FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong

C3 Paper E – Marking Guide

1.

$= \frac{x^2(2x+1)}{(x+2)(x-2)} \times \frac{x-2}{(2x+1)(x-3)}$ M1 A2

$= \frac{x^2}{(x+2)(x-3)}$ M1 A1 **(5)**

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2. *(a)* $\text{LHS} \equiv 2\sin x \cos x - \frac{\sin x}{\cos x}$ M1

$\equiv \frac{2\sin x\cos^2 x - \sin x}{\cos x}$ M1 A1

$\equiv \frac{\sin x(2\cos^2 x - 1)}{\cos x} \equiv \frac{\sin x}{\cos x} \times \cos 2x \equiv \tan x \cos 2x \equiv \text{RHS}$ M1 A1

(b) $\tan x \cos 2x = 2\cos 2x$

$\cos 2x (\tan x - 2) = 0$ M1

$\cos 2x = 0$ or $\tan x = 2$ A1

$2x = 90, 270$ or $x = 63.4$ B1

$x = 45°, 63.4°$ (1dp), $135°$ M1 A1 **(10)**

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3. *(a)* $f(1) = 2.30$, $f(1.5) = -18.5$ M1

sign change, $f(x)$ continuous ∴ root A1

(b) $x^2 + 5x - 2\sec x = 0 \Rightarrow x^2 + 5x = \frac{2}{\cos x}$ M1

$\cos x = \frac{2}{x^2+5x}$ M1

$x = \arccos \frac{2}{x^2+5x}$ ∴ $g(x) = \frac{2}{x^2+5x}$ A1

$x_1 = 1.3119$, $x_2 = 1.3269$, $x_3 = 1.3302$, $x_4 = 1.3310 = 1.331$ (3dp) M1 A2

(c) $f'(x) = 2x + 5 - 2\sec x \tan x$ M1

SP: $2x + 5 - 2\sec x \tan x = 0$

$f'(1.05345) = 0.00046$, $f'(1.05355) = -0.0022$ M1

sign change, $f'(x)$ continuous ∴ root ∴ x-coord of $P = 1.0535$ (5sf) A1 **(11)**

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4. *(a)* *(i)* $= \frac{1}{2}(1-\cos x)^{-\frac{1}{2}} \times \sin x = \frac{\sin x}{2\sqrt{1-\cos x}}$ M1 A2

(ii) $= 3x^2 \times \ln x + x^3 \times \frac{1}{x} = x^2(3\ln x + 1)$ M1 A2

(b) $\frac{dx}{dy} = \frac{1\times(3-2y)-(y+1)\times(-2)}{(3-2y)^2} = \frac{5}{(3-2y)^2}$ M1 A2

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{5}(3-2y)^2$ M1 A1 **(11)**

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5. *(a)* $\sqrt{3}\sin\theta + \cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$ ∴ $R = \sqrt{3+1} = 2$ M1 A1

$\tan\alpha = \frac{1}{\sqrt{3}}$, $\alpha = \frac{\pi}{6}$ ∴ $\sqrt{3}\sin\theta + \cos\theta = 2\sin(\theta + \frac{\pi}{6})$ M1 A1

(b) maximum = 2 B1

occurs when $\theta + \frac{\pi}{6} = \frac{\pi}{2}$, $\theta = \frac{\pi}{3}$ M1 A1

(c) $2\sin(\theta + \frac{\pi}{6}) + \sqrt{3} = 0$, $\sin(\theta + \frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$ M1

$\theta + \frac{\pi}{6} = -\frac{\pi}{3}, -\pi + \frac{\pi}{3} = -\frac{\pi}{3}, -\frac{2\pi}{3}$ B1 M1

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{2}$ A2 **(12)**

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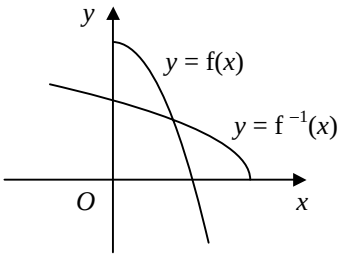
6. *(a)* $f(x) \leq 3$ B1

(b)



B3

(c) $y = 3 - x^2$

$x^2 = 3 - y$ M1

$x = \pm\sqrt{3-y}$

$f^{-1}(x) = \sqrt{3-x}\ ,\ x \in \mathbb{R},\ x \leq 3$ M1 A2

(d) $= f(\frac{4}{3}) = \frac{11}{9}$ M1 A1

(e) $\sqrt{3-x} = \frac{8}{3-x}$

$3 - x = \frac{64}{(3-x)^2}$ M1

$(3-x)^3 = 64$

$3 - x = 4$ M1

$x = -1$ A1 **(13)**

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7. *(a)* $t = 10, T = 18 \Rightarrow 18 = 5 + Ae^{-10k}$ (1)

$t = 60, T = 12 \Rightarrow 12 = 5 + Ae^{-60k}$ (2) M1

(1) $\Rightarrow A = \frac{13}{e^{-10k}} = 13e^{10k}$ M1

sub (2) $\Rightarrow 7 = 13e^{10k} \times e^{-60k}$

$e^{-50k} = \frac{7}{13}$ A1

$\therefore\ k = -\frac{1}{50}\ln\frac{7}{13} = 0.0124$ (3sf) M1 A1

$\therefore\ A = 13e^{10 \times 0.01238} = 14.7$ (3sf) A1

(b) $T = 5 + 14.71e^{-0.01238t}$

$\frac{dT}{dt} = -0.01238 \times 14.71\ e^{-0.01238t} = -0.1822e^{-0.01238t}$ M1 A1

when $t = 20$, $\frac{dT}{dt} = -0.1822e^{-0.01238 \times 20} = -0.142$ M1

$\therefore$ temperature decreasing at rate of 0.142 °C per minute (3sf) A1

(c) $T = 5 + 14.71e^{-0.01238(t-60)}$ M1

$= 5 + 14.71e^{0.7428 - 0.01238t}$

$= 5 + 14.71e^{0.7428} \times e^{-0.01238t}$ M1

$= 5 + 30.9e^{-0.01238t}$, $B = 30.9$ (3sf) A1 **(13)**

Total **(75)**

Performance Record – C3 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	rational expressions	trigonometry	numerical methods, differentiation	differentiation	trigonometry	functions	exponentials and logarithms, differentiation	
Marks	5	10	11	11	12	13	13	75
Student								