

GCE Examinations  
Advanced Subsidiary

## **Core Mathematics C4**

Paper D

### **MARKING GUIDE**

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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## C4 Paper D – Marking Guide

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                           |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| 1. | <p>(a) <math>= 2^{-3}(1 - \frac{3}{2}x)^{-3} = \frac{1}{8}(1 - \frac{3}{2}x)^{-3}</math></p> <p><math>= \frac{1}{8} [1 + (-3)(-\frac{3}{2}x) + \frac{(-3)(-4)}{2}(-\frac{3}{2}x)^2 + \frac{(-3)(-4)(-5)}{3 \times 2}(-\frac{3}{2}x)^3 + \dots]</math></p> <p><math>= \frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 + \frac{135}{32}x^3 + \dots</math></p> <p>(b) <math> x  &lt; \frac{2}{3}</math></p>                                                                                                                                                                                   | <p>B1</p> <p>M1</p> <p>A3</p> <p>B1 (6)</p>                                               |
| 2. | <p>(a) <math>2x + 3y + 3x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0</math></p> <p><math>\frac{dy}{dx} = \frac{2x+3y}{4y-3x}</math></p> <p>(b) <math>\text{grad} = \frac{6-6}{-8-9} = 0</math></p> <p><math>\therefore</math> normal parallel to y-axis <math>\therefore x = 3</math></p>                                                                                                                                                                                                                                                                                                       | <p>M1 A2</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (8)</p>                                      |
| 3. | <p>(a) <math>2x^3 - 5x^2 + 6 \equiv (Ax + B)x(x - 3) + C(x - 3) + Dx</math></p> <p><math>x = 0 \Rightarrow 6 = -3C \Rightarrow C = -2</math></p> <p><math>x = 3 \Rightarrow 15 = 3D \Rightarrow D = 5</math></p> <p>coeffs <math>x^3 \Rightarrow A = 2</math></p> <p>coeffs <math>x^2 \Rightarrow -5 = B - 3A \Rightarrow B = 1</math></p> <p>(b) <math>= \int_1^2 (2x + 1 - \frac{2}{x} + \frac{5}{x-3}) dx</math></p> <p><math>= [x^2 + x - 2 \ln x  + 5 \ln x-3 ]_1^2</math></p> <p><math>= (4 + 2 - 2 \ln 2 + 0) - (1 + 1 + 0 + 5 \ln 2)</math></p> <p><math>= 4 - 7 \ln 2</math></p> | <p>M1</p> <p>A1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A2</p> <p>M1</p> <p>A1 (10)</p>          |
| 4. | <p>(a) <math>\int x dx = \int k(5 - t) dt</math></p> <p><math>\frac{1}{2}x^2 = k(5t - \frac{1}{2}t^2) + c</math></p> <p><math>t = 0, x = 0 \Rightarrow c = 0</math></p> <p><math>t = 2, x = 96 \Rightarrow 4608 = 8k, k = 576</math></p> <p><math>t = 1 \Rightarrow \frac{1}{2}x^2 = 576 \times \frac{9}{2}, x = \sqrt{5184} = 72</math></p> <p>(b) 3 hours 5 mins <math>\Rightarrow t = 3.0833, x = \sqrt{12284} = 110.83</math></p> <p><math>\therefore \frac{dx}{dt} = \frac{576(5-3.0833)}{110.83} = 9.96, \frac{dx}{dt} &lt; 10</math> so she should have left</p>                   | <p>M1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (12)</p> |

5. (a)  $\begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ a \\ b \end{pmatrix} = 0 \quad \therefore 3 + 4a + 5b = 0$  M1 A1
- (b)  $4 + s = -3 + 3t \quad (1)$   
 $1 + 4s = 1 + at \quad (2)$   
 $1 + 5s = -6 + bt \quad (3)$  B1  
 $(1) \Rightarrow s = 3t - 7$  M1  
sub. (2)  $\Rightarrow 1 + 4(3t - 7) = 1 + at$   
 $12t - 28 = at, \quad t(12 - a) = 28, \quad t = \frac{28}{12 - a}$  M1 A1  
sub. (3)  $\Rightarrow 1 + 5(3t - 7) = -6 + bt$   
 $15t - 28 = bt, \quad t(15 - b) = 28, \quad t = \frac{28}{15 - b}$  A1  
 $\frac{28}{12 - a} = \frac{28}{15 - b}, \quad 12 - a = 15 - b, \quad b = a + 3$  M1  
sub (a)  $\Rightarrow 3 + 4a + 5(a + 3) = 0, \quad a = -2, \quad b = 1$  M1 A1
- (c)  $t = 2 \quad \therefore \mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}, \quad \therefore (3, -3, -4)$  M1 A1 (12)

6. (a)  $u^2 = 1 - x \Rightarrow x = 1 - u^2, \quad \frac{dx}{du} = -2u$  M1  
 $x = 0 \Rightarrow u = 1, \quad x = 1 \Rightarrow u = 0$  B1  
area  $= \int_0^1 x \sqrt{1 - x} \, dx = \int_1^0 (1 - u^2) \times u \times (-2u) \, du$  M1  
 $= \int_0^1 (2u^2 - 2u^4) \, du$  A1  
 $= [\frac{2}{3} u^3 - \frac{2}{5} u^5]_0^1$  M1 A1  
 $= (\frac{2}{3} - \frac{2}{5}) - (0) = \frac{4}{15}$  M1 A1
- (b)  $= \pi \int_0^1 x^2(1 - x) \, dx$  M1  
 $= \pi \int_0^1 (x^2 - x^3) \, dx$   
 $= \pi [\frac{1}{3} x^3 - \frac{1}{4} x^4]_0^1$  M1 A1  
 $= \pi \{(\frac{1}{3} - \frac{1}{4}) - (0)\} = \frac{1}{12} \pi$  M1 A1 (13)

7. (a)  $\frac{dx}{dt} = 6 \cos t \times (-\sin t), \quad \frac{dy}{dt} = 2 \cos 2t$  M1 A1  
 $\frac{dy}{dx} = \frac{2 \cos 2t}{-6 \cos t \sin t} = \frac{2 \cos 2t}{-3 \sin 2t} = -\frac{2}{3} \cot 2t$  M1 A1
- (b)  $-\frac{2}{3} \cot 2t = 0 \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow t = \frac{\pi}{4}, \frac{3\pi}{4}$  M1 A1  
 $\therefore (\frac{3}{2}, 1), (\frac{3}{2}, -1)$  A1
- (c)  $t = \frac{\pi}{6}, \quad x = \frac{9}{4}, \quad y = \frac{\sqrt{3}}{2}, \quad \text{grad} = -\frac{2}{3\sqrt{3}}$  B1  
 $\therefore y - \frac{\sqrt{3}}{2} = -\frac{2}{3\sqrt{3}}(x - \frac{9}{4})$  M1  
 $6\sqrt{3}y - 9 = -4x + 9$   
 $2x + 3\sqrt{3}y = 9$  A1
- (d)  $y^2 = \sin^2 2t = 4 \sin^2 t \cos^2 t = 4(1 - \cos^2 t) \cos^2 t$  M2  
 $\cos^2 t = \frac{x}{3} \quad \therefore y^2 = 4(1 - \frac{x}{3}) \frac{x}{3}, \quad y^2 = \frac{4}{9} x(3 - x)$  M1 A1 (14)

Total (75)

## Performance Record – C4 Paper D

[illegible]