FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Express

$$\frac{2x^3 + x^2}{x^2 - 4} \times \frac{x - 2}{2x^2 - 5x - 3}$$

as a single fraction in its simplest form. **(5)**

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2. *(a)* Prove that, for $\cos x \neq 0$,

$$\sin 2x - \tan x \equiv \tan x \cos 2x.$$ **(5)**

(b) Hence, or otherwise, solve the equation

$$\sin 2x - \tan x = 2 \cos 2x,$$

for x in the interval $0 \leq x \leq 180°$. **(5)**

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3. $$\mathrm{f}(x) = x^2 + 5x - 2 \sec x, \quad x \in \mathbb{R}, \quad -\tfrac{\pi}{2} < x < \tfrac{\pi}{2}.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root in the interval $[1, 1.5]$. **(2)**

A more accurate estimate of this root is to be found using iterations of the form

$$x_{n+1} = \arccos \mathrm{g}(x_n).$$

(b) Find a suitable form for $\mathrm{g}(x)$ and use this formula with $x_0 = 1.25$ to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 3 decimal places. **(6)**

The curve $y = \mathrm{f}(x)$ has a stationary point at P.

(c) Show that the x-coordinate of P is 1.0535 correct to 5 significant figures. **(3)**

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4. *(a)* Differentiate each of the following with respect to x and simplify your answers.

(i) $\sqrt{1 - \cos x}$

(ii) $x^3 \ln x$ **(6)**

(b) Given that

$$x = \frac{y + 1}{3 - 2y},$$

find and simplify an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. **(5)**

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5. *(a)* Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) State the maximum value of $\sqrt{3}\sin\theta + \cos\theta$ and the smallest positive value of θ for which this maximum value occurs. **(3)**

(c) Solve the equation

$$\sqrt{3}\sin\theta + \cos\theta + \sqrt{3} = 0,$$

for θ in the interval $-\pi \leq \theta \leq \pi$, giving your answers in terms of π. **(5)**

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6. The function f is defined by

$$f(x) \equiv 3 - x^2, \quad x \in \mathbb{R}, \quad x \geq 0.$$

(a) State the range of f. **(1)**

(b) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **(3)**

(c) Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$g(x) \equiv \frac{8}{3-x}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

(d) Evaluate fg(–3). **(2)**

(e) Solve the equation

$$f^{-1}(x) = g(x).$$ **(3)**

Turn over

7.

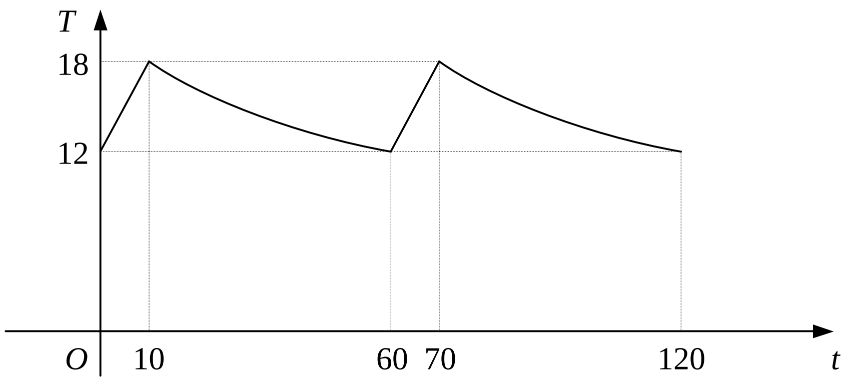

Figure 1

Figure 1 shows a graph of the temperature of a room, T °C, at time t minutes.

The temperature is controlled by a thermostat such that when the temperature falls to 12°C, a heater is turned on until the temperature reaches 18°C. The room then cools until the temperature again falls to 12°C.

For t in the interval $10 \leq t \leq 60$, T is given by

$$T = 5 + Ae^{-kt},$$

where A and k are constants.

Given that $T = 18$ when $t = 10$ and that $T = 12$ when $t = 60$,

(a) show that $k = 0.0124$ to 3 significant figures and find the value of A, **(6)**

(b) find the rate at which the temperature of the room is decreasing when $t = 20$. **(4)**

The temperature again reaches 18°C when $t = 70$ and the graph for $70 \leq t \leq 120$ is a translation of the graph for $10 \leq t \leq 60$.

(c) Find the value of the constant B such that for $70 \leq t \leq 120$

$$T = 5 + Be^{-kt}.$$ **(3)**

END