** to form a quadratic in $\sec\theta$.

360° not in interval.

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2 Review Exercise
Exercise A, Question 11

Question:

a Prove that $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$.

b Find all the values of x, in the interval $0 \le x \le 360^\circ$, for which
$\sec^4 2x = \tan 2x(3 + \tan^3 2x)$.
Give your answers correct to 1 decimal place, where appropriate.

Solution:

a

$\sec^4\theta - \tan^4\theta$

$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta)$

$= (1 + \tan^2\theta + \tan^2\theta)(1)$

$= 1 + 2\tan^2\theta$

Use $a^2 - b^2 = (a+b)(a-b)$.

Use $\sec^2\theta = 1 + \tan^2\theta$.

b

$\sec^4 2x = 3\tan 2x + \tan^4 2x$

$\Rightarrow \sec^4 2x - \tan^4 2x = 3\tan 2x$

$\Rightarrow 1 + 2\tan^2 2x = 3\tan 2x$

$\Rightarrow 2\tan^2 2x - 3\tan 2x + 1 = 0$

$\Rightarrow (2\tan 2x - 1)(\tan 2x - 1) = 0$

Use result in a with $\theta = 2x$.

$\tan 2x = \frac{1}{2}$ or $\tan 2x = 1$

$\tan 2x = 1 \Rightarrow 2x = 45^\circ, 225^\circ, 405^\circ, 585^\circ$

$x = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$

$\tan 2x = \frac{1}{2} \Rightarrow 2x = 26.6^\circ, 206.6^\circ, 386.6^\circ, 566.6^\circ$

$x = 13.3^\circ, 103.3^\circ, 193.3^\circ, 283.3^\circ$

$0 \le x \le 360^\circ$
$\Rightarrow 0 \le 2x \le 720^\circ$

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2 Review Exercise
Exercise A, Question 12

Question:

a Prove that

$\cot\theta - \tan\theta = 2\cot 2\theta,\ \theta \neq \frac{n\pi}{2}$.

b Solve, for $-\pi < \theta < \pi$, the equation

$\cot\theta - \tan\theta = 5$,

giving your answers to 3 significant figures.

Solution:

a

$$\begin{aligned} \text{LHS} &= \cot\theta - \tan\theta \\ &= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \\ &= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} \\ &= 2\cot 2\theta \end{aligned}$$

b Solve

$$\begin{aligned} 2\cot 2\theta &= 5 \\ \Rightarrow \cot 2\theta &= \frac{5}{2} \\ \tan 2\theta &= 0.4 \end{aligned}$$

$$\begin{aligned} -\pi &< \theta < \pi \\ -2\pi &< 2\theta < 2\pi \end{aligned}$$

$$\begin{aligned} \Rightarrow 2\theta &= -5.903, -2.761, 0.3805\ldots, 3.522 \\ \theta &= -2.95, -1.38, 0.190, 1.76 \text{ (3 s.f.)} \end{aligned}$$

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2 Review Exercise
Exercise A, Question 13

Question:

a Solve, in the interval $0 \le \theta \le 2\pi$, $\sec\theta + 2 = \cos\theta + \tan\theta(3+\sin\theta)$, giving your answers to 3 significant figures.

b Solve, in the interval $0 \le x \le 360^\circ$, $\cot^2 x = \operatorname{cosec} x(2-\operatorname{cosec} x)$, giving your answers to 1 decimal place.

Solution:

a

$$\begin{aligned}
\sec\theta + 2 &= \cos\theta + \tan\theta(3+\sin\theta)\\
\Rightarrow 1 + 2\cos\theta &= \cos^2\theta + 3\sin\theta + \sin^2\theta\\
\Rightarrow 1 + 2\cos\theta &= 1 + 3\sin\theta\\
\Rightarrow 3\sin\theta &= 2\cos\theta\\
\Rightarrow \tan\theta &= \frac{2}{3}\\
\Rightarrow \theta &= 0.588, 3.73 \text{ (3 s.f.)}
\end{aligned}$$

Multiply by $\cos\theta$.
Use $\sin^2\theta + \cos^2\theta = 1$.

b

$$\begin{aligned}
\cot^2 x &= \operatorname{cosec} x(2-\operatorname{cosec} x)\\
&= 2\operatorname{cosec} x - \operatorname{cosec}^2 x\\
\Rightarrow \operatorname{cosec}^2 x - 1 &= 2\operatorname{cosec} x - \operatorname{cosec}^2 x\\
\Rightarrow 2\operatorname{cosec}^2 x &- 2\operatorname{cosec} x - 1 = 0\\
\Rightarrow \operatorname{cosec} x &= \frac{2 \pm \sqrt{4+8}}{4}\\
&= \frac{1 \pm \sqrt{3}}{2}
\end{aligned}$$

Use $1 + \cot^2 x = \operatorname{cosec}^2 x$ to form a quadratic equation in $\operatorname{cosec} x$.

$\sqrt{12} = 2\sqrt{3}$

As $\operatorname{cosec} x \ge 1$ or $\operatorname{cosec} x \le -1$

$$\begin{aligned}
\operatorname{cosec} x &= \frac{1+\sqrt{3}}{2} = 1.366\ldots\\
\sin x &= 0.732\ldots\\
x &= 47.1^\circ, 132.9^\circ
\end{aligned}$$

$-1 \le \frac{1-\sqrt{3}}{2} \le 1$ so invalid.

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2 Review Exercise
Exercise A, Question 14

Question:

Given that

$y = \arcsin x, -1 \le x \le 1$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$,

a express $\arccos x$ in terms of y.

b Hence find, in terms of π the value of $\arcsin x + \arccos x$.

Given that

$y = \arccos x, -1 \le x \le 1$ and $0 \le y \le \pi$,

c sketch, on the same set of axes, the graphs of $y = \arcsin x$ and $y = \arccos x$, making it clear which is which.

d Explain how your sketches can be used to evaluate $\arcsin x + \arccos x$.

Solution:

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a $y = \arcsin x$

$\Rightarrow \sin y = x$

$x = \cos\left(\frac{\pi}{2} - y\right)$

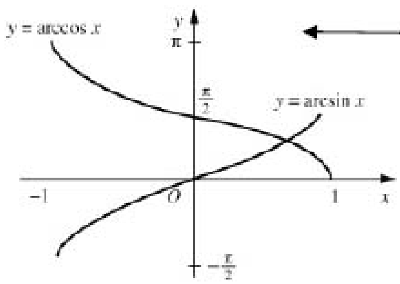
$\Rightarrow \frac{\pi}{2} - y = \arccos x$

Using $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$.

b

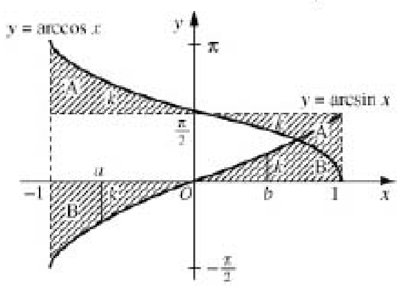
$$\arcsin x + \arccos x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$$

c



$y = \arccos x$ is a reflection in the line $y = x$ of $y = \cos x, 0 \le x \le \pi$.
$y = \arcsin x$ is a reflection in the line $y = x$ of $y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

d



The shaded areas A and B are congruent due to the symmetries of the graphs.
Consider $x = a$, where $-1 \le a < 0$

$\arccos x = \frac{\pi}{2} + h$, see diagram

$\arcsin x = -h$

$\Rightarrow \arcsin x + \arccos x = \frac{\pi}{2}$

Consider $x = b$, where $0 \le b \le 1$

$\arccos x = \frac{\pi}{2} - k$

$\arcsin x = k$

$\Rightarrow \arcsin x + \arccos x = \frac{\pi}{2}$

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2 Review Exercise
Exercise A, Question 15

Question:

a By writing $\cos 3\theta$ as $\cos(2\theta+\theta)$, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

b Given that $\cos\theta = \frac{\sqrt{2}}{3}$, find the exact value of $\sec 3\theta$.

Solution:

a

$$\begin{aligned}\cos(2\theta+\theta) &= \cos 2\theta\cos\theta - \sin 2\theta \sin\theta \\ &= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta) \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\ &= 4\cos^3\theta - 3\cos\theta\end{aligned}$$

Use $\cos(A+B) = \cos A\cos B - \sin A \sin B$ with $A = 2\theta, B = \theta$.

Use $\sin^2\theta = 1 - \cos^2\theta$.

b

$$\begin{aligned}\cos 3\theta &= 4\left(\frac{\sqrt{2}}{3}\right)^3 - 3\left(\frac{\sqrt{2}}{3}\right) \\ &= \frac{8\sqrt{2}}{27} - \frac{\cancel{3}\sqrt{2}}{\cancel{3}} \\ &= \frac{-19\sqrt{2}}{27}\end{aligned}$$

$$\Rightarrow \sec 3\theta = \frac{-27}{19\sqrt{2}} = \frac{-27\sqrt{2}}{38}$$

$(\sqrt{2})^3 = 2\sqrt{2}$

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2 Review Exercise

Exercise A, Question 16

Question:

Given that $\sin(x+30^\circ)=2\sin(x-60^\circ)$,

a show that $\tan x = 8+5\sqrt{3}$.

b Hence express $\tan(x+60^\circ)$ in the form $a+b\sqrt{3}$.

Solution:

a $\sin(x+30^\circ)=2\sin(x-60^\circ)$

$$\Rightarrow \sin x\cos 30^\circ + \cos x \sin 30^\circ = 2\sin x\cos 60^\circ - 2\cos x\sin 60^\circ$$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \sin x - \sqrt{3}\cos x$$

$$\Rightarrow \sqrt{3}\sin x + \cos x = 2\sin x - 2\sqrt{3}\cos x$$

$$\Rightarrow (2\sqrt{3}+1)\cos x = (2-\sqrt{3})\sin x$$

$$\Rightarrow \tan x = \frac{2\sqrt{3}+1}{2-\sqrt{3}}$$

$$= \frac{(2\sqrt{3}+1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{4\sqrt{3}+\sqrt{3}+2+6}{1}$$

$$= 8+5\sqrt{3}$$

$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

Use $\frac{\sin x}{\cos x} = \tan x$.

Rationalise the denominator.

b

$$\tan(x+60^\circ) = \frac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x}$$

$$= \frac{8+6\sqrt{3}}{1-8\sqrt{3}-15}$$

$$= \frac{8+6\sqrt{3}}{-8\sqrt{3}-14} = \frac{-4-3\sqrt{3}}{4\sqrt{3}+7}$$

$$= \frac{-(4+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$= \frac{-[(28-36)+(21\sqrt{3}-16\sqrt{3})]}{1}$$

$$= 8-5\sqrt{3}$$

$\tan 60^\circ = \sqrt{3}$

Use **a**.

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2 Review Exercise
Exercise A, Question 17

Question:

a Given that $\cos A = \frac{3}{4}$ where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$.

b **i** Show that

$$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) = \cos 2x$$

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

ii show that $\frac{dy}{dx} = \sin 2x$ ***E***

Solution:

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a

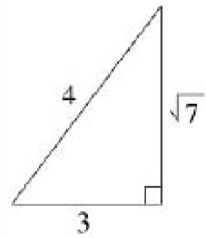
$$\cos A = \frac{3}{4}, \quad 270^\circ < A < 360^\circ$$

$$\Rightarrow \sin A = \frac{-\sqrt{7}}{4}$$

$$\text{so } \sin 2A = 2\sin A \cos A$$

$$= -2\frac{\sqrt{7}}{4} \times \frac{3}{4}$$

$$= \frac{-3\sqrt{7}}{8}$$



For $270^\circ < A < 360^\circ$ sin A is negative.

b i

$$\cos(2x + \frac{\pi}{3}) = \cos 2x \cos\frac{\pi}{3} - \sin 2x \sin\frac{\pi}{3}$$

$$= \frac{1}{2}\cos 2x - \frac{\sqrt{3}}{2}\sin 2x$$

$$\cos(2x - \frac{\pi}{3}) = \frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x$$

$$\text{so } \cos(2x + \frac{\pi}{3}) + \cos(2x - \frac{\pi}{3}) = \cos 2x$$

← Add two results above.

ii

$$y = 3\sin^2 x + \cos(2x + \frac{\pi}{3}) + \cos(2x - \frac{\pi}{3})$$

$$= 3\sin^2 x + \cos 2x$$

Use **b i**.

$$\frac{dy}{dx} = 3(2\sin x \cos x) - 2\sin 2x$$

$$= \sin 2x$$

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2 Review Exercise
Exercise A, Question 18

Question:

Solve, in the interval $-180^\circ \le x < 180^\circ$, the equations

a $\cos 2x + \sin x = 1$

b $\sin x(\cos x + \operatorname{cosec} x) = 2\cos^2 x$, giving your answers to 1 decimal place.

Solution:

a

$$\begin{aligned} \cos 2x + \sin x &= 1 \\ \Rightarrow (1 - \sin^2 x) + \sin x &= 1 \\ \Rightarrow 2\sin^2 x - \sin x &= 0 \\ \sin x(2\sin x - 1) &= 0 \\ \Rightarrow \sin x = 0 \quad \text{or } \sin x &= \frac{1}{2} \end{aligned}$$

Choose the appropriate form of $\cos 2x$ to give a quadratic in $\sin x$. Do NOT cancel $\sin x$, always factorise.

$$\Rightarrow x = -180^\circ, 0^\circ, 30^\circ, 150^\circ$$

b

$$\begin{aligned} \sin x \cos x + \sin x \cdot \frac{1}{\sin x} &= 2\cos^2 x \\ \Rightarrow \sin x \cos x &= 2\cos^2 x - 1 \\ \Rightarrow \frac{1}{2}\sin 2x &= \cos 2x \\ \Rightarrow \tan 2x &= 2 \\ \Rightarrow 2x &= -116.57^\circ, 63.43^\circ \\ x &= -58.3^\circ, 31.7^\circ \text{ (1 d.p.)} \end{aligned}$$

$$\begin{aligned} -180^\circ \le x &< 180^\circ \\ \Rightarrow -360^\circ \le 2x &< 360^\circ \end{aligned}$$

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2 Review Exercise
Exercise A, Question 19

Question:

a Prove that
$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}2\theta, \quad \theta\neq 90n^{\circ}.$$

b Sketch the graph of $y=2\operatorname{cosec}2\theta$ for $0^{\circ}<\theta<360^{\circ}$.

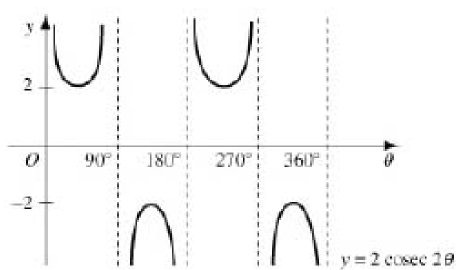
c Solve, for $0^{\circ}<\theta<360^{\circ}$, the equation
$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=3,$$
giving your answers to 1 decimal place. *E*

Solution:

a

$$\begin{aligned}\text{LHS} &= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\\ &= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\\ &= \frac{1}{\frac{1}{2}\sin 2\theta}\\ &= 2\operatorname{cosec}2\theta\end{aligned}$$

b



Note that maxima and minima are at −2 and +2 respectively.

c Solve

$$\begin{aligned}2\operatorname{cosec}2\theta &= 3\\ \text{i.e. } \sin 2\theta &= \frac{2}{3}\\ \Rightarrow 2\theta &= 41.8^{\circ}, 138.2^{\circ}, 401.8^{\circ}, 498.2^{\circ}\\ \theta &= 20.9^{\circ}, 69.1^{\circ}, 200.9^{\circ}, 249.1^{\circ}\ (1\text{ d.p.})\end{aligned}$$

$$\begin{aligned}0^{\circ}<\theta &< 360^{\circ}\\ \Rightarrow 0^{\circ}<2\theta &< 720^{\circ}\end{aligned}$$

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2 Review Exercise
Exercise A, Question 20

Question:

a Express $3\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

b Hence find the greatest value of $(3\sin x + 2\cos x)^4$.

c Solve, for $0 < x < 2\pi$, the equation
$3\sin x + 2\cos x = 1$, giving your answers to 3 decimal places. ***E***

Solution:

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a Set

$$3\sin x + 2\cos x \equiv R\sin(x+\alpha)$$
$$\equiv R\sin x\cos x + R\cos x\sin x$$

$\Rightarrow R\cos\alpha = 3$

$R\sin\alpha = 2$

(Comparing $\sin x$. Comparing $\cos x$.)

$\Rightarrow \tan\alpha = \dfrac{2}{3}$ (Divide.)

$\Rightarrow \alpha = 0.588\ldots$

(α should be in radians as $0 < \alpha < \dfrac{\pi}{2}$.)

$R = \sqrt{13}$

(For R: either square and add $R^2(\sin^2\alpha + \cos^2\alpha) = 3^2 + 2^2$, $R > 0$ or use $R\cos\alpha = 3$ or $R\sin\alpha = 2$ with α found above.)

$\Rightarrow 3\sin x + 2\cos x = \sqrt{13}\sin(x + 0.588\ldots)$

b The maximum value of $\sqrt{13}\sin(x+0.588\ldots)$ is $\sqrt{13}$ and occurs when $\sin(x+0.588\ldots) = 1$ so maximum value of

$$\left\{\sqrt{13}\sin(x+0.588)\right\}^4 = (\sqrt{13})^4 = 169$$

c Solve

$\sqrt{13}\sin(x+0.588\ldots) = 1$

$\Rightarrow \sin(x+0.588\ldots) = \dfrac{1}{\sqrt{13}}$

($0 < x < 2\pi$ $\Rightarrow 0.588 < x + 0.588 < 6.87$)

$\Rightarrow x + 0.588\ldots = \pi - 0.2810\ldots\ 2\pi + 0.2810\ldots$

$= 2.8606, 6.5642\ldots$

$\Rightarrow x = 2.273, 5.976$ (3 d.p.)

($\sin^{-1}\dfrac{1}{\sqrt{13}} = \sin^{-1}0.277 = 0.2810\ldots$ is outside above interval for $x + 0.588$.)

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2 Review Exercise
Exercise A, Question 21

Question:

The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$.

The x-coordinate of P is 3.
Find an equation of the normal to the curve at the point P in the form
$y = ax + b$, where a and b are constants. ***E***

Solution:

$$y = \ln\left(\frac{1}{3}x\right)$$

$$x = 3, y = \ln 1 = 0, \text{ so } P \equiv (3, 0)$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Remember that $\log ax = \log a + \log x$ in any base
so $\frac{d}{dx}(\ln ax) = \frac{d}{dx}(\ln x)$

At P, gradient of tangent $= \frac{1}{3}$

so gradient of normal $= -3$

Use $m_1 m_2 = -1$ for perpendicular lines.

Equation of normal is

$$y - = -3(x - 3)$$

$$y = -3x + 9$$

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2 Review Exercise
Exercise A, Question 22

Question:

a Differentiate with respect to x

i $3\sin^2 x + \sec 2x$,

ii $\{x + \ln(2x)\}^3$,

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}, x \neq -1$,

b show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$ *E*

Solution:

a i

$$y = 3\sin^2 x + \sec 2x$$
$$\frac{dy}{dx} = 6\sin x\cos x + 2\sec 2x\tan 2x$$
$$= 3\sin 2x + 2\sec 2x\tan 2x$$

$\dfrac{d}{dx}(\sin x)^2 = 2\sin x\cos x$

$\dfrac{d}{dx}\sec ax = a\sec ax\tan a\,x$

ii

$$y = \{x + \ln(2x)\}^3$$
$$\frac{dy}{dx} = 3\{x + \ln(2x)\}^2\left\{1 + \frac{1}{x}\right\}$$

$y = u^3$ where $u = f(x)$

$\dfrac{dy}{dx} = 3u^2\dfrac{du}{dx}$

Note $\dfrac{d}{dx}(\ln ax) = \dfrac{1}{x}$

b

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \quad x \neq -1$$
$$\frac{dy}{dx} = \frac{(x-1)^2(10)(x-1) - (5x^2 - 10x + 9)(2)(x-1)}{(x-1)^4}$$
$$= \frac{10(x^2 - 2x + 1) - 2(5x^2 - 10x + 9)}{(x-1)^3}$$
$$= \frac{10x^2 - 20x + 10 - 10x^2 + 20x - 18}{(x-1)^3}$$
$$= \frac{-8}{(x-1)^3}$$

Use quotient rule

$u = 5x^2 - 10x + 9$

$\dfrac{du}{dx} = 10x - 10$

$v = (x-1)^2$

$\dfrac{dv}{dx} = 2(x-1)$

Be careful to use brackets and signs appropriately.

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2 Review Exercise
Exercise A, Question 23

Question:

Given that $y = \ln(1 + e^x)$,

a show that when $x = -\ln 3, \frac{dy}{dx} = \frac{1}{4}$

b find the exact value of x for which $e^y \frac{dy}{dx} = 6$.

Solution:

$y = \ln(1 + e^x)$

a $\frac{dy}{dx} = \frac{e^x}{1 + e^x}$

If $y = \ln u$ where $u = f(x)$
$\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx}$

when $x = -\ln 3$

$e^x = e^{-\ln 3} = e^{\ln 3^{-1}}$
$= \frac{1}{3}$

Remember that $e^{\ln k} = k$

so $\frac{dy}{dx} = \frac{\frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4}$

b

$e^y \frac{dy}{dx} = 6$

$\Rightarrow (1 + e^x)\frac{e^x}{1 + e^x} = 6$

$\Rightarrow e^x = 6$

$\Rightarrow x = \ln 6$

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2 Review Exercise
Exercise A, Question 24

Question:

a Differentiate with respect to x

i x^2e^{3x+2},

ii $\dfrac{\cos(2x^3)}{3x}$

b Given that $x = 4\sin(2y+6)$, find $\dfrac{dy}{dx}$ in terms of x. ***E***

Solution:

a i

$$y = x^2e^{3x+2}$$
$$\frac{dy}{dx} = x^2 \cdot 3e^{3x+2} + 2xe^{3x+2}$$
$$= (3x+2)xe^{3x+2}$$

Use the product rule with
$u = x^2 \Rightarrow \dfrac{du}{dx} = 2x$
$v = e^{3x+2} \Rightarrow \dfrac{dv}{dx} = 3e^{3x+2}$

ii

$$y = \frac{\cos(2x^3)}{3x}$$
$$\frac{dy}{dx} = \frac{3x[-6x^2\sin(2x^3)] - 3\cos(2x^3)}{9x^2}$$
$$= \frac{-[6x^3\sin(2x^3) + \cos(2x^3)]}{3x^2}$$

Use the quotient rule with
$u = \cos(2x^3)$
$\Rightarrow \dfrac{du}{dx} = -6x^2\sin(2x^3)$
$v\ 3x = \Rightarrow \dfrac{dv}{dx} = 3$

b

$$x = 4\sin(2y+6)$$
$$\Rightarrow \frac{dx}{dy} = 8\cos(2y+6)$$
$$\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$$
$$= \pm\frac{1}{8\sqrt{1-\sin^2(2y+6)}}$$
$$= \pm\frac{1}{8\sqrt{1-\frac{x^2}{16}}}$$
$$= \pm\frac{1}{2\sqrt{16-x^2}}$$

If
$y = \sin(ax+b)$
$\dfrac{dy}{dx} = a\cos(ax+b)$

$\dfrac{dy}{dx}$ is in terms of y.

Use $\sin^2\theta + \cos^2\theta = 1$
$\Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$

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2 Review Exercise
Exercise A, Question 25

Question:

Given that $x = y^2 e^{\sqrt{y}}$,

a find, in terms of y, $\frac{dx}{dy}$

b show that when $y = 4, \frac{dy}{dx} = \frac{e^{-2}}{12}$.

Solution:

$x = y^2 e^{\sqrt{y}}$

a

$$\frac{dx}{dy} = y^2 \frac{1}{2\sqrt{y}} e^{\sqrt{y}} + 2y e^{\sqrt{y}}$$

$$= \frac{1}{2} y^{\frac{3}{2}} e^{\sqrt{y}} + 2y e^{\sqrt{y}}$$

$$= \frac{y}{2} e^{\sqrt{y}} (\sqrt{y} + 4)$$

Use the product rule with

$u = y^2 \Rightarrow \frac{du}{dy} = 2y$

$v = e^{\sqrt{y}} \Rightarrow \frac{dv}{dy} = \frac{1}{2\sqrt{y}} e^{\sqrt{y}}$

Remember
If

$y = e^{f(x)}$

$\frac{dy}{dx} = f'(x) e^{f(x)}$

b When $y = 4$

$$\frac{dx}{dy} = \frac{4}{2} e^2 (2 + 4)$$

$$= 12e^2$$

$$\text{so } \frac{dy}{dx} = \frac{1}{12e^2} = \frac{e^{-2}}{12}$$

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2 Review Exercise
Exercise A, Question 26

Question:

a Given that $y=\sqrt{1+x^2}$, show that $\frac{dy}{dx}=\frac{\sqrt{3}}{2}$ when $x=\sqrt{3}$.

b Given that $y=\ln\{x+\sqrt{(1+x^2)}\}$, show that $\frac{dy}{dx}=\frac{1}{\sqrt{(1+x^2)}}$.

Solution:

a

$$y = \sqrt{1+x^2}=(1+x^2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}\cdot 2x = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$$

When $x=\sqrt{3}$

$$\frac{dy}{dx} = \frac{\sqrt{3}}{\sqrt{1+3}} = \frac{\sqrt{3}}{2}$$

b

$$y = \ln\{x+\sqrt{1+x^2}\}$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\cdot\frac{d}{dx}\left\{x+\sqrt{1+x^2}\right\}$$

$$= \frac{1}{x+\sqrt{1+x^2}}\times\left\{1+\frac{x}{\sqrt{1+x^2}}\right\}$$

$$= \frac{1}{\cancel{x+\sqrt{1+x^2}}}\cdot\frac{(\cancel{\sqrt{1+x^2}+x})}{\sqrt{1+x^2}}$$

$$= \frac{1}{\sqrt{1+x^2}}$$

Note that this cannot be simplified in particular, $\ln\left\{x+\sqrt{1+x^2}\right\}\neq\ln x+\ln\sqrt{1+x^2}$

If
$y = \ln f(x)$
$\frac{dy}{dx} = \frac{1}{f(x)}f'(x)$

$$\frac{d}{dx}(1+x^2)^{\frac{1}{2}} = \frac{1}{2}(1+x^2)^{-\frac{1}{2}}\cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

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2 Review Exercise
Exercise A, Question 27

Question:

Given that $f(x) = x^2e^{-x}$,

a find $f'(x)$, using the product rule for differentiation

b show that $f''(x) = (x^2 - 4x + 2)e^{-x}$.

A curve C has equation $y = f(x)$.

c Find the coordinates of the turning points of C.

d Determine the nature of each turning point of the curve C.

Solution:

a

$$f(x) = x^2e^{-x}$$
$$f'(x) = x^2(-e^{-x}) + 2xe^{-x}$$
$$= e^{-x}[-x^2 + 2x]$$

Use the product rule.

b

$$f''(x) = e^{-x}[-2x+2] - e^{-x}[-x^2+2x]$$
$$= e^{-x}[x^2 - 4x + 2]$$

c Turning points when $f'(x) = 0$

i.e. $x(2-x)e^{-x} = 0$

$\Rightarrow x(2-x) = 0$ As $e^{-x} \neq 0$

So turning points when $x = 0, y = 0$

$x = 2, y = 4e^{-2}$

d When $x = 0, f''(0) = +2 > 0$

So (0, 0) is a minimum point

When $x = 2, f''(2) = e^{-2}(4 - 8 + 2) < 0$

So $(2, 4e^{-2})$ is a maximum point

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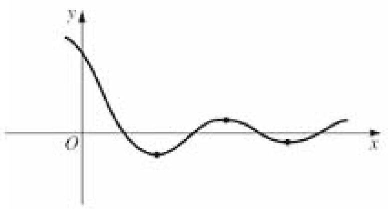
2 Review Exercise
Exercise A, Question 28

Question:

a Express $(\sin 2x + \sqrt{3}\cos 2x)$ in the form $R\sin(2x + k\pi)$, where $R > 0$ and $0 < k < \frac{1}{2}$.



The diagram shows part of the curve with equation
$y = e^{-2\sqrt{2}x}(\sin 2x + \sqrt{3}\cos 2x)$.

b Show that the x-coordinates of the turning points of the curve satisfy the equation
$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

Solution:

a $\sin 2x+\sqrt{3}\cos 2x = R\sin 2x\cos k\pi + R\cos 2x\sin k\pi$

$$\Rightarrow R\cos k\pi = 1$$
$$R\sin k\pi = \sqrt{3}$$
$$\tan k\pi = \sqrt{3} \Rightarrow k = \frac{1}{3}$$
$$R = 2$$

b

Use product rule.

$$y = 2e^{-2\sqrt{2}x}\sin(2x+\frac{\pi}{3})$$
$$\frac{dy}{dx} = 2e^{-2\sqrt{2}x}2\cos(2x+\frac{\pi}{3}) - 4\sqrt{2}e^{-2\sqrt{2}x}\sin(2x+\frac{\pi}{3})$$
$$= 4e^{-2\sqrt{2}x}\left[\cos(2x+\frac{\pi}{3}) - \sqrt{2}\sin(2x+\frac{\pi}{3})\right]$$
$$\frac{dy}{dx} = 0$$

$$\Rightarrow \cos(2x+\frac{\pi}{3}) - \sqrt{2}\sin(2x+\frac{\pi}{3}) = 0$$
$$\Rightarrow \tan(2x+\frac{\pi}{3}) = \frac{1}{\sqrt{2}}$$

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2 Review Exercise
Exercise A, Question 29

Question:

The curve C has equation $y = x^2\sqrt{\cos x}$. The point P on C has x-coordinate $\frac{\pi}{3}$.

a Show that the y-coordinate of P is $\frac{\sqrt{2}\,\pi^2}{18}$.

b Show that the gradient of C at P is 0.809, to 3 significant figures.

In the interval $0 < x < \frac{\pi}{2}$, C has a maximum at the point A.

c Show that the x-coordinate, k, of A satisfies the equation $x\tan x = 4$.

The iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{4}{x_n}\right), \quad x_0 = 1.25,$$

is used to find an approximation for k.

d Find the value of k, correct to 4 decimal places.

Solution:

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a

$$y = x^2\sqrt{\cos x}$$

$$x = \frac{\pi}{3} \Rightarrow y = \frac{\pi^2}{9}\sqrt{\frac{1}{2}} = \frac{\sqrt{2}\pi^2}{18}$$

$$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Use product rule

$u = x^2 \quad \frac{du}{dx} = 2x$

$v = \sqrt{\cos x}, \frac{dv}{dx} = \frac{1}{2}(\cos x)^{-\frac{1}{2}}(-\sin x)$

b $\frac{dy}{dx} = 2x\sqrt{\cos x} + \frac{x^2(-\sin x)}{2\sqrt{\cos x}}$

When $x = \frac{\pi}{3}$

$$\frac{dy}{dx} = \left[\frac{2\pi}{3}\sqrt{\frac{1}{2}} + \frac{\pi^2}{18}\left(-\frac{\sqrt{3}}{2}\right)\cdot\sqrt{2}\right]$$

$= 0.8094\ldots$

$= 0.809$ (3 s.f.)

c Setting $\frac{dy}{dx} = 0$

$\Rightarrow \frac{4x\cos x - x^2\sin x}{2\sqrt{\cos x}} = 0$

$\Rightarrow 4\cos x - x\sin x = 0$

$\Rightarrow 4\cos x = x\sin x$

$\Rightarrow x\tan x = 4$

Divide both sides by $\cos x$.

Ensure your calculator is in radian mode.

d $x_{n+1} = \tan^{-1}\left(\frac{4}{x_n}\right), x_0 = 1.25$

$\Rightarrow x_1 = 1.26791\ldots$

$x_2 = 1.26383\ldots$

$x_3 = 1.26476\ldots$

$x_4 = 1.26455\ldots$

$x_5 = 1.26460\ldots$

$x_6 = 1.26459\ldots$

$x_7 = 1.26459\ldots$

$x_8 = 1.26459\ldots$

$\Rightarrow x = 1.2646$ (4 d.p.)

You can verify this by considering the sign of $f(x) = 4\cos x - x\sin x$ in interval [1.26455, 1.26465]

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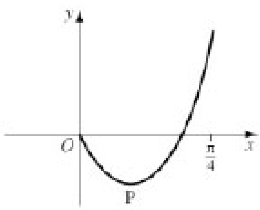
2 Review Exercise
Exercise A, Question 30

Question:



The figure shows part of the curve with equation

$y = (2x - 1)\tan 2x, \quad 0 \le x < \frac{\pi}{4}.$

The curve has a minimum at the point P. The x-coordinate of P is k.

a Show that k satisfies the equation

$4k + \sin 4k - 2 = 0.$

The iterative formula

$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$

is used to find an approximate value for k.

b Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

c Show that $k = 0.277$, correct to 3 significant figures. ***E***

Solution:

a

$$y = (2x-1)\tan 2x$$

$$\frac{dy}{dx} = (2x-1)2\sec^2 2x + 2\tan 2x$$

Setting $\frac{dy}{dx} = 0$

$\Rightarrow (2x-1)\sec^2 2x + \tan 2x = 0$

$\Rightarrow (2x-1)\frac{1}{\cos^2 2x} + \frac{\sin 2x}{\cos 2x} = 0$

$\Rightarrow (2x-1) + \sin 2x \cos 2x = 0$

$\Rightarrow 4x - 2 + 2\sin 2x \cos 2x = 0$

$\Rightarrow 4x + \sin 4x - 2 = 0$

$\cos 2x \neq 0$ in $0 \leq x < \frac{\pi}{4}$

Use $\sin 2A = 2\sin A \cos A$ with $A = 2x$

so k satisfies $4x + \sin 4x - 2 = 0$

b

$x_1 = 0.2670$ (4 d.p.)

$x_2 = 0.2809$ (4 d.p.)

$x_3 = 0.2746$ (4 d.p.)

$x_4 = 0.2774$ (4 d.p.)

Work in radian mode.

c Consider $f(x) = 4x + \sin 4x - 2$

$f(0.2775) = 0.00569\ldots$

$f(0.2765) = -0.000087\ldots$

so k is 0.277 (3 s.f.)

As there is a sign change in the interval, k lies between the two values.