

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C3

Paper D

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.



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1. The function  $f$  is defined by

$$f(x) \equiv 2 + \ln(3x - 2), \quad x \in \mathbb{R}, \quad x > \frac{2}{3}.$$

(a) Find the exact value of  $ff(1)$ . (2)

(b) Find an expression for  $f^{-1}(x)$ . (3)

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2. Find, to 2 decimal places, the solutions of the equation

$$3 \cot^2 x - 4 \operatorname{cosec} x + \operatorname{cosec}^2 x = 0$$

in the interval  $0 \leq x \leq 2\pi$ . (6)

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3. (a) Given that  $y = \ln x$ , find expressions in terms of  $y$  for

(i)  $\log_2 x$ ,

(ii)  $\ln \frac{x^2}{e}$ . (4)

- (b) Hence, or otherwise, solve the equation

$$\log_2 x = 4 - \ln \frac{x^2}{e},$$

giving your answer to 2 decimal places. (4)

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4. (a) Use the identities for  $(\sin A + \sin B)$  and  $(\cos A + \cos B)$  to prove that

$$\frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y} \equiv \tan(x + y). \quad (4)$$

- (b) Hence, show that

$$\tan 52.5^\circ = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2. \quad (5)$$

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5. 
$$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2-5x-3}, \quad x \in \mathbb{R}, \quad x < -1.$$

(a) Show that

$$f(x) = \frac{4x-1}{2x+1}. \quad (5)$$

(b) Find an equation for the tangent to the curve  $y = f(x)$  at the point where  $x = -2$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (5)

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6. A curve has the equation  $y = e^{3x} \cos 2x$ .

(a) Find  $\frac{dy}{dx}$ . (2)

(b) Show that  $\frac{d^2y}{dx^2} = e^{3x}(5 \cos 2x - 12 \sin 2x)$ . (3)

The curve has a stationary point in the interval  $[0, 1]$ .

(c) Find the  $x$ -coordinate of the stationary point to 3 significant figures. (4)

(d) Determine whether the stationary point is a maximum or minimum point and justify your answer. (2)

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7. (a) Sketch on the same diagram the graphs of  $y = 4a^2 - x^2$  and  $y = |2x - a|$ , where  $a$  is a positive constant. Show, in terms of  $a$ , the coordinates of any points where each graph meets the coordinate axes. (6)

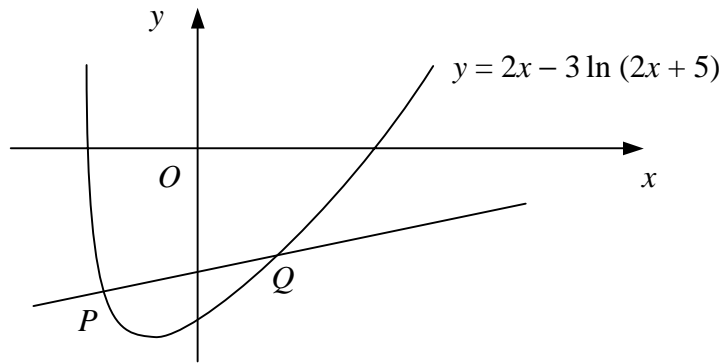
(b) Find the exact solutions of the equation

$$4 - x^2 = |2x - 1|. \quad (6)$$


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*Turn over*

8.



**Figure 1**

Figure 1 shows the curve with equation  $y = 2x - 3 \ln(2x + 5)$  and the normal to the curve at the point  $P(-2, -4)$ .

- (a) Find an equation for the normal to the curve at  $P$ . (4)

The normal to the curve at  $P$  intersects the curve again at the point  $Q$  with  $x$ -coordinate  $q$ .

- (b) Show that  $1 < q < 2$ . (3)

- (c) Show that  $q$  is a solution of the equation

$$x = \frac{12}{7} \ln(2x + 5) - 2. \quad (2)$$

- (d) Use the iterative formula

$$x_{n+1} = \frac{12}{7} \ln(2x_n + 5) - 2,$$

with  $x_0 = 1.5$ , to find the value of  $q$  to 3 significant figures and justify the accuracy of your answer. (5)

**END**